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New PN Even Balanced Sequences for Spread-Spectrum Systems

J. A. L. Inácio
Instituto de Engenharia de Sistemas e Computadores (INESC), Rua Alves Redol 9, 1000 Lisboa Codex, Portugal
ENIDH, Avenida Bonneville Franco, Paço D'Arcos, 2780 Oeiras, Portugal
Email: jali@mail.telepac.pt

J. A. B. Gerald
Instituto de Engenharia de Sistemas e Computadores (INESC), Rua Alves Redol 9, 1000 Lisboa Codex, Portugal
Instituto Superior Técnico (IST) Universidade Técnica de Lisboa, Avenida Rovisco Pais, 1000 Lisboa, Portugal
Email: jabg@inesc.pt

M. D. Ortigueira
Instituto de Engenharia de Sistemas e Computadores (INESC) Rua Alves Redol 9, 1000 Lisboa Codex, Portugal
UNINOVA, Campus da FCT da UNL, Quinta da Torre, Monte da Caparica, 2825 - 114 Caparica, Portugal
Email: mdo@dee.fct.unl.pt

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A new class of pseudonoise even balanced (PN-EB) binary spreading sequences is derived from existing classical odd-length families of maximum-length sequences, such as those proposed by Gold, by appending or inserting one extra-zero element (chip) to the original sequences. The incentive to generate large families of PN-EB spreading sequences is motivated by analyzing the spreading effect of these sequences from a natural sampling point of view. From this analysis a new definition for PG is established, from which it becomes clear that very high processing gains (PGs) can be achieved in band-limited direct sequence spread-spectrum (DSSS) applications by using spreading sequences with zero mean, given that certain conditions regarding spectral aliasing are met. To obtain large families of even balanced (i.e., equal number of ones and zeros) sequences, two design criteria are proposed, namely the ranging criterion (RC) and the generating ranging criterion (GRC). PN-EB sequences in the polynomial range $3 \leq n \leq 6$ are derived using these criteria, and it is shown that they exhibit secondary autocorrelation and cross-correlation peaks comparable to the sequences they are derived from. The methods proposed not only facilitate the generation of large numbers of new PN-EB spreading sequences required for CDMA applications, but simultaneously offer high processing gains and good despreading characteristics in multiuser SS scenarios with band-limited noise and interference spectra. Simulation results are presented to confirm the respective claims made.

Keywords and phrases: even balanced spreading sequences, PN sequences, processing gain, direct sequence spread spectrum.

1. INTRODUCTION

In the first half of the 20th century, spread-spectrum (SS) systems were conceived in order to guarantee privacy and low probability of interception (LPI) in military communications. Later, their applications began to include most of the tactical military communications (as in, e.g., location and positioning monitoring, weapons and missile armament control, electronic warfare, etc.). However, their properties, such as resistance against intentional or inadvertent jammers and the ability to accommodate multiple users in the same frequency band, made SS systems an attractive choice for commercial communication applications too [1, 2]. One very important application of SS systems is found in power line communications (PLC), where CDMA turned out to be particularly useful by virtue of its robustness against noise and by its interference suppression features [3, 4]. Recently, wireless local-area network (WLAN) standards such as the IEEE 802.11x family exploited this modulation technique and gained unprecedented popularity in last mile wireless Internet access solutions. However, the largest commercial application of DSSS assumes undoubtedly the form of W-CDMA, which is the predominant technology used in present 3G wireless cellular systems, as embodied in the IMT-2000 (UMTS and 3GPP) standards [5]. In CDMA applications, the choice and availability of large

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families of spreading sequences with good correlation properties remains a primary design consideration.

Amongst the well-known forms of SS, such as direct sequence SS (DSSS), frequency hopping (FH), time hopping (TH), linear FM (chirp), and hybrid methods [4, 5, 6, 7, 8], this paper will focus on DSSS communication systems. In DSSS systems employing BPSK modulation, spreading is achieved by multiplying or modulating a low symbol (bit) rate binary information sequence with a pseudonoise (PN) signal generated by means of a shift register running at a considerably higher symbol or "chip" rate ($1/\tau$ chips/s) than the information bit rate (bps). The resulting frequency spread signal occupies a much wider bandwidth than the original BPSK signal. The increase in the output signal-to-noise ratio (SNR), as a result of the spectral spreading process, is known as processing gain (PG). Since the primary objective of the paper is an analysis of the spreading effect achieved by the PN spreading sequences, PSK modulation will henceforth be neglected and only a baseband DSSS system will be considered.

The outline of the paper is as follows. In Section 2, the spreading process is analyzed in detail. Unlike the classic approach that assumes the spreading signal to be a binary stochastic process (almost true for long period sequences, which is useful when secrecy is required), the interest here is mainly in achieving acceptable performance for the system from a communication theory point of view, namely, to ensure a high signal-to-noise ratio at the demodulator output. As a consequence, the spreading signal is treated as periodic and an understanding of particular features of the spreading process is investigated within the framework of the sampling theory, in an attempt to gain a better insight into the process [9]. It is found that the PG is not only a simplistic relation between the chip and bit rates, but also a function of the channel noise and interference. Consequently, in Section 3, a new definition for processing gain (PG) is formulated, and it is shown that the classical definition is a special case of the new one [9]. A close look into the spreading process, as well as the new PG definition reveals that very high PG can be obtained under certain circumstances; one of them is the use of zero-mean spreading sequences. This observation motivated a search for zero-mean spreading sequences. So, Section 4 introduces methods to generate new families of pseudonoise even balanced (PN-EB) sequences obtained from existing odd-length maximum-length sequences (so-called m-sequences) by appending or inserting an extra chip to yield an even length sequence with an equal number of "ones" and "zeros." However, this modification has to be done in such a way that the new PN-EB sequences retain the good correlation properties of the sequences they are derived from. To achieve this goal, two design criteria are introduced, namely the ranging criterion (RC) [10] and the generating ranging criterion (GRC) [11]. These criteria are demonstrated to yield large numbers of balanced sequences exhibiting low levels of secondary cross-correlation and "out-of-phase" autocorrelation peaks. Simulation results are presented in Section 5 to verify and confirm the correctness of the proposed RC and GRC criteria. Finally, concluding remarks are presented in Section 6.

2. CONSIDERING DSSS FROM A NATURAL SAMPLING PERSPECTIVE

Let $x(t)$ be a signal, not necessarily band-limited, with Fourier transform (FT), $X(\omega)$. The natural sampling of $x(t)$ comprises the multiplication of $x(t)$ by a periodic rectangular pulse sequence, $r(t)$, which is assumed to have unit amplitude, a T seconds period, and pulse width τ given by

$$r(t) = \sum_{i=-\infty}^{+\infty} p_\tau(t - iT), \tag{1}$$

where $p_\tau(t)$ is the rectangular pulse (chip). $r(t)$ can be represented by the Fourier series (FS) with coefficients

$$r_n = \frac{\tau}{T} \cdot \operatorname{sinc}\left(\frac{\tau}{T}n\right) \cdot e^{-j\pi(\tau/T)n}, \quad n \neq 0, \tag{2}$$

$r_0 = \tau/T$ and it has an FT

$$R(\omega) = 2\pi \sum_{-\infty}^{+\infty} r_n \delta\left(\omega - \frac{2\pi}{T}n\right). \tag{3}$$

It can be seen that

(i) the number of spectral lines in a given band depends only on the sampling interval, T;
(ii) r_n as a function of "n" has infinite lobes;
(iii) the number of spectral lines in the main lobe depends on the duty cycle of the pulses, expressed as τ/T.

