The Random Walk Model Kalman Filter in Multichannel Active Noise Control

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***Abstract*—In order to adapt the Kalman filter (KF), the RLS algorithm or other, to single channel active noise control (ANC) the MFx structure is used. But, this is more complex in the multichannel case. The RLS and others have been adapted to multi-channel ANC. However, there has been no adaptation of the KF based on a random walk state space model. This is similar to the RLS but has some advantages. The RLS algorithm tries to invert the autocorrelation matrix of the reference signal, and can become unstable if this is singular. The same is not true for random walk based KF. In the paper we present an adaptation of the KF to multi-channel ANC.**

***Index Terms*—Active Noise Control, Kalman Filter, Multichannel, RLS, Random Walk.**

I. Introduction

In active noise control (ANC) [1]–[9] a sound wave of opposite phase to the noise wave is generated (anti-noise) that interferes destructively with the noise, reducing its level.

A typical feedforward ANC system is represented in Fig. 1. In this there are the reference microphones that measures the reference signals, $u_i(n)$, cancelation loudspeakers that generates the anti-noise wave, and error microphones that measures the error signals, $e_k(n)$. The cancelation loudspeakers are fed by the anti-noise signals, $y_j(n)$. The error microphones capture the primary noise. The reference signals and the error signals are processed by the ANC system to generate the anti-noise signals. There are I reference signals, J anti-noise signals and K error signals. The paths from reference signals to the error signals are called the primary paths, $P_{i,k}$, and the paths from anti-noise signals to the error signals are called the secondary paths, $S_{j,k}$.

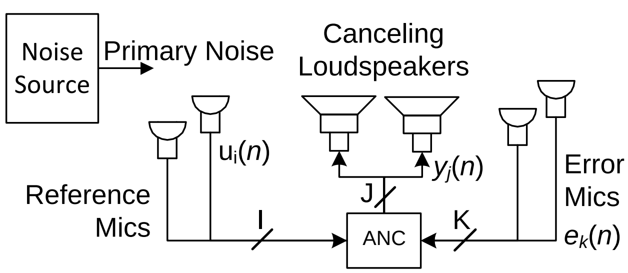

Fig. 1. Multichannel feedforward ANC system.

The LMS algorithm is known slow convergence problems [10], namely high sensitivity to eigenvalue spread. This is even more important in multichannel system, where the multiple references signals can be highly correlated. In order to overcome these problems the RLS algorithm or others are used. However, the use of the RLS algorithm in ANC is not straightforward, it requires the use of the MFx structure [11]. The case of multichannel system is still more demanding because, for instance, the number of filtered reference signals of the MFx is usually not equal to the number of reference signals. Each input sample in RLS algorithm will become a matrix, $\mathbf{V}(n)$, as proposed in [11, (20)]. However, the RLS algorithm tries to invert the autocorrelation matrix of the reference signal, and this can cause problems when this matrix is singular or close to singular, for instance in the case of narrowband signals. One possible solution to this is to add a small amount of white noise at the inputs of the RLS algorithm. This will cause a small bias in the solution but will solve the numerical problems. Another solution is to use an implementation of the Kalman Filter, as proposed in this paper. Note that it is well known that the RLS is an implementation of the Kalman Filter with carefully chosen state space model [12], but different state space models can be used for application of the Kalman Filter in adaptive filtering, for instance a random walk state space model. This will result in a different algorithm that has better properties in the case of close to singular autocorrelation matrices. Also, the application of the Kalman Filter to multichannel ANC is conceptually simpler than the direct application of the RLS.

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II. State of the Art

There are a number of papers on fast converging algorithms for ANC, but most of them are based on the RLS and so require the adding of white noise at the reference signals in order to solve the problem of inverting close to singular reference signal autocorrelation matrices. Previous work on the use of the random walk Kalman filter model for single channel ANC is in [13]. The RLS algorithm for multichannel ANC is presented in [11].

