

# Order Reduction Techniques for Coupled Multi-Domain Electromagnetic based Models

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**Abstract.** This work presents a comprehensive flow able to efficiently generate reduced order models for realistic, hierarchy aware, Electromagnetic (EM) based models. Knowledge of the structure of the problem is explicitly exploited using domain partitioning and novel electromagnetic connector modeling techniques to generate a hierarchically coupled representation. This enables the efficient use of structure preserving block model order reduction techniques to generate block-wise compressed models that satisfy overall requirements, and provide cheap evaluation and simulation accurate approximations of the complete EM behaviour.

## 1 Introduction

The performance of each device in currently designed, complex integrated circuits, is strongly affected by the environment surrounding it. The behaviour of each circuit part or subsystem depends not only on its own physical and electrical characteristics, but also on the devices to which it is directly connected to or coupled with, including air and substrate. Analyzing such behaviour leads us to the need for treating the complete nanoscale RF blocks as a whole. Such blocks, composed of several elements or sub-systems, need to be accurately modeled, including the unintended EM couplings existing between the different elements. The EM based modeling procedures usually rely on a discretization of the governing equations, in this case Maxwell's equations, on the domain of interest. However, integrated components and systems with complex structures generate complex EM field problems that are difficult to solve. An efficient approach to manage this complexity is to apply a divide and conquer principle and decompose the computational domain in sub-domains, each of which generates a simpler field problem. This approach is in fact not dissimilar from the usual integrated circuit decomposition into active and passive components. The EM interactions or couplings between sub-domains, which can be either of electric or magnetic nature, can be modeled via a consistent mathematical formulation proposed for the first time in [1] and used as a method for domain partitioning in [2, 3]. In this formulation, the interactions, entitled *hooks* or *connectors*, can be understood as ports of different sub-systems, which model the effect the electric and magnetic field has in the mathematical model of the sub-domain.

Nonetheless, the models thus generated are still too large to allow them to be included efficiently into any simulation flow. To overcome this issue, Model Order Reduction (MOR) procedures have been advocated along the past decade [4, 5]. These algorithms have now reached maturity, and some of the better known techniques [6–9] are routinely used inside modeling and simulation environments. Furthermore, the inner model structure provided by hierarchical modeling approaches allow to get some advantages that can be exploited advantageously to achieve additional compression.

This paper presents a comprehensive flow able to efficiently generate reduced models for the RF blocks related to interconnects and designed-in passives. The method takes into account parasitic effects caused by EM couplings, and generates reduced models amenable for efficient simulation. The paper is structured as follows: in Section 2 we present an overview of the main features of the models under study, and a set of guidelines for their reduction. In Section 3 the proposed methodology is presented and discussed. In Section 4 several examples are shown that illustrate the efficiency of the proposed technique, and in Section 5 conclusions are drawn.

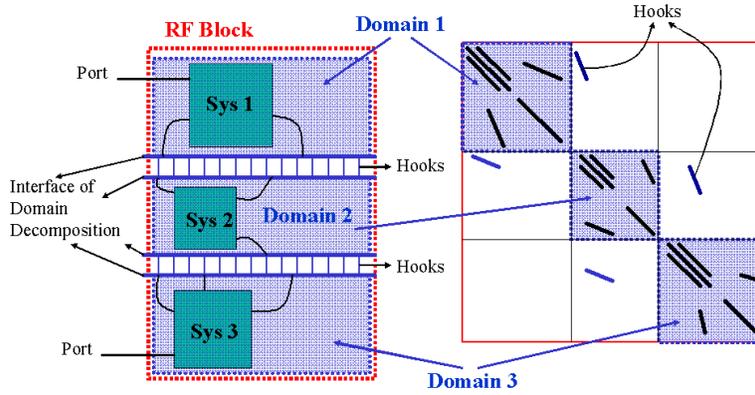
## 2 Background

### 2.1 Hierarchical EM Modeling

In this section we briefly introduce the concepts of hooks or connectors, and their applicability to Domain Decomposition (DD) in EM modeling. The numerical approach pursued is based on the domain decomposition of a RF block into its active and passive components, as well as in the environmental components, for instance the substrate and the upper air. Each of these simple connected sub-domains satisfies a set of Electro-Magnetic Circuit Element (EMCE) [1] boundary conditions and can be interconnected or coupled with the rest. Each sub-domain can then be individually modeled and the resulting equivalent circuits or descriptions are reconnected together to generate a model of the entire RF block. The coupling of an integrated component with its environment is realized in one or more of three basic ways: conductive, capacitive and inductive, by means of: electric interconnect terminals, electric virtual connectors and magnetic terminals.