Let $x_s(t)$ be the natural sampling of $x(t)$ obtained by the product of $x(t)$ by $r(t)$. Its spectrum is

$$X_s(\omega) = \sum_{-\infty}^{+\infty} r_n X\left(\omega - \frac{2\pi}{T}n\right). \tag{4}$$

As this sampling operation is equivalent to a uniform and weighted repetition of $X(\omega)$, $x_s(t)$ is a spread-spectrum signal. In the particular case of no aliasing, one can recover a weighted version of $x(t)$ by means of a lowpass filtering operation. In the general case, with aliasing, there is no possibility of removing the spreading effect (despreading).

Consider, now, a high-order sampling scheme in which $x(t)$ is sampled by several delayed sampling series, as shown in Figure 1.

Before the sampled signals are added, they are multiplied by coefficients a_i $(i = 0, 1, \ldots, N-1)$ that can only take on two values, that is, $a_i \in \{-1, +1\}$. If $N = T/\tau$ is the number of sampled versions of $x(t)$, the output $s_p(t)$ results in the weighted sum of N signals $s_i(t)$:

$$s_i(t) = x(t) \sum_{n=-\infty}^{+\infty} r_n \cdot e^{-j(2\pi/N)in} \cdot e^{j(2\pi/T)nt}, \quad i = 0, 1, \ldots, N-1. \tag{5}$$

As each $s_i(t)$ signal is a sampled version of $x(t)$, it is also an SS signal. Note that, during each time interval τ of a pe-

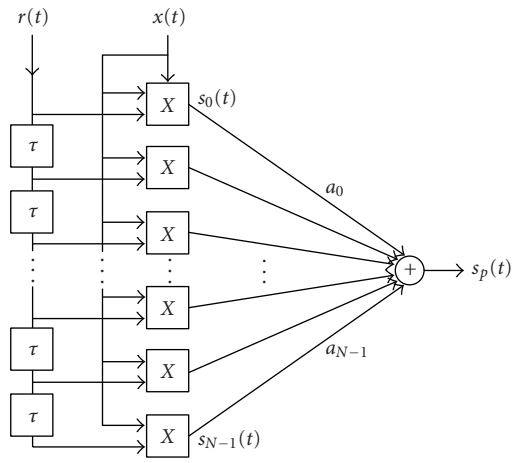


Figure 1: Construction of the SS signal from the sampled signals.

riod T, only one of the N delays has a nonzero value. In some particular combinations of weights, the spreading is destroyed (as, e.g., when all $a_i = 1$ or all $a_i = -1$ ($i = 0,\dots,N-1$), which will result in a $s_p(t)$ signal equal to $x(t)$ or $-x(t)$, resp.). However, in general, the signal $s_p(t)$, being a linear combination of SS signals, is an SS signal, too. In fact, it follows from (5) that

$$\begin{aligned} s_p(t) &= \sum_{i=0}^{N-1} a_i s_i(t) \\ &= x(t)\cdot \sum_{n=-\infty}^{\infty} r_n \cdot \left[\sum_{i=0}^{N-1} a_i e^{-j(2\pi/N)in}\right]\cdot e^{j(2\pi/T)nt} \\ &= x(t)\cdot b(t), \end{aligned} \tag{6}$$

where $b(t)$ is a periodic signal with period T and Fourier coefficients given by

$$b_n = r_n A(n), \tag{7}$$

where $A(n)/N$ is the DFT of the sequence $a_k, k = 0,\dots,N-1$. The existence, or not, of spreading is determined by $A(n)$. In the referred trivial cases where all $a_i = 1$ or all $a_i = -1$, $i = 0,\dots,N-1$, $A(n) = \pm N\delta(n) (\mathrm{mod}\, N)$, and so, the coefficients corresponding to indices $n \neq 0$ are null. If $a_{j\neq} a_i$ for at least one pair ($i \neq j$), spreading will occur. For nontrivial cases, the SS signal $s_p(t)$ can be obtained by multiplying $x(t)$ by a spreading function $b(t)$, periodic with period T. Each period of $b(t)$ is a linear combination of rectangular pulses of duration τ:

$$b(t) = \sum_{i=0}^{N-1} a_i \cdot p_\tau(t - i\tau). \tag{8}$$

The analysis of (4) and (6) shows that $S_p(\omega)$ can be thought of as composed of an infinite number of replicas of $X(\omega)$,

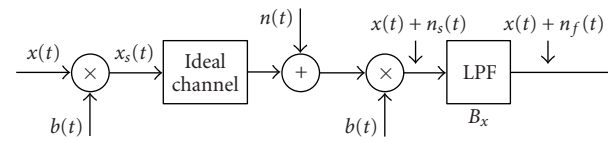


Figure 2: Ideal DS communication system.

with N replicas at each spectral line, $\omega = (2\pi n)/T$, $n = -\infty,\dots,0,\dots,+\infty$. Each replica is weighted by a coefficient

$$b_{ni} = a_i r_n \cdot e^{-j(2\pi/N)in}, \quad i = 0,\dots,N-1. \tag{9}$$

The signal $s_p(t)$ is called a direct sequence (DS) spread-spectrum signal. The despreading operation is accomplished by multiplying $s_p(t)$ by a synchronous version of the spreading sequence:

$$s_p(t)\cdot b^*(t) = x(t)\cdot \left|b(t)\right|^2 = x(t) \tag{10}$$

if $|b(t)|^2 = 1$. Thus, the despreading process results in all repetitive spectral information terms, $X(\omega)$, to be translated back to zero frequency, where they add up to reconstruct $x(t)$. For a given $r(t)$, the spreading sequence term $|A(n)|^2$ {see (7)}, which is periodic with period N, determines the smearing of the signal energy over the entire spectrum. $|A(n)|^2$ is the sampled version of the spectrum of the a_i's sequence and also the DFT of the autocorrelation, $R_a(n)$, of the a_i, $i = 0,\dots,N-1$. An interesting case is the classical SS [12]:

$$\left|A(n)\right|^2 = \begin{cases} 1 & \text{if } n = 0, \\ N+1 & \text{if } n \neq 0, \end{cases} \tag{11}$$

which corresponds to a pseudonoise sequence.