Modified implementations of the Kalman filter in ANC that incorporates the state of the secondary path in the state space model, but that are still implementations of the RLS, for the single and multichannel cases are presented in [14] and in [15] respectively. Fast array implementations are also presented, but have numerical problems. A fast array version of the algorithms proposed in this paper is not known.

Fast implementations of RLS in Active noise control are presented in [11], [16].

There is also work on the use of adaptive lattices in ANC. In [17] and [18], it was proposed the use of adaptive lattices based on the Gradient Adaptive Lattice (GAL) algorithm and the

TABLE I
THE KALMAN FILTER.

$$
\begin{aligned}
\boldsymbol{\alpha}(n) &= \mathbf{z}(n) - \mathbf{H}(n)\bar{\mathbf{x}}_{|n-1}(n) && (3)\\
\mathbf{K}(n) &= \mathbf{\Sigma}_{x|n-1}(n)\mathbf{H}^H(n) && (4)\\
&\quad (\mathbf{H}(n)\mathbf{\Sigma}_{x|n-1}(n)\mathbf{H}^H(n) + \mathbf{Q}_{vv}(n))^{-1} &&\\
\bar{\mathbf{x}}_{|n}(n) &= \bar{\mathbf{x}}_{|n-1}(n) + \mathbf{K}(n)\boldsymbol{\alpha}(n) && (5)\\
\mathbf{\Sigma}_{x|n}(n) &= \mathbf{\Sigma}_{x|n-1}(n) - \mathbf{K}(n)\mathbf{H}(n)\mathbf{\Sigma}_{x|n-1}(n) && (6)\\
\bar{\mathbf{x}}_{|n}(n+1) &= \mathbf{F}(n)\bar{\mathbf{x}}_{|n}(n) && (7)\\
\mathbf{\Sigma}_{x|n}(n+1) &= \mathbf{F}(n)\mathbf{\Sigma}_{x|n}(n)\mathbf{F}^H(n) + \mathbf{Q}_{nn}(n), && (8)
\end{aligned}
$$

DCFx structure for single channel ANC. Chen [19] proposed and adaptation of the GAL algorithm to multichannel active control, but it only supported one reference signal, or multiple references combined into a single one. These papers were based on the GAL algorithm, other use a Least Squares Lattice (LSL) [10]. Yu and Bouchard [20] proposed the use of QR decomposition LSL, in a full multichannel system using the MFx structure. This has better numerical properties than LSL algorithm, but a higher computational complexity.

There is also research in the use of the AP algorithm in ANC for instance in [21]–[23] but they give lower performance than RLS implementation based approaches.

III. The Kalman Filter

The Kalman filter [10], [24] estimates the state, $\mathbf{x}(n)$, of a liner system given a set of measurements, $\mathbf{z}(n)$. The equations for the system are,

$$
\begin{aligned}
\mathbf{x}(n+1) &= \mathbf{F}(n)\mathbf{x}(n) + \mathbf{n}(n) && (1)\\
\mathbf{z}(n) &= \mathbf{H}(n)\mathbf{x}(n) + \mathbf{v}(n), && (2)
\end{aligned}
$$

where $\mathbf{F}(n)$ is the state transition matrix, $\mathbf{n}(n)$ is the state noise, $\mathbf{H}(n)$ is the measurement matrix, and $\mathbf{v}(n)$ is the measurement noise. The measurement noise is uncorrelated to the state noise, $\mathrm{E}[\mathbf{n}(n)\mathbf{n}^{\mathrm{T}}(n)] = \mathbf{Q}_{nn}(n)$ and $\mathrm{E}[\mathbf{v}(n)\mathbf{v}^{\mathrm{T}}(n)] = \mathbf{Q}_{vv}(n)$. The solution to the estimation problem is given by the set of equations in Table I, where $\alpha(n)$ is the innovations term, $\mathbf{K}(n)$ is the Kalman gain, $\bar{\mathbf{x}}_{|n}(n)$ is the current state estimate given measurements up to time n, and $\mathbf{\Sigma}_{x|n}(n)$ is the current value of the covariance matrix of state estimate given measurements up to time n.

