On the Efficient Reduction of Complete EM based Parametric Models

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***Abstract*—**
Due to higher integration and increasing frequency based effects, full Electromagnetic Models (EM) are needed for accurate prediction of the real behavior of integrated passives and interconnects. Furthermore, these structures are subject to parametric effects due to small variations of the geometric and physical properties of the inherent materials and manufacturing process. Accuracy requirements lead to huge models, which are expensive to simulate and this cost is increased when parameters and their effects are taken into account. This paper presents a complete procedure for efficient reduction of realistic, hierarchy aware, EM based parametric models. Knowledge of the structure of the problem is explicitly exploited using domain partitioning and novel electromagnetic connector modeling techniques to generate a hierarchical representation. This enables the efficient use of block parametric model order reduction techniques to generate block-wise compressed models that satisfy overall requirements, and provide accurate approximations of the complete EM behaviour, which are cheap to evaluate and simulate.

I. Introduction

New coupling and loss mechanisms, including Electromagnetic field coupling and substrate noise as well as process-induced variability, are becoming too strong and relevant to be neglected, whereas more traditional coupling and loss mechanisms are more difficult to describe given the wide frequency range involved and the greater variety of structures to be modeled in currently designed systems.

The performance of each device in a system is strongly affected by the environment surrounding it. In other words, the behaviour of each circuit part depends not only on its own physical and electrical characteristics, but also on the devices to which it is directly connected to or coupled with, leading to the treatment of complete nanoscale RF blocks as a whole. Such blocks, composed of several elements or sub-systems, need to be accurately modeled, including the unintended EM couplings existing between the different elements. The EM based modeling procedures usually rely on a discretization of the governing equations, in this case Maxwell equations, on the domain of interest. However, integrated components and systems with complex structures generate complex EM field problems that are difficult to solve. An efficient approach to manage this complexity is to apply a divide and conquer principle and decompose the computational domain in sub-domains, each of which generates a simpler field problem. This approach is in fact not dissimilar to the usual integrated circuit decomposition into active and passive components. The EM interactions between sub-domains, which can be either of electric or magnetic nature, can be modeled via a consistent mathematical formulation proposed for the first time in [1] and used as a method for domain partitioning in [2], [3]. In this formulation, the interactions, entitled *hooks* or *connectors*, can be understood as ports of different sub-systems, which model the effect the electric and magnetic field has in the mathematical model of the sub-domain.

However, these models of RF blocks are also under the influence of variations, both due to the imperfections in the process of generating the physical devices, or to intentional variations of the design process. At nanoscales, such variations, start to have larger effects on the performance of the physical elements, and thus must be captured in the characterization and modeling stages. This leads to the generation of models depending on a set of parameters. In consonance, their reduction must take into account such parametric variations [4], [5]. Furthermore, the reduced models must maintain a similar parametric dependence in order to be efficiently used inside simulation environments, providing the designers with reliable tools for their IC performance prediction. On the other hand, inside the complete RF-block, the parameters may affect different sub-domains, as well as the interactions between them.

This paper presents a comprehensive flow able to efficiently generate reduced models for the RF blocks related to interconnects and designed-in passives. The method takes into account effects both caused by EM couplings and parametric variations, and generates reduced models amenable to be efficiently simulated. The paper is structured as follows: in Section II we present an overview of the main features of the models under study, and a set of guidelines for their reduction. We also present an introduction to the Parametric Model Order Reduction (pMOR) paradigm and the existing techniques for such task, along with a discussion of their pros and cons. In Section III the proposed methodology is presented and discussed. In Section IV several examples are shown that illustrate the efficiency of the proposed technique, and in Section V conclusions are drawn.

II. Background

A. Hierarchical EM Modeling and Reduction

In this section we briefly introduce the concepts of hooks or connectors, and their applicability to Domain Decomposition (DD) in EM modeling. The discussion of the underlying theory referring to these issues is beyond the scope of the paper, and the objective of this section is to give an introductory overview of the physical significance, and how they affect the description of the system to be reduced (for more details see [2], [3], [1]).