3. THE NEW INTERPRETATION AND DEFINITION OF PROCESSING GAIN

In Figure 2 an ideal DS-SS communication system is represented. The only information needed about the modulating signal is its bandwidth, which is needed to fix the bandwidth of the output lowpass filter, B_x. The system output is the sum of the information signal $x(t)$ and a noise signal $n_f(t)$, which is a filtered version of the spread noise $n_s(t)$.

The output signal power does not depend on the performed spreading, because, ideally, the original signal must be recovered. However, the output noise power depends on the spreading. In general, the output signal-to-noise ratio with, and without, spreading, will be different. So, it makes sense to define the *processing gain* (PG) as the quotient between the output signal-to-noise ratios, with and without spreading, according to [9]:

$$\mathrm{PG} = \frac{S_s/N_s}{S_0/N_0}. \tag{12}$$

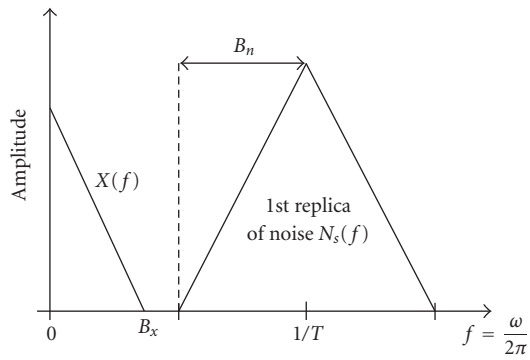


FIGURE 3: Reference band B_x and noise band B_n.

Once $S_s = S_0$, it follows that

$$\mathrm{PG} = \frac{N_0}{N_s}. \tag{13}$$

The processing gain is the most important feature used in literature to qualify the performance of an SS system. The PG is usually defined as the quotient between the spread signal and signal bandwidths. In the case of a binary signal source, this gain is the quotient between the spreading chip rate and the source bit rate [5, 6, 8]. Later it will be seen how to make this definition compatible with (12). To compute the PG, as defined in (12), it is necessary to analyze the effect of spreading of the noise. Consider first the non-band-limited noise and assume, by simplicity, that $n(t)$ has a constant spectrum, $N(\omega) = 1$; in this case, it follows that $N_f(\omega) = N(\omega)$ (note that $\sum_{-\infty}^{\infty} r_n A(n) = 1$). This result confirms the usual affirmation: "the DS-SS has no effect on the white noise" [13, 14, 15]. For a non-band-limited and nonflat spectrum, the analysis is more involved and complex. A discussion of this case is presented in a later section. However, assuming that the spectrum decreases with the frequency, it is not difficult to see that there will be an energy concentration at the lower frequencies and the PG will be near 1. For a band-limited noise, the situation is quite different. Consider any noise signal, $n(t)$, with FT $N(\omega)$ and bandwidth B_n. After spreading, the FT of the noise $n_s(t)$ is a sequence of repetitions of $N(\omega)$ located at multiple frequencies of $2\pi/T$. In the reference band, B_x, the total number of replicas decreases with T, and this decreasing stops when T is less than $1/(B_n + B_x)$. Below this limit there is only one replica of the original noise signal, located at the zero frequency, as illustrated in Figure 3.

This is the best situation. The ratio between the noise power before and after spreading, inside the baseband B_x, is given by

$$\mathrm{PG} = \frac{1}{\left[r_0 A(0)\right]^2} = \frac{1}{\left[(\tau/T)A(0)\right]^2}. \tag{14}$$

In the case of a PN spreading sequence, $|A(0)| = 1$, and with $T/\tau = N$, it results in

$$\mathrm{PG} = N^2. \tag{15}$$

If T is greater than $1/(B_n + B_x)$, there is aliasing inside the reference band which will increase the output noise power. In the limit, as T increases indefinitely, the number of spectral lines inside the band B_x increases and the spectrum becomes almost continuous. It is like a sliding, into the interval $[-B_x, B_x]$, of $2B_xT$ spectral lines and corresponding replicas of the noise with a power (see (2) and (11)) equal to $2(N+1)B_xT/N^2 \approx 2B_xT/N = 2B_x\tau$.[1] So, the PG will be

$$\mathrm{PG} = \frac{1}{2B_x\tau}. \tag{16}$$

As $1/\tau$, the chip rate, is equal to twice the spreading sequence bandwidth B_s, the PG can be written as

$$\mathrm{PG} = \frac{B_s}{B_x}, \tag{17}$$

which is the classic PG definition [16]. Recall that the best situation occurs when $T \le 1/(B_x + B_n)$, requiring a PG given by (13), which is different from the classic definition in (17).

A close look into (14) reveals a way to increase the PG. In an ideal situation, using a spreading signal with a zero-mean value, the PG will be infinite. This may seem rather strange, but not impossible to conceive. There are two ways to achieve this:

(i) using odd balanced sequences by increasing the period with one "zero" chip, in order to make the number of ones and zeros equal;

(ii) using a Manchester pulse shape instead of a rectangular one, at the price of an increase (doubling) of the required information bandwidth. For this reason, this method was not used in this paper.

This means that with a small enough spreading sequence period (usually used in narrowband applications) that leads to spectral lines far apart and a zero-mean sequence, a band without noise will appear. In the modulating case, this band will originate the lower and upper sidebands. Simulation results are presented in Figures 12, 13, 14, and 15 of Section 5.5.

4. GENERATION OF PN-EB SEQUENCES

4.1. *Even sequences*

The new definition of processing gain points out the advantage of generating families of zero-mean spreading sequences to maximize the processing gain of a DSSS system. These spreading sequences should also have good auto- and cross-correlation properties to make them suitable for CDMA applications. Thus, it is required that the proposed family of spreading sequences have very low secondary correlation peaks.

[1] The zeroth term will be negligible (see (11)).

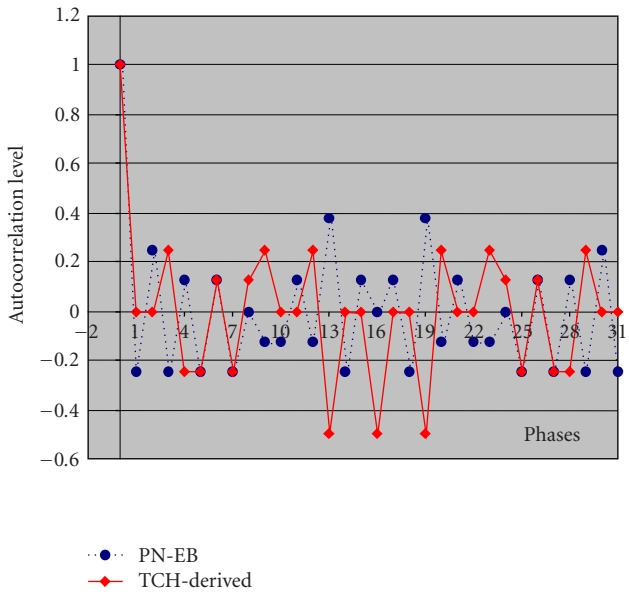


FIGURE 4: Autocorrelation level of TCH-derived and PN-EB sequences.