IV. The Proposed Algorithm

The proposed algorithm is based on the Kalman filter, with a random walk space state model. This is different to the model used to derive the RLS algorithm [12]. The random walk space state model consists simply in making $\mathbf{F}$ identity in (1). The state is then constant apart from a random walk due to state noise. As usual in adaptive filtering, the state will consist in the coefficient of the filters to be determined, w_m. Let's consider a multi-channel ANC system with I reference sensors, J anti-noise sources, and K error sensors. Also let N_P, N_S and N_W be the primary, secondary and controller filter length. The equations for the multi-channel ANC system are,

TABLE II
STATE SPACE MODEL FOR MFXMK

$\mathbf{x}(n)$	$W_{(j,i,m)}$
$\mathbf{F}(n)$	$\mathbf{I}$
$\mathbf{Q}_{nn}$	$q_{nn}\mathbf{I}$
$\mathbf{z}(n)$	d_k^2
$\mathbf{H}(n)$	$u'_{k,(j,i,m)}$
$\mathbf{Q}_{vv}$	$q_{vv}\mathbf{I}$

$$y_j(n) = -\sum_{i=0,m=0}^{I-1,N_W-1} W_{j,i,m}\, u_i(n-m) \qquad (9)$$

$$\hat{d}_k(n) = -\sum_{j=0,l=0}^{J-1,N_S-1} S_{k,j,l}\, y_j(n-l) \qquad (10)$$

$$d_k(n) = \sum_{i=0,l=0}^{J-1,N_P-1} P_{k,i,l}\, u_i(n-l) + r_j(n) \qquad (11)$$

$$e_k(n) = d_k(n) - \hat{d}_k(n) \qquad (12)$$

where i, j and k are the indexes for the reference sensor, anti-noise source and error sensor respectively, W, is the controller filter that generates the anti-noise, S, is the secondary path and, P, is the primary path.

Assuming that W and S are constant or slowly varying, their order can be exchanged resulting in,

$$\hat{d}_k(n) = \sum_{j=0,i=0,m=0}^{J-1,I-1,N_W-1} W_{j,i,m}\, u'_{k,j,i}(n-m) \qquad (13)$$

$$u'_{k,j,i}(n) = \sum_{l=0}^{N_S-1} S_{k,j,l}\, u_i(n-l) \qquad (14)$$

that can be checked by combining (9) and (10).

Ideally, one should have $d_k(n) = \hat{d}_k(n)$ apart from any measurement noise, in order to minimize $e_k(n)$. One can use the modified Fx structure (MFx) [25] [26] [27] [11] to estimate the desired signal, $d_k(n)$. Namely, the estimate, $\tilde{d}_k(n)$, can be obtained using,

$$\tilde{d}_k(n) = e(n) - \sum_{j=0,l=0}^{J-1,N_S-1} \hat{S}_{k,j,l}\, y_j(n-l) \qquad (15)$$

where $\hat{S}_{k,j,l}$ is an estimate of the secondary path. So (13) corresponds to the measurement equation, (2), resulting in the state space model presented in table II. In table II, $u'_{k,j,i,m} = u'_{k,j,i}(n-m)$, $\mathbf{I}$ is the identity matrix and the parentis in the indexes represents joining indexes to form a single one. For instance (j,i,m) converts to $i_2 = j + i \times J + m \times J \times I$. So, $W_{(j,i,m)}$ is a vector and $u'_{k,(j,i,m)}$ is a matrix. This can be implemented by a reshape of the multi-dimensional array in some software packages.

The proposed algorithm is obtained by using MFx structure, ((14) and (15)), and the Kalman filter, ((3) to (8)), with the

TABLE III
STATE SPACE MODEL FOR MFXMK2

$\mathbf{F}(n)$	$\lambda\mathbf{I}$
$\mathbf{Q}_{nn}$	$\sigma^2(1-\lambda^2)\mathbf{I}$

identities presented in table II. We call this algorithm MFx Multichannel Kalman (MFxMK).