The numerical approach pursued is based on the domain decomposition of the RF block in its active and passive

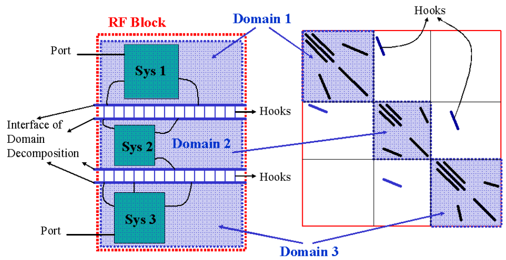

Fig. 1. Domain Decomposition, Hooks, and matricial representation.

components, as well as in the environmental components, for instance the substrate and the upper air. Each of these simple connected sub-domains is supposed to comply with the Electro-Magnetic Circuit Element (EMCE) [1] boundary conditions, which can be interconnected or coupled with the rest. The individual equivalent circuits for each sub-domain are re-connected together to generate the model of the entire RF block. The coupling of an integrated component with its environment is realized in three basic ways: conductive, capacitive and inductive, by means of: electric interconnect terminals, electric virtual connectors and magnetic terminals.

The hook or connector [1] is conceptually realized as a boundary condition in the associated EM field problem. In order to keep the general context, the hook will be considered an object containing a series of associated sub-concepts and quantities. They may be:

- electric or magnetic terminals on the boundary surface.
- a spatial domain.
- a pair of scalar quantities, which will complementarily describe the intensity of the coupling.
- a parameter (independent of the states in the linear materials) describing the strength of the coupling.

After the modeling stage of the individual sub-domains, the global state space descriptor for the complete domain can be obtained by connecting those sub-domains via the hooks [2]. A more graphical depiction of the DD paradigm and the hook concept can be seen in Figure 1.

The DD provides a set of interconnected smaller models [3], but on the other hand, in order to maintain the global accuracy and keep the EM interactions between sub-domains, requires the use of a large number of hooks, which from the mathematical point of view can be seen as an increase of the number of ports for each sub-domain.

From the MOR point of view, these models enhanced with a large number of hooks can be considered as massive MIMO systems, with lots of inputs and outputs, and so the independent reduction of each of them may become an issue. Most of the known MOR techniques are inefficient when attempting to reduce such models. A set of procedures that addresses this issue can be found in the literature (as an example we refer the reader to [6]). Most of them are devoted to the reduction of digital circuits, with lots of real physical ports excited with known waveform patterns. Assumptions about the correlation between the ports may help in the reduction stage. Regrettably, this is not necessarily the case when discussing the role of the hooks. The EM nature of these connectors and the unknown waveform patterns of RF designs make these MOR approaches fairly useless in the case under study.

An alternative is based on Block Structure Preserving (BSP) approaches [7], [8]. These methods are aimed at the maintaining the inner block structure of the matrices in projection frameworks. Figure 1 shows that the complete system can be represented as global state space with a inner structure, where sub-system matrices are placed in the main diagonal, whereas the connections between them (given by the hooks) are placed in the off-diagonal blocks. The use of the global ports for building the projection subspace leads to more compressed reduced models, independently of the number of hooks. The models also maintain the block hierarchy of the original matrices. And although the large global matrices increase the computational cost, the smaller number of inputs and outputs avoids numerical errors and higher computational effort on the orthonormalization of the generated basis, which may compensate the use of those larger yet sparse matrices.