In a first step, the pseudonoise even balanced (PN-EB) sequences were developed from the Gold sequences [17], by appending one extra-zero to each original Gold sequence. The reason for this choice of the "extra-zero" insertion process lies in its easy implementation. We will call them PN-EB original sequences. As known [18, 19, 20], the Gold sequences are generated from pairs of m-sequences. It is possible, for each pair of nth-order m-sequences, to generate 2^n+1 Gold sequences. In order to present the PN-EB sequences, Gold sequences of length 31 (polynomial generator with degree $n = 5$) were used. In this case there are 396 balanced sequences [18, 19, 20]. According to their properties, they present three levels of autocorrelation [21, 22, 23, 24].

Another viable alternative is to use the TCH-derived sequences presented in [25], which also are m-sequences with length 2^n and assume low correlation levels. Figure 4 shows identical autocorrelation peak values for TCH-derived and PN-EB original sequences.

A comparative analysis of the highest secondary peak level of the autocorrelation function between these two types of even sequences shows that both the processes present similar levels (see Table 1).

In Table 1, the number of sequences by levels of the highest secondary peak of the autocorrelation of the known TCH-derived and PN-EB original sequences are presented.

However, when doing a comparison between TCH-derived and PN-EB sequences (with the same period), the TCH-derived sequences present the disadvantage of existing in a smaller number (for identical values of the autocorrelation function). The number of available sequences is very important when we want to accommodate a great number of individual users [4, 18].

In a second step we studied the advantage that results from adding the extra-zero to the longest run of "0's" [26] in the worst autocorrelation cases (absolute levels of normalized correlation ≥ 0.5). This is the Rees criterion [27, 28] (note that this criterion was only applied by Rees in the m-sequences case [27]). This procedure led to the results expressed in Table 2, assigned as "PN-EB Rees." We applied the Rees criterion to the 83 worst cases of secondary autocorrelation peaks of the PN-EB original sequences, indicated in Table 1 (values ≥ 0.5). This procedure led to a significant reduction in the number of worst cases. Although this new set of PN-EB sequences presents a higher number of sequences with lower correlation levels, 21 sequences still remain without improvement.

A study of the Rees criterion [29] leads to the conclusion that when applied to any m-sequence generated by a primitive polynomial, it produces a minor change in the secondary autocorrelation levels.

In the case of Gold sequences, it produces an improvement on the secondary correlation levels in 62 sequences.

With the main goal of improving the 21 still remaining cases, the two following methods were formulated, as solutions for the extra-zero positioning problem: (i) to optimize the positioning process through the analysis of the autocorrelation function; (ii) to apply the Rees criterion, not to the Gold sequences, but to one of the two m-sequences generators, in this case, the primary sequence[2] [30, 31] (the one that is characterized by its characteristic phase [32]).

This led to the formulation of two different approaches to generate the PN-EB sequences as described next.

4.2. *The ranging criterion*

In the ranging criterion (RC), the extra-zero positioning process is sequential and it is described by the following sequential steps:

(1) the extra-zero will be inserted at the end of the sequence period;
(2) in all those sequences that still present a high level of correlation (e.g., > 0.5), the extra-zero will be inserted in the bigger run of 0's;
(3) for those sequences whose improvement was not reached with the steps (1) and (2), the extra-zero is placed at the *run* of 0's, nearest to the $[(N+1)/2+1]$th symbol position;
(4) for sequences whose improvement was not obtained with the steps (2) and (3), the extra-zero should be placed at last *run* of 0's.

The first step results from the most simple and immediate process. However, from the derived sequences, some of them present an undesirable increase of the autocorrelation secondary peak values regarding the original Gold sequences. For these, step (2) (the Rees criterion [27]) is applied for its efficiency and simplicity (the resulting sequences are named PN-EB Rees sequences). Nevertheless, this process may not

[2] This concept of "primary sequence generator" and "secondary sequence generator" was introduced to allow discriminating the "*characteristic* m-sequence" generator (see [30]) from the secondary m-sequence generator "out-of-the-phase."

Table 1: Comparison between known TCH-derived and PN-EB original sequences.

	Number of sequences by correlation levels						
Correlation levels	0.75	0.625	0.50	0.375	0.25	0.125	Total
PN-EB	3	2	78	189	123	1	396
TCH	0	0	22	62	41	3	128

Table 2: Improvement of the PN-EB sequences using the Rees criterion to the 83 worst "PN-EB original" sequences.

	Number of sequences by correlation levels						
Correlation levels	0.75	0.625	0.50	0.375	0.25	0.125	Total
PN-EB original	3	2	78	189	123	1	396
PN-EB Rees	2	0	19	219	155	1	396

Table 3: Improvement with the RC and the GRC.

	Number of sequences by correlation levels						
Correlation levels	0.75	0.625	0.5	0.375	0.25	0.125	Total
PN-EB original	3	2	78	189	123	1	396
PN-EB w/RC	0	0	0	233	162	1	396
PN-EB w/GRC	0	0	0	231	164	1	396

resolve all the undesirable cases. The partial results regarding these two initial steps, for the case $n = 5$, are presented in Section 5.1.

In order to proceed with the worst cases, the analysis of the autocorrelation function was performed to determine the optimum localization for the extra-zero to reach low values for the secondary peak levels of the autocorrelation function. A detailed analysis is presented in [30].

4.3. The generating ranging criterion

The second criterion considered here uses both the process of positioning the extra-zero in a sequential mode and the primary m-sequence generator to determine the final phase for the extra-zero, thus this criterion was called the *generators ranging criterion* (GRC) [11]. Starting from Gold balanced m-sequences, this criterion is summarized in the following steps:

(1) the extra-zero will be inserted at the end of the sequence period;
(2) in all those sequences that still present a high level of correlation (e.g., > 0.5), the extra-zero will be inserted in the bigger run of 0's;
(3) if still there are sequences whose improvement was not reached with steps (1) and (2), then, the extra-zero will be inserted next to the 1st or the 2nd bigger run of 0's of the primary m-sequence generator.

The development of the generators ranging criterion was based on

(a) the fact that the Rees criterion does not make significant alterations in the values of the secondary peaks of autocorrelation function;
(b) a simple characterization of the position of the extra-zero symbol.

Remark that, it is implied in the third step that an extra-zero in the same phase position is also inserted in the secondary m-sequence generator.

The option for choosing the position using the primary m-sequence generator relatively to the secondary sequence generator is justified by the simplicity of the Gold criteria relatively to the definition of the initial conditions for this shift register generator [30].

The generators ranging criterion is based on procedures that Rees developed for m-sequences and are applied here to the Gold sequences. Although the extra-zero will appear in the generated Gold sequence, the localization of the extra-zero symbol will now be chosen by the region of the 1st or the 2nd bigger run of 0's of the "primary m-sequence generator."

Note that, in all the considered cases, only five (out of all the three hundred and ninety six) produced better results using the 2nd run of 0's, regarding the autocorrelation levels.