These connections are achieved through the concept of hook or connector, which is conceptually realized as a boundary condition in the associated EM field problem [1, 2]. In order to keep the general context, the hook will be considered an object containing a series of associated sub-concepts and quantities. They may be:

- Electric or magnetic terminals (surfaces - part of the component boundary which act as a "window" for the electric and magnetic field, respectively)
- A spatial domain - support for the interaction called inter-connector (if it is intentional, e.g. a metallic wire) or interactor (if it is parasitic, for instance the substrate).
- A pair of scalar quantities, one of "longitudinal" nature (a line integral along the interaction path) and another "transversal" (a surface integral across the component terminal), such as voltage and current. Both quantities will describe, from complementary perspectives, the "intensity" of coupling;



**Fig. 1.** Domain Decomposition, Hooks, and matricial representation.

- A parameter which, in the linear materials, does not depend on the states, describing the "strength" of the coupling. For instance, any entry in the hybrid matrix is a transfer function describing the interaction between two terminals, such as impedance or admittance.

A more graphical depiction of the DD paradigm and the hook concept can be seen in Figure 1.

## 2.2 Hierarchy Aware Reduction

The DD provides a description of a large system as a set of interconnected smaller models [3]. On the other hand, in order to maintain the global accuracy and keep the EM interactions between sub-domains, requires the use of a large number of hooks, which from the mathematical point of view can be seen as an increase of the number of ports for each sub-system. Each of the sub-systems is usually represented as a state-space descriptor

$$C\dot{x} + Gx = Bu, \quad y = Lx \quad (1)$$

where  $C, G \in \mathbb{R}^{n \times n}$  are respectively the dynamic and static matrix descriptors,  $B \in \mathbb{R}^{n \times m}$  is the matrix that relates the input vector  $u \in \mathbb{R}^m$  to the states' vector  $x \in \mathbb{R}^n$ , and  $L \in \mathbb{R}^{p \times n}$  is the matrix that links those inner states to the outputs  $y \in \mathbb{R}^p$ .

This state space descriptor has an associated frequency transfer function

$$H(s) = L(sC + G)^{-1}B. \quad (2)$$

The objective of MOR techniques is to generate a reduced order approximation, able to accurately capture the input-output behavior of the system for any point frequency space,

$$\hat{H}(s) = \hat{L}(s\hat{C} + \hat{G})^{-1}\hat{B}, \quad \hat{H}(s) \approx H(s) \quad (3)$$

where  $\hat{C}, \hat{G} \in \mathbb{R}^{q \times q}$ ,  $\hat{B} \in \mathbb{R}^{q \times m}$ , and  $\hat{L} \in \mathbb{R}^{p \times q}$ , with  $q$  the reduced order. In most cases [6, 9] this is achieved by means of congruence transformation on the system matrices,

which project the large dimension states' vector  $x$  into a much smaller dimensional space,  $\hat{x} = V^T x$ , with  $\hat{x} \in \mathbb{R}^q$ ,  $q \ll n$ . Techniques based on this approach are denoted as projection based, and differ in the choice of the subspace  $V$  they use for the projection, and how they build such a subspace.

From the MOR point of view, the models generated by DD and enhanced with a large number of hooks can be considered as massive MIMO systems, with many inputs and outputs, and therefore the independent reduction of each of them may become an issue. Most of the known MOR techniques are known to be somewhat inefficient when attempting to reduce such models. A set of procedures that addresses this issue can be found in the literature (as an example we refer the reader to [10]). Most of those frameworks are devoted to the reduction of digital circuits, with lots of real physical ports and excited with known waveform patterns. A knowledge or study of correlation between the ports may help in the reduction stage. Regrettably, this is not necessarily the case when discussing the role of the hooks. The EM nature of these connectors and the unknown waveform patterns of RF designs make these MOR approaches fairly useless in the case under study.