This algorithm is less sensitive to singular or close to singular autocorrelation matrices than the RLS and its' derivatives. However it propagates the covariance matrix of the state estimate, $\mathbf{\Sigma}_{x|n}(n)$. If there is no information about a part of the state, $\mathbf{\Sigma}_{x|n}(n)$ will grow indefinitely with the state noise, each time $\mathbf{Q}_{nn}$ is added in (8). However, this increase is linear, and not exponential as in the RLS algorithm. This is easily seen when the input signal is zero. In this case in (8) $\mathbf{Q}_{nn}(n)$ will be repeatedly added to $\mathbf{\Sigma}_{x|n}(n)$ while (6) keeps it constant, because $\mathbf{H}(n)$ is zero. So $\mathbf{\Sigma}_{x|n}(n)$ will grow without bound but linearly. In the RLS there is a similar effect but in this case $\mathbf{P}$ will be repeatedly multiplied by $1/\lambda$, so $\mathbf{P}$ will grow exponentially. The proposed algorithm may take a very long time to become unstable, and this could not be reached in our simulations results.

In order to solve the possible, very long time instability problem a slightly modified algorithm is proposed. This is called MFxMK2 and it is equal to MFxMK with the differences presented in table III. The goal it to stop the state to drift indefinitely as can happen in the simple random walk model. Instead it is assumed that the state changes are dictated by first order process, with a forgetting factor given by, λ, and that the steady state variance of the state is equal to σ, namely, $x_i(n+1) = \lambda x_i(n) + n(n)$, with $\mathrm{E}[x_i^2(\infty)] = \sigma^2$, resulting in $q_{nn} = \sigma^2(1-\lambda^2)$.

V. COMPUTATIONAL COMPLEXITY

The computational complexity of the proposed algorithm (MFxMK) is,

$$3I^2J^2KN_W^2 + 2IJK^2N_W + IJKN_S + 2IJKN_W + JKN_S + \mathrm{INV}(K\times K) \quad (16)$$

multiplications, where $\mathrm{INV}(K\times K)$ stands for the number of multiplications for a $K\times K$ matrix inversion. The expression is dominated by the first term in the sum. Note that the term $\mathbf{H}(n)\mathbf{\Sigma}_{x|n-1}(n)$ can be reused in the equations for the Kalman filter. Also note that in the case of no numerical errors $(\mathbf{H}(n)\mathbf{\Sigma}_{x|n-1}(n))^H = \mathbf{\Sigma}_{x|n-1}(n)\mathbf{H}^H(n)$, because $\mathbf{\Sigma}_{x|n-1}(n)$ is hermitian, but that this was not assumed in our implementation to make it more robust to these errors [10].

The computational complexity of the modified filtered-x RLS algorithm [11] is exactly the same if we don't count the multiplications by λ.

The computational complexity of the modified filtered-x FTF algorithm [11] is,

$$N_W\left(IJK^2 + 2IJK + IJ\right) + (N_W-1)7I^2J^2K + N_S(IJK+JK) + 2I^2J^2K + I^2J^2 + K + \mathrm{INV}(IJ\times IJ) + \mathrm{INV}(K\times K). \quad (17)$$

TABLE IV
NUMBER OF MULTIPLICATIONS PER SAMPLE FOR A SYSTEM WITH $I = K = 2$, $J = 3$, $N_W = N_S = 32$.

Proposed	224064 + INV(2, 2)
RLS	224064 + INV(2, 2)
FTF	18110 + INV(2, 2) + INV(6, 6)
LMS	1538

We present this as an example of the computational complexity of a fast implementation of the RLS, although this algorithm has numerical problems.

Finally, the computational complexity of the multichannel modified filtered-x LMS algorithm is,

$$N_W(2IJK + IJ) + N_S(IJK + JK) + K \quad (18)$$

As an example the number of multiplications for a system with $I = K = 2$, $J = 3$, $N_W = N_S = 32$ are presented in table IV for the referred algorithms. It can be seen that the extra performance of the RLS and the proposed algorithm is only achieved with a great increase in computational complexity.