5. SIMULATION RESULTS

5.1. Autocorrelation levels in PN-EB

Table 3 presents some results concerning the application of both the ranging criterion and the generators ranging criterion to the balanced Gold sequences of period 31. As it can be seen, it is possible to derive even balanced sequences with good secondary autocorrelation peak levels, even for those PN-EB original sequences that presented the worst correlation values.

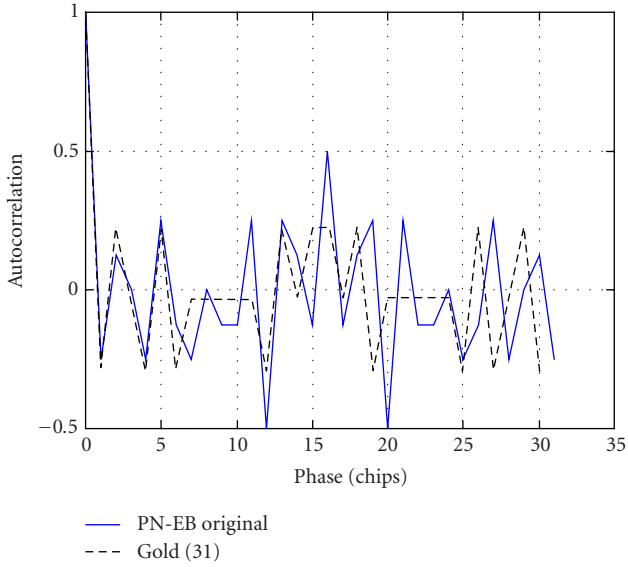


Figure 5: Autocorrelations of a Gold sequence and the corresponding PN-EB original.

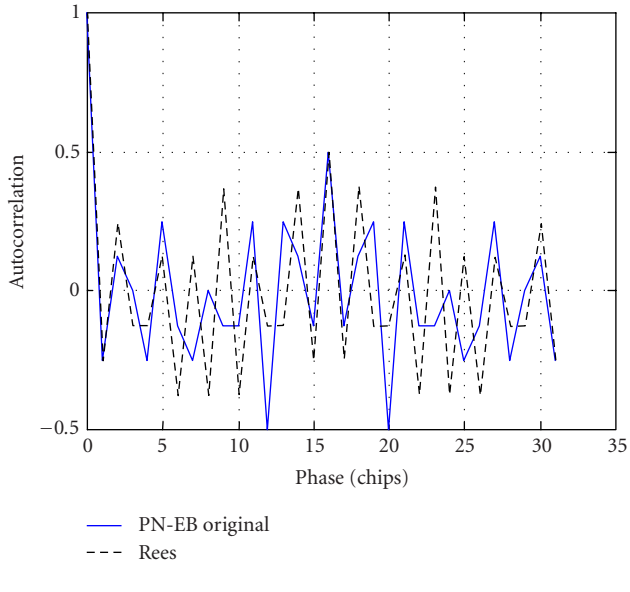


Figure 6: Autocorrelation of the PN-EB original and corresponding PN-EB Rees.

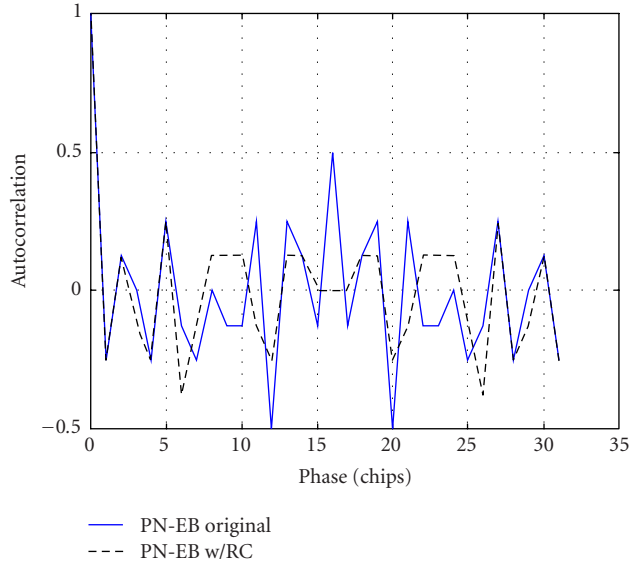


Figure 7: Autocorrelation of the PN-EB original and the corresponding PN-EB with RC.

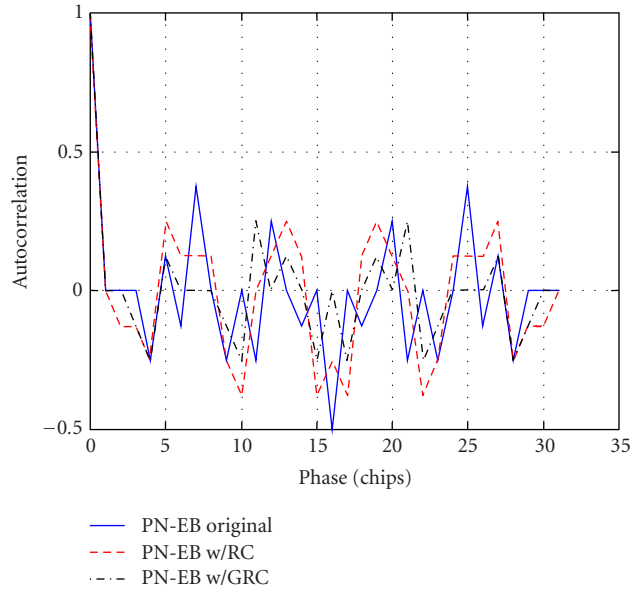


Figure 8: Autocorrelation of the PN-EB original and the corresponding PN-EB with RC and PN-EB with GRC.

From Table 3, one can also conclude that the GRC leads to similar results to those of the RC. Remark that, in face of the RC in this example of $n = 31$, the GRC still allows, in some cases, smaller autocorrelation secondary peak levels.

To characterize the evolution of the autocorrelation values along the different steps of the proposed positioning methodology, in Figures 5, 6, 7, and 8 the autocorrelations are shown for one of the worst cases in Table 2. The figures illustrate the steps from the originally Gold sequence to the final PN-EB sequence.

In Figure 5 one can observe that the result of the extra-zero inserted at the end of the Gold sequence period, generating the PN-EB original sequence, implies a certain degradation of the secondary peak levels of its autocorrelation function.

In this case there are three visible peaks of normalized correlation whose absolute value is 0.5, whereas the remaining peaks have levels similar to those of the Gold sequence, [21, 22, 23, 24, 33]. The application of the Rees method to this case led to some improvement, as shown in Figure 6.

Table 4: Improvement with the RC and the GRC.

	Number of sequences by correlation levels							
Correlation levels	0.5	0.437	0.375	0.312	0.25	0.187	0.125	Total
PN-EB original	2	1	7	20	16	1	2	49
PN-EB w/RC	0	0	0	20	26	1	2	49
PN-EB w/GRC	0	0	0	11	33	3	2	49

However, the Rees method did not correct all the higher peaks. Figure 7 shows that the RC method allows correcting this situation, changing the maximum limit of the correlation secondary peaks to values of 0.375.