An alternative is based on Block Structure Preserving (BSP) approaches [11, 12]. These methods are aimed at maintaining the inner block structure of the matrices in projection frameworks. Figure 1 shows that the complete system can be represented as global state space with a inner structure, where sub-system matrices are placed in the main diagonal, whereas the connections between them (given by the hooks) are placed in the off-diagonal blocks. The use of the global ports for building the projection subspace leads to more compressed reduced models, independent of the number of hooks. The models also maintain the block hierarchy of the original matrices. However, the use of the large global matrices increases the computational cost. On the other hand, the use of a smaller number of inputs and outputs reduces numerical errors and minimizes the added computational effort from the orthonormalization of the generated basis, which may compensate the use of larger matrices.

### 3 Proposed Methodology and Computational Issues

Inside the MOR realm, the moment matching algorithms based on single point expansion may not be able to capture the complete behaviour along the wide frequency range required for common RF systems, or may lead to excessively large models. Therefore the most suitable techniques for the reduction seem to be the multipoint ones. Among those techniques, PMTBR [9] offers a reliable framework with some interesting features that can be exploited, such as the inclusion of a trade off between size and error, which allows for some control of the error via analysis of the singular values related to the dropped vectors. On the other hand, it requires higher computational effort than the moment matching approaches, as it is based on sampling schemes and Singular Value Decomposition (SVD), but the compression ratio and reliability that it offers compensates this drawback. Furthermore, this technique offers some extra advantages when combined with block structured systems, such as the block-wise error control with respect to the global input-output behaviour, which can be applied to improve the

efficiency of the reduction. This means that each block can be reduced to a different order depending on its relevance in the global response.

The proposed flow starts from a state-space descriptor, such as (1), which exhibits a multi-level hierarchy. The matrices of size  $n$  have a structure that reveals  $K$  domains, each with size  $n_i$ , ( $i = 1, \dots, K$ ,  $n = \sum_i n_i$ ). For instance, for the static part,

$$G = \begin{bmatrix} G_{11} & \dots & G_{1K} \\ \vdots & \ddots & \vdots \\ G_{K1} & \dots & G_{KK} \end{bmatrix} \quad C = \begin{bmatrix} C_{11} & \dots & C_{1K} \\ \vdots & \ddots & \vdots \\ C_{K1} & \dots & C_{KK} \end{bmatrix} \quad B = \begin{bmatrix} B_1 \\ \vdots \\ B_K \end{bmatrix} \quad (4)$$

Sampling in the frequency can then be performed and for each point one generates a vector  $z_j$ ,

$$z_j = (G + s_j C)^{-1} B, \quad (5)$$

where  $C$  and  $G$  are the global matrices of the complete domain, with  $n$  degrees of freedom (dofs). To generate the vector  $z_j \in \mathbb{R}^{n \times m}$ , with  $m$  the number of global ports, we can apply a direct procedure, or in cases when a direct method may be too expensive, an iterative procedure (e.g. GMRES).

The choice of the sampling points may be an issue, as there is no clear scheme or procedure that is known to provide an optimal solution. However, as stated in [9], the accuracy of the method does not depend strictly on the accuracy of the quadrature (and thus in the sampling scheme), but on the subspace generated. After generating  $P$  samples (and thus  $Pm$  vectors, with  $m$  the number of global ports), the next step is the orthonormalization, via SVD, of such vectors for generating a basis of the subspace in which to project the matrices. Here an **independent basis**  $V_i$ ,  $i \in \{1 \dots K\}$ , can be generated for each  $i$ -th sub-domain. To this end the vectors  $z_j$  are split following the block structure present in the system matrices (i.e. the  $n_i$  rows for each block,  $n = \sum_i n_i$ ), and an SVD is performed on each of these sets of vectors. For each block, the independent SVD allows us to drop the less relevant vectors for the global response (estimated by the magnitude dropped singular value ratio, as presented in [13]). This step generates a set of projectors,  $V_i \in \mathbb{R}^{n_i \times q_i}$ , with  $q_i \ll n_i$  the reduced size for the  $i$ -th block of the global system matrix. These projectors can be placed in the diagonal blocks of an overall projector, that can be used for reducing the initial global matrices to an order  $q = \sum_i q_i$ ,

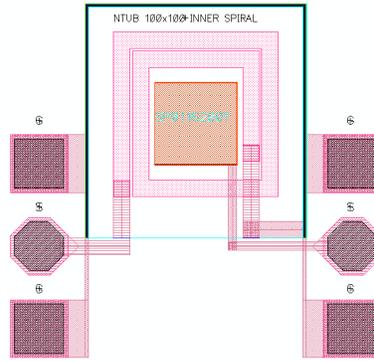
$$\check{V} = \begin{bmatrix} V_1 & & \\ & \ddots & \\ & & V_K \end{bmatrix}, \quad (6)$$