B. Parametric Model Representation and Reduction

Variability in actual fabrication of physical devices leads to a dependence of the extracted circuit elements on several parameters, of electrical or geometrical origin, that must be accounted for. This leads to a parametric dependent frequency transfer function

$$H(s,\lambda) = L(sC(\lambda) + G(\lambda))^{-1}B, \tag{1}$$

usually represented as a parametric state-space descriptor

$$\begin{aligned} &C(\lambda)\dot{x}(\lambda) + G(\lambda)x(\lambda) = Bu \\ &y = Lx(\lambda) \end{aligned} \tag{2}$$

where $C, G \in \mathbb{R}^{n\times n}$ are respectively the dynamic and static matrix descriptors, $B \in \mathbb{R}^{n\times m}$ is the matrix that relates the input vector $u \in \mathbb{R}^m$ to the state $x \in \mathbb{R}^n$, and $L \in \mathbb{R}^{p\times n}$ is the matrix that links those inner states to the outputs $y \in \mathbb{R}^p$. The elements of C and G, as well as the states x, depend on a set of P parameters $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_P] \in \mathbb{R}^P$ which model the effects of the mentioned uncertainty. Usually the system is formulated so that the input (B) and output (L) matrices do not depend on the parameters.

The objective of pMOR techniques is to generate a reduced order approximation, able to accurately capture the input-output behavior of the system for any point in the joint frequency-parameter space,

$$\hat{H}(s,\lambda) = \hat{L}(s\hat{C}(\lambda) + \hat{G}(\lambda))^{-1}\hat{B}, \tag{3}$$

where $\hat{C}, \hat{G} \in \mathbb{R}^{q\times q}$, $\hat{B} \in \mathbb{R}^{q\times m}$, and $\hat{L} \in \mathbb{R}^{p\times q}$, with q the reduced order. In general, one attempts to generate a ROM whose structure is as similar to the original as possible, i.e. exhibiting a similar parametric dependence allowing more control within analysis and optimization frameworks. The most common procedure to achieve this goal is to use some form of projection scheme. Once a suitable subspace basis is computed, the system can be projected into that subspace, and a reduced model such as (3) can be obtained, that captures the behavior of the system under parameter variations.

In the past few years several techniques have emerged in order to tackle this problem. The most common and effective ones appear to be extensions of the basic MOR algorithms [9], [10] to handle parameterized descriptions. An example of these are multiparameter moment-matching pMOR methods [4] which can generate accurate reduced models that capture both

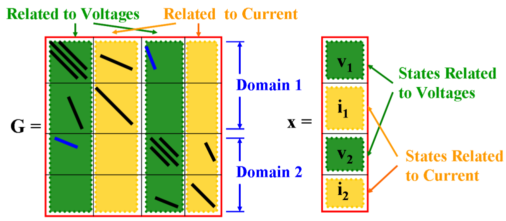

Fig. 2. Two-level hierarchy: Domain level (given by the numbers, 1 and 2) and Variable level (voltages u and currents i).

frequency and parameter dependence. The idea is to match, via different approaches, generalized moments of the parametric transfer function, and build an overall projector. This can be accomplished by accounting for the functional dependence on both the frequency as well as the parameters and matching moments of the joint space [4], [5], or simply matching the moments of the individual parameter spaces [11]. However, the structure of such methods may present some computational problems, and the resulting system models usually suffer from oversize when the number of moments to match is high, either because high accuracy (order) is required or because the number of parameters is large. Sample-based techniques have been proposed in order to contain the large growth in model order for multiparameter, high accuracy systems. In this scenario the method in [12] recursively generates the same moments and choses the most relevant, to later project the original matrices into the subspace these moments span. The Variational PMTBR [13] is an extension of the techniques based on approximate balancing [10], to account for variability. It relies on sampling of the multi-dimensional frequency and parameters space. This approach allows the inclusion of *a priori* knowledge of the parameter variation, and provides some error estimation capabilities.

III. Methodology and Computational Issues

This section describes a complete procedure which combines the pMOR approaches with the Block Structure Preserving reduction in order to generate reduced yet equivalent models in the presented scenario.