Another illustrative example from the 21 worst cases of Table 2 can be observed in Figure 8, where the extra-zero was placed at the end of another Gold sequence.

This originated a degradation of some secondary peak levels of the autocorrelation function. Both the RC and the GRC overcame this problem, reducing that value to 0.375.

One conclusion to extract from these figures is that, in contrast with what happens with the primitive sequence whose boundary values of the secondary peaks correlation do not exceed the absolute value of 0.25, the application of the Rees criterion to the Gold sequences is not always successful. However, the application of the RC or the GRC process leads to an improvement of the worst autocorrelation levels, producing sequences with low secondary peak levels, suitable for most cases of use.

Next, some results are presented that characterize and confirm the good performance of the new PN-EB sequences derived through the RC and GRC criteria.

More significant results can be obtained with the degree $n = 6$ as shown in Table 4.

It is significant to refer that preferred pairs do not exist for this case (for $n = 5$ there is a single preferred pair). So, the secondary peaks of the autocorrelation are not expected to have low levels. All the simulations performed with other pairs of sequences generated by 6th-degree polynomials gave results in agreement with the presented examples.

For sequences generated from polynomials with a degree higher than $n = 6$, there are no studies yet. However, these sequences lead to situations with more "aliasing" tendency, which becomes disadvantageous in the context of this study.

5.2. The PN-EB sequences cross-correlation

Another important aspect not yet considered is the performance of the cross-correlation function. All the simulations show that the cross-correlation behaviour for the PN-EB sequences is similar to that verified with the autocorrelation function out-of-phase, as can be observed in the example illustrated in Figure 9.

This result is still similar to that obtained with classic odd sequences. For these, theory predicts the worst values for cross-correlation function to be similar to the autocorrelation secondary peak levels [33]. Figure 9 shows the values of the cross-correlation function for three PN-EB pairs.

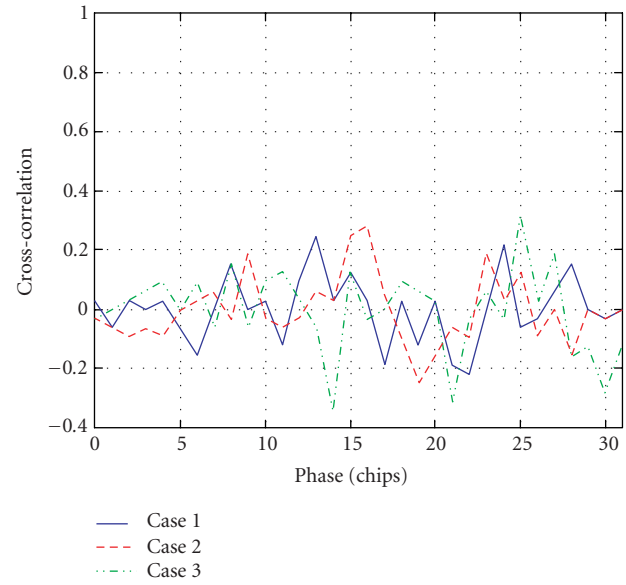


Figure 9: Examples of three cases of cross-correlation between PN-EB w/GRC sequences.

5.3. Number of available sequences

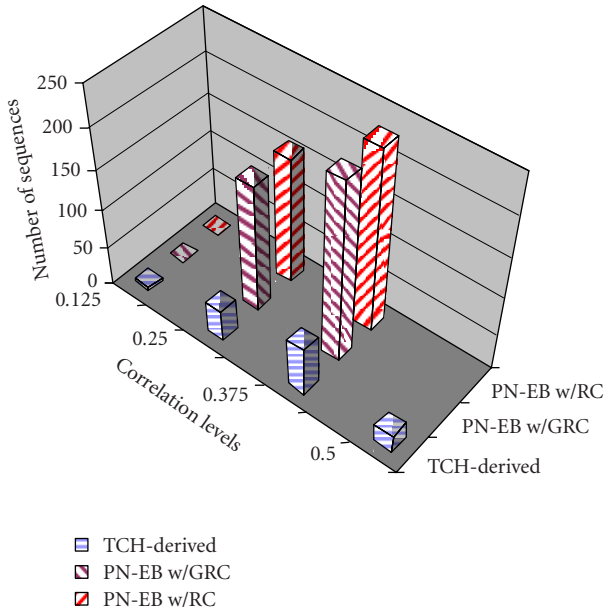
Comparing the number of PN-EB sequences (and their distribution according to the secondary autocorrelation peak levels) with those obtained using another class of even length sequences (the TCH), the improvement obtained with the proposed sequences is clear (see Figure 10).

As one can see, besides showing a better distribution of the PN-EB sequences by the values of the secondary peaks of the autocorrelation function, it illustrates the greater number of the PN-EB sequences available. This fact is also important in the case of polynomials of degree n=6. This study constitutes an area for exploring.

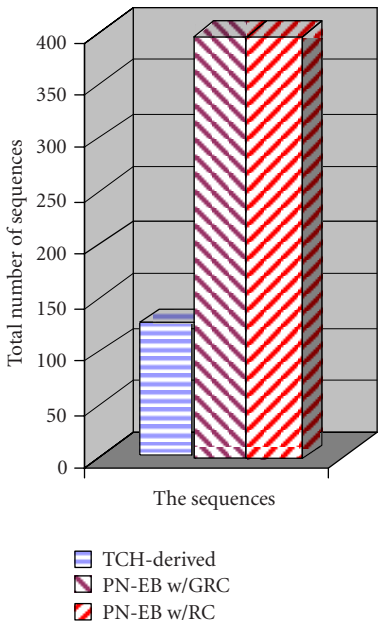
5.4. Processing gain of PN-EB compared to Gold sequences

To illustrate the assertion done in Section 3, some simulation results obtained with odd, even, and zero-mean odd PN sequences using Manchester pulses are going to be presented. A 1 kHz bandwidth noise was used, obtained by lowpass filtering white noise. A reference bandwidth of $B_x = 1$ kHz was used (signal bandwidth). The processing gain was calculated according to (13). The results are shown in Figure 11, where the processing gain is plotted against the sequence length, N.

The chip rate was 0.5 Mchip/s. In the referred conditions, the aliasing begins at a sequence period length around 250.



(a)



(b)

FIGURE 10: Total number of the TCH-derived and the PN-EB sequences with length 32.

The odd sequences (difference between the number of zeros and ones equal to 1) had lengths 5, 7, 15, 31, 63, 127, 255, 511, 1023, and 2047, and the even ones had lengths 8, 16, 18, 32, 34, 42, 64, 102, 128, 170, 256, 512, 1024, and 2048. As it can be seen, even for the *m*-sequences (odd) while there is no aliasing, the processing gain still increases with N^2 and decreases with the aliasing. The used even sequences are balanced, so, they present a null mean value theoretically able to lead to an infinite PG. As it can be seen, the gain is very high while there is no aliasing. This fact disappears with the aliasing: the gain decreases to values comparable to those obtained with odd sequences. The zero-mean odd sequences are *m-sequences* (with lengths ranging from 7 to 2047) using Manchester pulses. As it can be seen, without aliasing the gain is very high, as expected. Also, Manchester pulse sequences present the higher PG when in aliasing conditions.