This block diagonal projector enables block structure (and thus sub-domain) preservation, increasing the sparsity of the ROM with respect to that of the standard projection

$$\hat{G}_{ij} = V_i^T G_{ij} V_j \quad \hat{C}_{ij} = V_i^T C_{ij} V_j \quad \hat{L}_i = L_i V_i \quad \hat{B}_i = V_i^T B_i \quad (7)$$

**Table 1.** Characteristics of the Examples

| Example | Domain   | # Dofs | # Terminals<br>(EH,MH,IT) | ROM # Dofs<br>BSP PRIMA | ROM # Dofs<br>BSP PMTBR |
|---------|----------|--------|---------------------------|-------------------------|-------------------------|
| 4.1     | Top      | 34595  | 466 (257, 207, 2)         | 80                      | 49                      |
|         | Bottom   | 16397  | 464 (257, 207, 0)         | 80                      | 31                      |
|         | Complete | 50992  | 2 (0, 0, 2)               | 160                     | 80                      |
| 4.2     | Left     | 85616  | 938 (473, 464, 1)         | 600                     | 54                      |
|         | Middle   | 8027   | 1874 (946, 928, 0)        | 600                     | 59                      |
|         | Right    | 85629  | 938 (473, 464, 1)         | 600                     | 87                      |
|         | Complete | 179272 | 2 (0, 0, 2)               | 1800                    | 200                     |

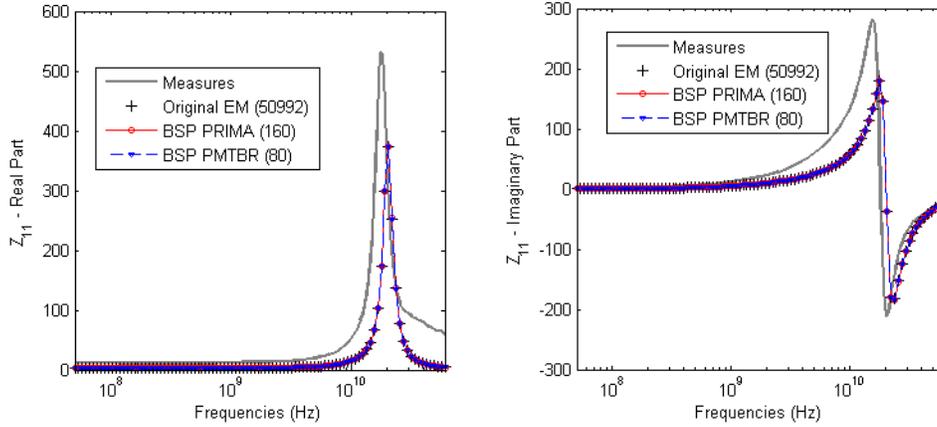
**Fig. 2.** Layout of the proposed benchmark: Spiral over N-Well.

## 4 Simulation Results

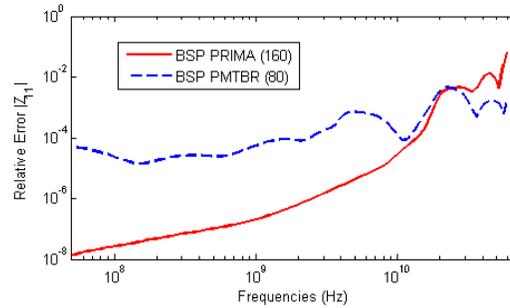
### 4.1 Spiral over N-WELL

For our first example, we use an industrial circuit, composed by a square integrated spiral inductor over a N-Well (See Figure 2). In this case the domain is divided into two sub-domains. The first one, the top one, includes the air, the layers in which the spiral is included, and the N-Well within the upper part of the substrate. The second sub-domain, the bottom one, includes the lower bulk part of the substrate. FIT [2] is used as EM modeling technique, leading to a complete system with a total of 50992 dofs. Both sub-domains are coupled via hooks, and only the top domain has external ports.