Inside the pMOR realm, the moment matching algorithms based on single point expansion may not be able to capture the complete behaviour along the large frequency range required for common RF systems, and may lead to excessively large models if many parameters are taken into account. Therefore the most suitable techniques for the reduction seem to be the multipoint ones. Among those techniques, Variational PMTBR [13] offers a reliable framework with some interesting features that can be exploited, such as the inclusion of probabilistic information and the trade off between size and error, which allows for some control of the error via analysis of the singular values related to the dropped vectors. On the other hand, it requires a higher computational effort than the multi-dimensional moment matching approaches, as it is based on multidimensional sampling schemes and Singular Value Decomposition (SVD), but the compression ratio and reliability that it offers compensates this drawback. The effort spent in the generation of such models can be amortized when the reduced order model generated is going to be used multiple times. This is usually the case of parametric models,

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Algorithm I: Efficient BSP pMOR for EM Models

Starting from a Block Structured System C, G, B, L with K blocks and depending on a parameter set, λ:

1: Precondition of matrices for numerical accuracy
2: Generate the required sample vectors z_j in the $\{s \times \lambda\}$ space
3: Split the z_j vectors row-wise into K blocks
4: For each block, apply SVD on the set of vectors
5: For each block, drop vectors whose singular values fall below a desired tolerance, and keep the rest in a block projector V_i
6: Generate a global projector V with V_i in its diagonal blocks
7: Perform a congruent transformation with V on C, G, B, L

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as the designer may require several evaluations for different parameter sets (e.g. in the case of Monte Carlo simulations, or optimization steps). Furthermore, this technique offers some extra advantages when combined with block structured systems [14], such as the block-wise error control with respect to the global input-output behaviour, which can be applied to improve the efficiency of the reduction. This means that each block can be reduced to a different order depending on its relevance in the global response.

An important point to recall here is that the block division may not reflect different sub-domains. Different subdivisions can be done to address different hierarchical levels. For instance, it may be interesting to divide the complete set in sub-domains connected by hooks, which generates a block structured matricial representation. But inside each block corresponding to a sub-domain, another block division may be done, corresponding either to smaller sub-domains or to a division related to the different kind of variables used to model each domain (for example, in a simple case, currents and voltages). This variable related block structure preservation has already been advocated in the literature [7] and may help the synthesis of and equivalent SPICE-like circuit [15] in the case that is required.

Figure 2 shows a more intuitive depiction of the previous statements, in which a two domain example is shown with its hierarchy, and each domain has also some inner hierarchy related to the different kind of variables (in this case, voltages and currents, but it can also be related to the electric and magnetic variables, depending on the formulation and method used for the generation of the system matrices).

The proposed flow (see Algorithm I) starts from a parametric state-space descriptor, such as (2), which exhibits a multi-level hierarchy, and a block parametric dependence (as different parameters may affect different sub-domains). The matrices of size n have K domains, each with size n_i, $n = \sum_i n_i$. For instance, for the static part,

$$G = \begin{bmatrix} G_{11}(\lambda_{\{11\}}) & \dots & G_{1K}(\lambda_{\{1K\}}) \\ \vdots & \ddots & \vdots \\ G_{K1}(\lambda_{\{K1\}}) & \dots & G_{KK}(\lambda_{\{KK\}}) \end{bmatrix}, \quad (4)$$

where $\lambda_{\{ij\}}$ is the set of parameters affecting $G_{ij} \in \mathbb{R}^{n_i \times n_j}$. Then we perform the multidimensional sampling, both in the frequency and the parameter space. For each point we generate a vector z_j,

$$z_j = (C(\lambda_j)s_j + G(\lambda_j))^{-1} B, \quad (5)$$

where $C(\lambda)$ and $G(\lambda)$ are the global matrices of the complete domain, with n degrees of freedom (dofs). To generate the vector $z_j \in \mathbb{R}^{n \times m}$, with m the number of global ports, we can apply a direct procedure, meaning a factorization (at cost $O(n^\beta)$, with $1.1 \leq \beta \leq 1.5$ for sparse matrices) and a solve (at cost $O(n^\alpha)$, with $1 \leq \alpha \leq 1.2$ for sparse matrices). In cases when a direct method may be too expensive, an iterative procedure (e.g. GMRES) may be used.