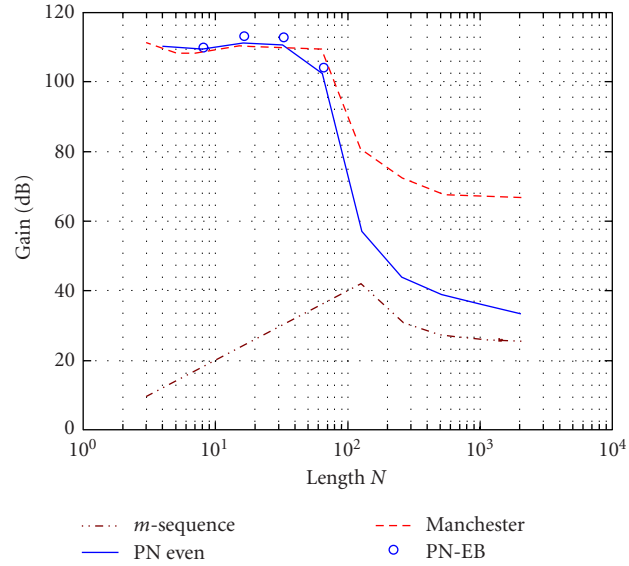


FIGURE 11: Comparison of the PG with the PN-EB and classic sequences versus length of the sequence.

This can be explained attending to their spectral power density distribution, which is very low near the origin, resulting in low-weighted replicas invading the reference band for the aliasing cases illustrated.

Remark that, essentially, these results concern baseband noise signals. The passband case will be discussed next.

5.5. *Spreading sequence PSD analysis: the bandpass-filtered case*

The theoretical study of the bandpass noise case is somehow involved. Since a theoretical analysis of the bandpass noise case is quite complex, simulations were carried out to evaluate the performance in the presence of bandpass noise. Recall that at the receiver, despreading is accomplished by multiplying the received channel signal with a local PN spreading sequence and retaining only that part of the recovered information signal (and interference) that falls within the data signal bandwidth, B_x. Compared to the original (unspread) spectrum of the received noise, the despread noise contribution within the signal bandwidth B_x will be significantly reduced, resulting in a large PG. To obtain the actual SNR in the information band B_x, the contribution of all band-limited noise replicas (at multiples of the "sampling" or chip frequency) that remain in the information detection band after despreading must be determined.

From the above considerations, in order to compare the expected performance (PG) of both the new PN-EB sequences and the original classic ones, under the same bandpass noise conditions, it seems reasonable to compare their maximum spectrum values.

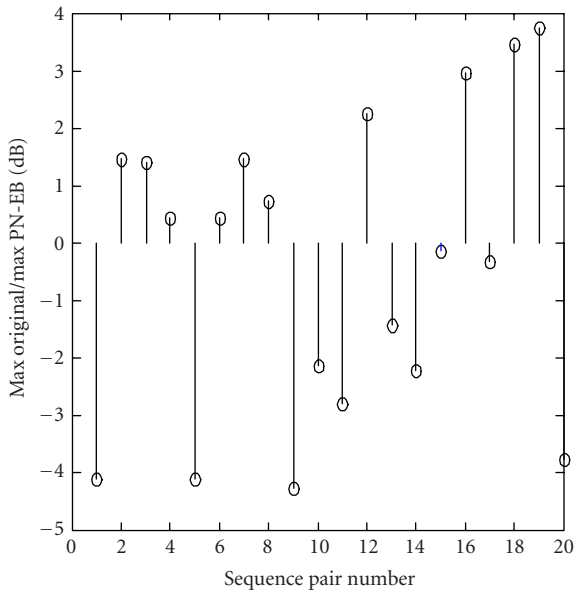


Figure 12: Twenty examples of Gold sequence spectral maximum value/PN-EB spectral maximum value.

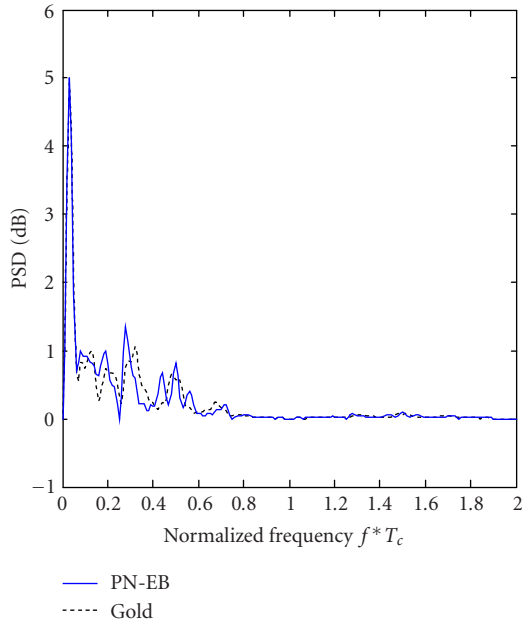


Figure 13: Spectra for pair number 15.

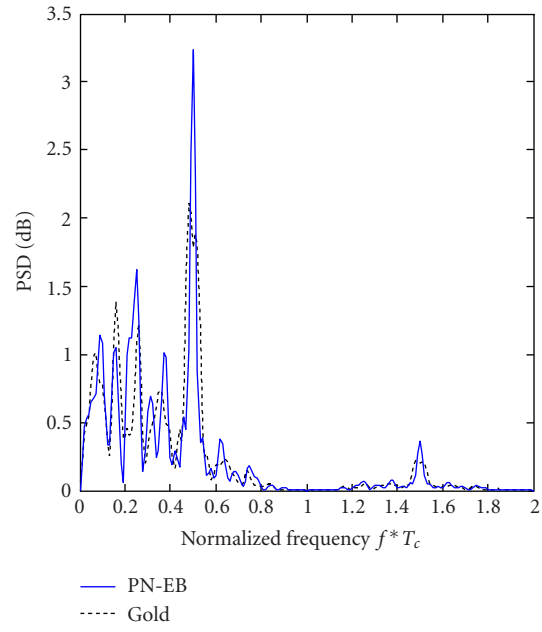


Figure 14: Spectra for pair number 9.