Notice that the methodology is a prototype tool, and the grid used is not refined, in order to generate a tractable model for standard computers, so the models may not have a perfect accuracy in comparison with measurements. Table 1 shows the characteristics of the original system and the reduction with the proposed methodology. These



**Fig. 3.** Real (Left) and Imaginary (Right) Part of  $Z_{11}$  for the Measurements, the EM original model, and the ROMs: BSP PRIMA (red), and BSP PMTBR (blue).



**Fig. 4.** Relative Error in  $Z_{11}$  of the ROMs with respect to the original EM model.

include the number of dofs of each model, the number of terminals (including electrical, EH, and magnetic, MH, hooks, as well as ports, P). The reduction is carried with BSP PRIMA [12], matching 40 moments, which generates a 160-vector BSP projector, and the proposed BSP PMTBR, with 50 frequency samples and a relative tolerance of  $1e-4$ , with generates a 49-dofs top model, and a 31-dofs bottom model (and a global model of size 80). It is important to recall that the reduction procedure is independent of the number of hooks (464). The proposed BSP PMTBR applies different compression ratio to both domains, as they have different relevance in the global response (the bottom domain does not have external terminals, and thus only has parasitic effects on the top domain). The frequency results can be seen in Figure 3. Measurements have been include for comparison with the EM model. Although the model is obtained via a prototype tool, and the mesh is relatively coarse, very good results are achieved. The ROM models capture the EM original model behaviour perfectly, and the curves are indistinguishable. PRIMA ROM requires a higher order, and is less accurate at higher

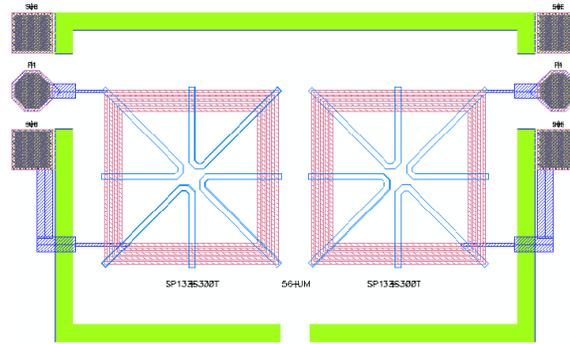


Fig. 5. Layout configuration of the Double Spiral example.

frequencies (due to single point expansion), whereas PMTBR manages to maintain the accuracy with higher compression (see Figure 4). After the reduction, the model still maintains its block hierarchy, and the two coupled domains still can be distinguished and recovered.

## 4.2 Double Spiral

Our second example is another industrial circuit, composed of two coupled integrated planar spirals. Each of the spiral ends represent one port, having one terminal voltage excited (intentional terminal, IT) and one terminal connected to ground. The complete domain, of size 179272, includes substrate and upper air, and is partitioned into three sub-domains, each of them connected to the others via a set of hooks (both electric, EH, and magnetic, MH). In one sub-domain we have the left spiral, in the middle domain it is the area between both spirals, whereas in the right domain we have the right spiral. Each sub-domain includes the corresponding substrate and upper air layer. The Full Wave EM model was obtained via Finite Integration Technique (FIT) [2], and its matrices present a Block Structure that follows the domain partitioning (see Table 1 for details).

Notice that the number of connectors or hooks for each domain is 938, whereas the global ports are only 2. Trying to reduce the domains independently with standard projection methodologies would lead to huge models (each block vector generated would have 938 vectors), and would not provide us with any simulation advantage. On the other hand, the BSP methodologies manage to obtain very compressed models. PMTBR based approach generates different orders for each domain, according to its complexity and relevance on the external ports. For the Krylov based approach, 200 moments are matched at a single expansion point, to generate a 1200-dofs ROM. Both methodologies preserve the block structure (provided by the domain decomposition) after the reduction stage.

The frequency results for the impedance at one port, and the S-parameters, can be seen in Figure 6, in which the experimental measurements up to 40GHz, and the EM results using the comercial MOMENTUM tool (up to 15GHz) are included. The

proposed methodology provides a very good agreement, and both BSP PRIMA and and BSP PMTBR provide very compressed and accurate models.

## 5 Conclusions

This paper presents a complete procedure for efficient generation of reduced order models of passive systems, which preserve the original coupled multi-domain structure. Starting from the electromagnetic description of their behavior, the modeling techniques and ensuing reduction are detailed, leading to small systems amenable to be included inside simulation environments for coupled analysis with other linear and non-linear devices. The methodology can be combined with any EM modeling technique, and takes advantage of the hierarchical information provided by either the topology of the domain or the EM description (in particular, from divide and conquer approaches that lead to sub-domain division and connection by means of hooks).