VI. SIMULATION RESULTS

In this section simulation results comparing the proposed algorithm and other algorithms in the literature are presented.

In Fig. 2 the following algorithm are compared: the proposed algorithm (MFxMK); the adaptation of the RLS to multichannel ANC using the MFx structure in [11] (RLS); and the algorithm proposed in [15] that incorporates the state of the secondary path in the state of the system (Ophem). Also shown is the noise level with ANC off (Off). Fast, lattice implementation or affine projections algorithms, are not compared because they are just efficient implementation of the RLS and suffer from similar problems as the RLS, when trying to invert the autocorrelation matrix. Also, there are no known fast implementations of the proposed algorithm.

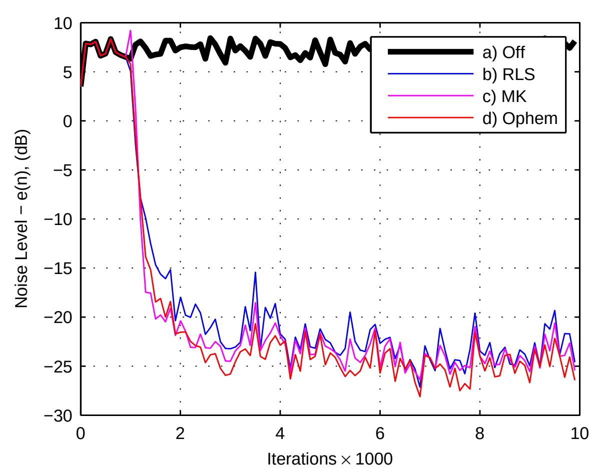

Fig. 2. Comparing the proposed (MFxMK), RLS and Ophem algorithms for broadband reference signals.

The system simulated had 2 independent reference signals, 3 anti-noise sources and 2 error sensors. The reference signals were obtained by filtering white Gaussian noise through a FIR

TABLE V
PARAMETERS USED BY THE ALGORITHMS FOR BROADBAND SIGNALS

MFxMK	$q_{vv} = 0.1$, $\lambda = 0.9999$, $\pi_0 = 0.1$
RLS	$\lambda = 0.9999$, $\pi_0 = 0.01$
Ophem	$\lambda = 0.9999$, $q_{vv} = q_{nn} = 0.1$, $\pi_0 = 1$

TABLE VI
PARAMETERS USED BY THE ALGORITHMS FOR NARROWBAND SIGNALS

MFxMK	$q_{vv} = 10^{-10}$, $\lambda = 0.9$, $\pi_0 = 1$
RLS	$\lambda = 0.9$, , $\pi_0 = 0.01$
Ophem	$\lambda = 0.9$, $q_{vv} = q_{nn} = 0.1$, $\pi_0 = 1$

filter with two levels of 1 and 0.3, in order to obtain colored noise. The secondary paths were non-minimum phase system formed by 3 fractional delays implemented by sinc filters with amplitudes 1.3, -0.3 and 0.4. The fractional delays values were random numbers selected form the range from 10 to 20. The primary paths were similar but with different values for the amplitudes and with fractional delays selected form the range from 20 to 30. This allowed the simple generation of the 6 secondary paths and 4 primary paths involved. The controller filters had length 32.

The parameters used by the algorithms are presented in table V. The value of λ was selected as a large value to make the Ophem algorithm stable. Other parameters were selected by trial and error in order to achieve fast convergence and low excess noise. The value of $\pi_0\mathbf{I}$ is the initial value for $\mathbf{\Sigma}$, in the Ophem and MFxMK algorithm and the initial value for $\mathbf{P}$ in the RLS algorithm. In order to make a fair comparison of the algorithms, the value of the state noise, q_{nn}, of the MFxMK algorithm was adjusted so that the excess mean square error was approximately the same as the RLS and Ophem. This was achieved by making $q_{nn} = (1-\lambda)^2\, q_{vv}/P_x$, where P_x is the power of the reference signal. This is an approximate expression that can be obtained for slow changes and a white reference signal.