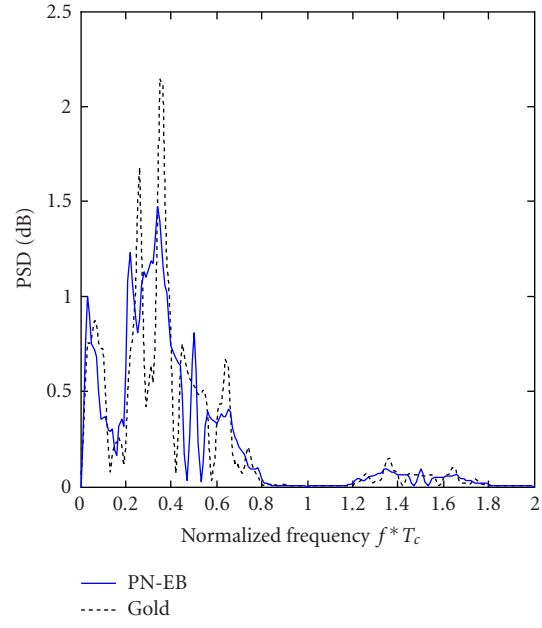


Figure 15: Spectra for pair number 19.

A large difference means a significantly better/worse performance of the new PN-EB regarding the original one; on the other hand, a small difference means that, in the worst conditions, the expected performance of the new PN-EB and that of the classic sequence from which it was derived is likely to be the same. Simulation results show that the spectra of the original Gold sequence and that of the PN-EB sequence are not very different, presenting the respective maxima a difference of a few dBs, as illustrated in the examples of Figure 12. In Figure 12 the difference between the maximum spectral values of the original Gold sequence and that of the corresponding PN-EB is shown.

In Figures 13, 14, and 15, one can see spectra of pairs (PN-EB sequence-Gold sequence). Figure 13 corresponds to a pair of sequences with a very similar (difference of maxima of 0 dB) spectrum (pair number 15 in Figure 12).

Figure 14 corresponds to spectra with difference of maxima ≈ -4.5 dB (pair number 9 in Figure 12) and Figure 15 corresponds to spectra with difference of maxima $\approx +3.7$ dB (pair number 19 in Figure 12).

As illustrated, the spectra of a new PN-EB and that of the Gold sequence from which it was obtained do not present much differences, namely in their maxima. As a result, in the worst cases of bandpass noise (noise centred

at the replica frequency), the difference of PG is almost negligible.

6. CONCLUSIONS

In this paper, a new look into the DS spread-spectrum was proposed. A novel definition of processing gain was introduced. This allowed concluding that a higher signal-to-noise output ratio can be obtained provided that a zero-mean spreading sequence is used. This led to a search for sequences with such characteristic obtained from pre-existing ones. To achieve this goal a new class of pseudonoise even balanced binary spreading sequences was derived from existing classical odd-length families of maximum-length sequences, such as those proposed by Gold, by appending or inserting one extra-zero symbol to the original sequences. To obtain large families of balanced sequences, two design criteria were proposed, namely the ranging criterion (RC) and the generating ranging criterion (GRC). PN-EB sequences in the polynomial range order $3 \leq n \leq 6$ were derived using these criteria. It was shown that they exhibit secondary auto- and cross-correlation peaks comparable to the sequences they are derived from.

The most important reasons for the generation of PN-EB sequences are summarized in the following points: (i) their performance is comparable to those with pulses of Manchester, without the complexity of the latter; (ii) their correlation properties are at least comparable to the Gold ones; (iii) there is the possibility of generating a high number of sequences.

The proposed methods are not only suitable for CDMA applications, but simultaneously offer high processing gains and good despreading characteristics in multiuser SS scenarios with band-limited noise and interference spectra. Simulation results were presented to confirm the theoretical results.

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J. A. L. Inácio was born in Lisbon in 1942. In 1967, he received the Bachelor degree in electrotechnics and machines from ISEL, Lisbon. In 1977, he received the Electrical Engineering (Telecomunications) degree, in 1990 the Master of Science degree, and in 2001 the Ph.D. degree in electrical engineering and computers, all from Instituto Superior Técnico (IST), Universidade Técnica de Lisboa, Lisbon, Portugal. From 1962 to 1965, he was a Radar Engineer at the Portuguese Air Force Army. In 1966, he joined ITT, Lisbon, as a Production Engineer and in 1967 held the position of Quality Control Engineer at the same company. In 1968 he became Development Engineer with the development of Telephone 7P. In 1970, he joined Plessey (PAEP), Lisbon, as Technical Admnistrator Engineer in the R & D Engineering Department and from 1976 to 1977, he was the Design and Development Engineer Manager. From 1978 to 1981, he worked as Technical Manager and Consultant in Lisbon factories. In 1982 he began teaching science, electronics, and telecommunications engineering of undergraduate level. In 1985 his teaching activities included both undergraduate and graduate programs at the ENIDH till now, and in 1990 he gave lectures at IPS, Setúbal. Since 1988, he has been a Research Engineer at the Instituto de Engenharia de Sistemas e Computadores-Investigação e Desenvolvimento (INESC-ID), Lisbon, working on sinal detection systems (radar and sonar), spread-spectrum systems, and power line communications.

J. A. B. Gerald was born in Lisbon in 1956. He received the Diploma degree in electrical engineering from the Instituto Superior Técnico (IST), Universidade Técnica de Lisboa, Lisbon, Portugal, in 1980. In 1992 he received the Ph.D. degree from IST, with a thesis on adaptive recursive structures for data communication systems. Since 1980, he has been with IST in the Electrical Engineering Department, where he is now Assistant Professor. He has been teaching both electronics and telecommunication engineering. From 1980 to 1988 he was a Research Engineer at Centro de Electrónica Aplicada (CEAUTL), Universidade Técnica de Lisboa, Lisbon, Portugal, where he worked on digital signal processing, modems, and adaptive systems. Since 1988, he has been a Research Engineer at the Instituto de Engenharia de Sistemas e Computadores (INESC), Lisbon, presently Instituto de Engenharia de Sistemas e Computadores-Investigação e Desenvolvimento (INESC-ID), where he is working on signal processing for modems, and spread-spectrum systems. His research interests include the areas of adaptive filtering, spread-spectrum communications, and power line communications. Recently he was also involved in a wireless visual system project for cortical neuroprodthesis.

M. D. Ortigueira was born in Seia, Portugal, in 1949. He received the Electrical Engineer (Telecomunications) degree in 1975, the Ph.D. degree in electrical engineering in 1984, and the title of "Agregado" in electrical engineering all from the Instituto Superior Técnico (IST), Universidade Técnica de Lisboa, Lisbon, Portugal. He was Assistant Professor of IST till 2001 when he joined the Electrical Engineering Department, Faculty of Sciences and Technology, Universidade Nova de Lisboa. He currently teaches signals and systems theory and digital signal processing. In 1985, he joined the Digital Signal Processing Group at the Instituto de Engenharia de Sistemas e Computadores (INESC), Lisbon, and in 1997 the Center for Intelligent Robotics, UNINOVA. From 1988 to 1990, he was on leave at the Telecomunication School, Catalonia Polithecnic University, Barcelona, Spain, where he worked on radar simulation and moving source detection. His scientific interests include spectral estimation and identification of time series (ARMA models), spread-spectrum communications, biomedical signal processing, and signal processing applications of fractional calculus.