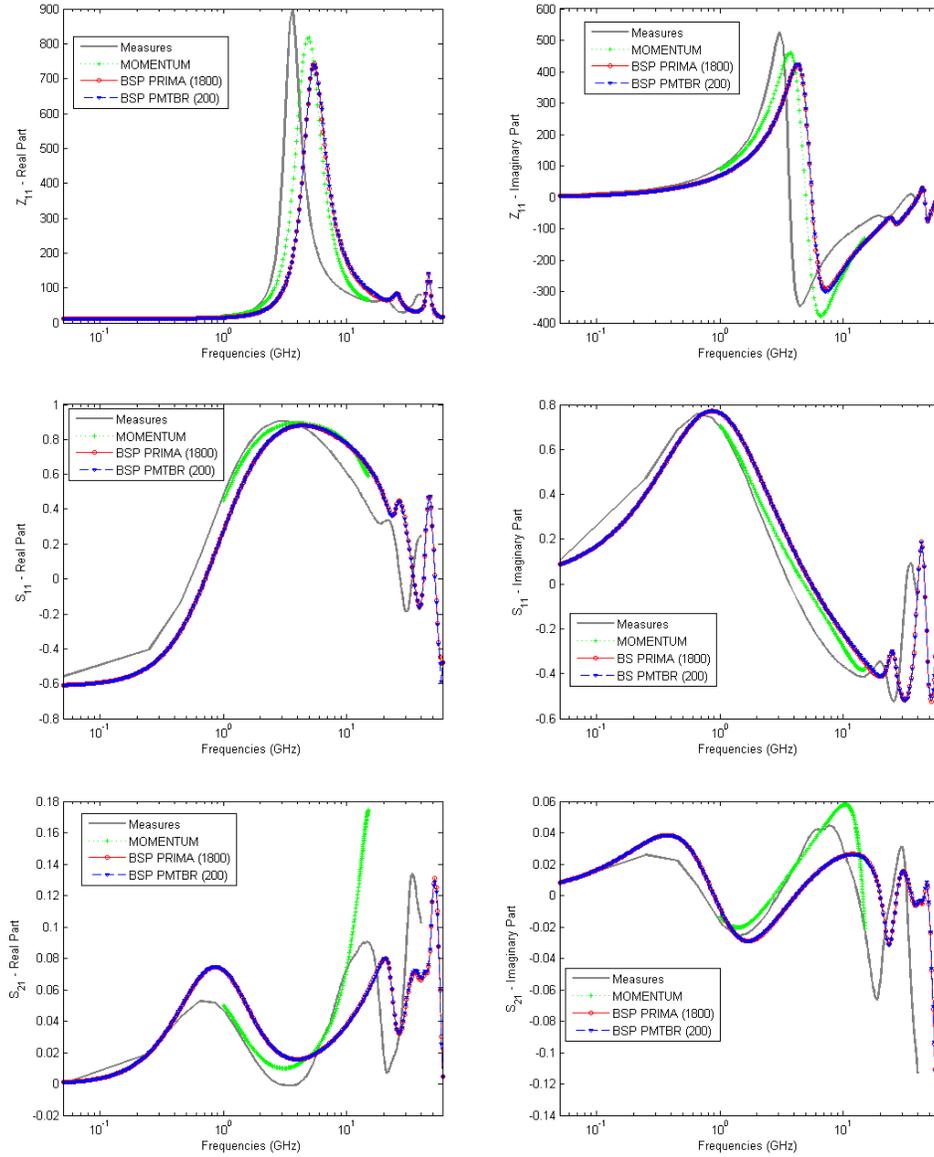
Noticeable advantages are different compression order for each block based on its relevance in the global behavior, higher degree of sparsification of the nominal matrices, and the maintenance of the block domain hierarchy after reduction, which can lead to simulation advantages.

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10 Jorge Fernández Villena, Wil H.A. Schilders, and L. Miguel Silveira

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**Fig. 6.** Coupled Double Spiral: Comparison of ROMs versus Experimental measurements and MOMENTUM results. (Top) Real (left) and Imaginary (Right) part of  $Z_{11}$ . (Center) Real (left) and Imaginary (Right) part of  $S_{11}$ . (Bottom) Real (left) and Imaginary (Right) part of  $S_{21}$ .