In Fig. 2 it can be seen that for the case of broadband reference signal all algorithms converge rapidly to a near optimum solution, and that all are numerically stable.

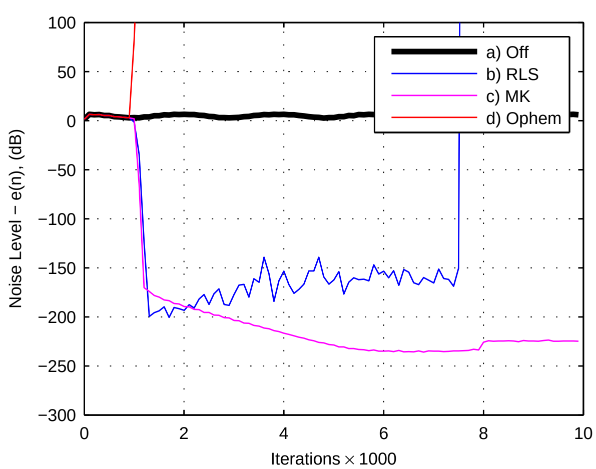

Fig. 3. Comparing the proposed (MFxMK), RLS and Ophem algorithms for narrowband reference signals.

Fig. 3 represents the same simulation but for narrowband reference signals. Each reference signal was composed of three sinusoids with amplitudes 1, 0.3 and -0.3 and with frequencies chosen randomly between 100 and 400 Hz, with a sampling frequency of 2000 Hz. The parameters used by the algorithms in this case are presented in table VI. In this case λ was select much lower in order to improve convergence and tracking and the other parameters were again selected by trial and error. Only the proposed algorithm is stable in this case. The Ophem algorithm diverges immediately after it is turned on at 1000 iterations, and the RLS algorithm diverges after 7500 iterations. The RLS algorithm can be stabilized by adding a small amount of white noise at its inputs. The Ophem algorithm on the other hand is only stable for larger values of λ.

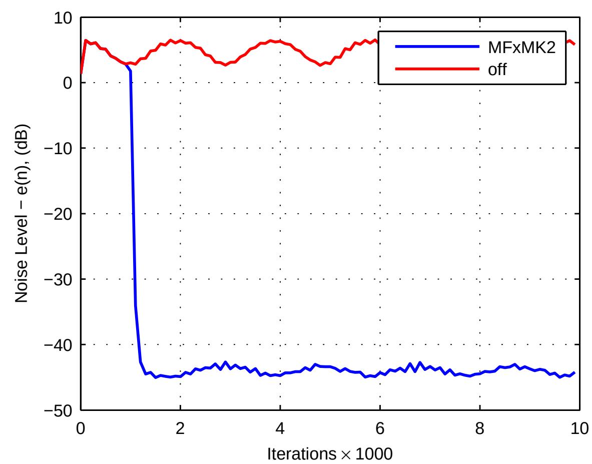

Fig. 4. Performance of the MFxMK2 algorithm for narrowband reference signals.

In Fig. 4 it is presented the convergence curve for the MFxMK2 for narrowband reference signals. The parameters of the algorithm were $q_{vv} = 0.1$ $\lambda = 0.9999$, $\pi_0 = 100$, $\sigma_x^2 = 1$. These were selected by trial and error, with a large value for λ in order to have low leakage in the coefficients of the filter. It can be seen that the algorithm is stable and fast converging, but that the noise reduction is limited to about 45 dB. This value is more than enough for most practical applications.

VII. CONCLUSION

The Kalman Filter (KF) can be used to derive the RLS algorithm using an adequate state space model. However there are other state space models that can be used with the KF in adaptive filtering, namely the random walk model, with some advantages. The resulting algorithm is less sensitive to numerical problems in inverting singular reference signal autocorrelation matrices. In the paper the KF with a random walk state space model was adapted to multi-channel ANC. Simulations confirm its better numerical properties. The technique described can also be used to adapt the RLS algorithm to multi-channel ANC in a simpler way.

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