The choice of the sampling points may be an issue, as there is no clear scheme or procedure that is known to provide an optimal solution. However, as stated in [13], the accuracy of the method does not depend on the accuracy of the quadrature (and thus in the sampling scheme), but on the subspace generated. For this reason, a good sampling scheme is to perform samples in the frequency for the nominal system, and around these nominal samples, perform some parametric random sampling in order to capture the vectors that the perturbed system generates. The reasoning behind this scheme is that for small variations, such as the ones resulting from process parameters, the subspace generated along the frequency is generally more dominant than the one generated by the parameters. Furthermore, under small variations, the nominal sampling can be used as a good initial guess for an iterative solver to generate the parametric samples. For the direct solution scheme, to generate P samples (and thus Pm vectors) for the global system has a cost of $O(Pn^\alpha + Pn^\beta)$. Note that since m is the number of global (or external) ports, the number of vectors is smaller than if we take all the hooks into account.

The next step is the orthonormalization, via SVD, of the Pm vectors for generating a basis of the subspace in which to project the matrices. Here an **independent basis** V_i, $i \in \{1 \dots K\}$, can be generated for each i-th sub-domain. To this end the vectors z_j are split following the block structure present in the system matrices (i.e. the n_i rows for each block), and SVD is perform on each of these set of vectors, at a cost of $O(n_i(Pm)^2)$, where n_i is the size of the corresponding block, and $n = \sum_i n_i$. For each block, the independent SVD allows to drop the vectors less relevant for the global response (estimated by the dropped singular value ratio, as presented in [13]). This step generates a set of projectors, $V_i \in \mathbb{R}^{n_i \times q_i}$, with $q_i \ll n_i$ the reduced size for the i-th block of the global system matrix. These projectors can be placed in the diagonal blocks of an overall projector, that can be used for reducing the initial global matrices to an order $q = \sum_i q_i$. This block diagonal projector allows a block structure (and thus sub-domain) preservation, increasing the sparsity of the ROM with respect to that of the standard projection. This sparsity increase is particularly noticeable in the case of the sensitivities (if a Taylor Series is used as base representation), as the block parameter dependence is maintained (e.g. in the static matrix)

$$\hat{G}_{ij}(\lambda_{\{ij\}}) = V_i^T G_{ij}(\lambda_{\{ij\}}) V_j. \quad (6)$$

The total cost for the procedure can be approximated by

$$O(Pn^\alpha + Pn^\beta + \textstyle\sum_i n_i(Pm)^2). \quad (7)$$

IV. Simulation Results

A. Spiral over N-Well

This is an industrial example, composed by a square integrated spiral inductor over a N-Well (See Figure 3). In this case, no parameter is taken into account, but the domain is divided into two sub-domains. The first one, the top one, includes the air, the layers in which the spiral is included, and the N-Well within the upper part of the substrate. The second sub-domain, the bottom one, includes the lower bulk part of the substrate. FIT [2] is used as EM modeling technique with a total of 50992 dofs. Interactions between both sub-domains are characterized via hooks. Table I shows the characteristics of the original system and the reduction with the proposed methodology. The reduction is carried with BSP PRIMA [8], matching 40 moments, which generates a 160-vector BSP projector, and the proposed BSP PMTBR (no variability is used here), with 30 frequency samples and a relative tolerance of $1e$-3, with generates a 42-dofs top model, and a 26-dofs bottom model. It is important to recall that the reduction procedure is independent of the number of hooks (464). The proposed BSP PMTBR applies different compression ratio to both domains, as they have different relevance in the global response (bottom domain does not have external terminals, and thus only has parasitic effects on the top domain). The frequency results can be seen in Figure 4. PRIMA ROM loses accuracy at high frequencies (due to single point expansion), whereas PMTBR manages to maintain the accuracy with higher compression.

TABLE I
Characteristics of the Examples

Ex	Domain	Dofs	Terminals (EH,MH,IT)	ROM Dofs
A	Top	34595	466 $(257, 207, 2)$	42
	Bottom	16397	464 $(257, 207, 0)$	26
	Complete	50992	2 $(0, 0, 2)$	68
B	Left	785	77 $(42, 34, 1)$	85
	Middle	645	152 $(84, 68, 0)$	90
	Right	785	77 $(42, 34, 1)$	85
	Complete	2215	2 $(0, 0, 2)$	260
C	Var_1	49125	2 $(0, 0, 2)$	142
	Var_2	54977	2 $(0, 0, 2)$	165
	Complete	104102	2 $(0, 0, 2)$	307

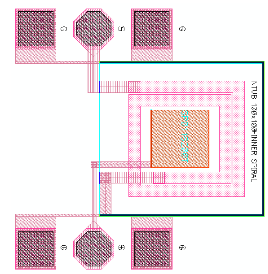

Fig. 3. Layout of the proposed benchmark: Spiral over N-Well.

B. U Coupled

This is a simple test case, which has two U-Shape conductors, and each of the conductors ends represent one port, having one terminal voltage excited (intentional terminal, IT) and one terminal connected to ground. The distance (d) separating the conductors and the thickness (h) of the corresponding metal layer are parameterized. The complete domain is partitioned into three sub-domains, each of them

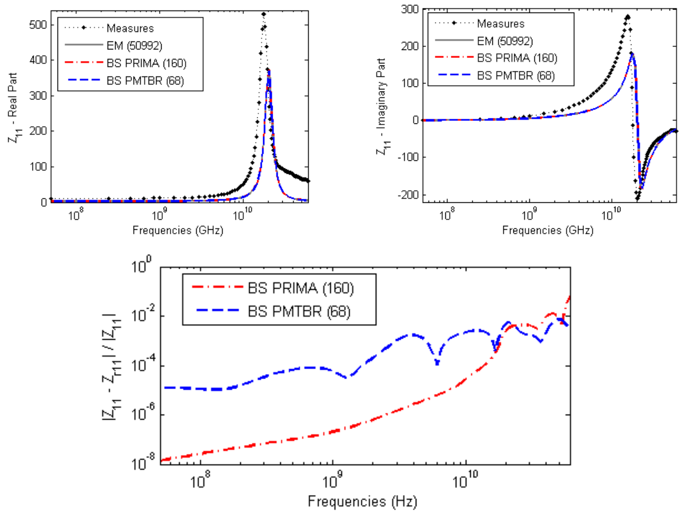

Fig. 4. Z_{11}(Up) and Relative Error (Down). Include Measurements, EM Model, and BSP PRIMA and BSP PMTBR ROMs.

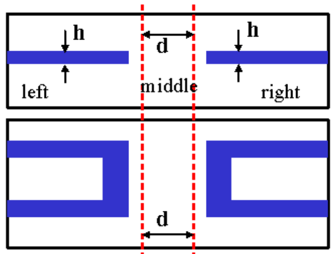

Fig. 5. Topology of the U-Shape: (Up) cross view, (Down) top view. Parameters: distance between conductors, d, and thickness of the metal, h.

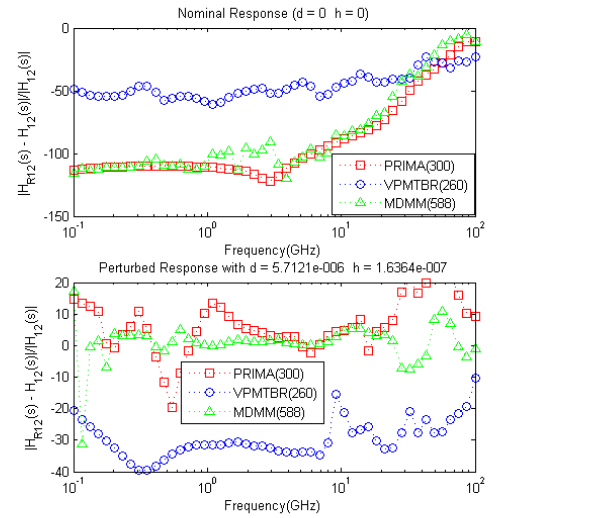

Fig. 6. U Coupled: Relative Error (dB) in $|H_{12}(s)|$ for (Up) the nominal response, and (Down) the perturbed response at a single parameter set. The curves represent: BSP PRIMA, BSP VPMTBR, and BSP MDMM.

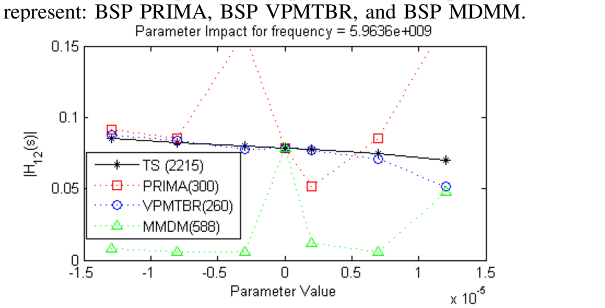

Fig. 7. U Coupled: Variation of $|H_{12}|$ vs. the variation of the parameter d at 59.6 GHz for the original TS and the three BSP ROMs.

connected to the others via a set of hooks (both electric, EH, and magnetic, MH). The domain hierarchy and parameter dependence are kept after the reduction. The Full Wave EM model was obtained via Finite Integration Technique (FIT) [2], and its matrices present a Block Structure that follows the domain partitioning. Table I shows the characteristics of the original system. A clear illustration of the setting is given by Figure 5. Each sub-domain is affected by a parameter. The left and right sub-domains contain the conductors, and thus are affected by the metal thickness h. The middle domain width varies with the distance between the two conductors, and thus is affected by parameter d. For each parameter the first order sensitivity is taken into account, and a first order Taylor Series (TS) formulation is taken as the original system.

For the reduction we apply three techniques. First a Nominal Block Structure Preserving (BSP) PRIMA [8], with a single expansion point and matching 50 moments. This leads to a 100-vector generated basis, that after BSP expansion produces a 300-dofs Reduced Order Model (ROM). A BSP procedure coupled with a Multi-Dimensional Moment Matching (MDMM) approach [11]. The basis will match 40 moments with respect to the frequency, and 30 moments with respect to each parameter. The orthonormalized basis has 196 vectors, that span a BSP ROM of size 588. And the proposed BSP VPMTBR, with 60 multidimensional samples and a relative tolerance of $1e$-3 for each block. This process generates different reduced sizes for each block: 85, 90 and 85, with a global size of 260. Figure 6 shows the relative error in the frequency transfer function at a parameter set point for the three ROM w.r.t. the Taylor Series. PRIMA and MDMM approaches fail to capture the behaviour with the order set, but the proposed approach performs much better even for a lower order. Figure 7 shows the response change with the variation of parameter d at a single frequency point (Parameter Impact). PRIMA and MDMM only present accuracy for the nominal point, whereas the proposed method maintains the accuracy for the parameter range.

C. Double Spiral

This is an industrial example, composed by two square integrated spiral inductors in the same configuration as the previous example (See Figure 8). The complete domain has two ports, and 104102 Dofs. The example also depends on the same two parameters, the distance d between spirals, and

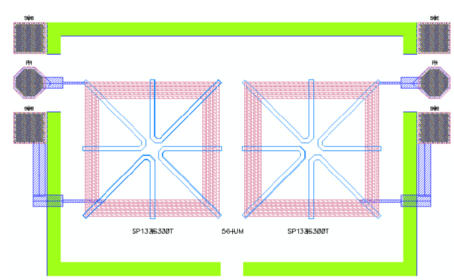

Fig. 8. Layout configuration of the Double Spiral example.

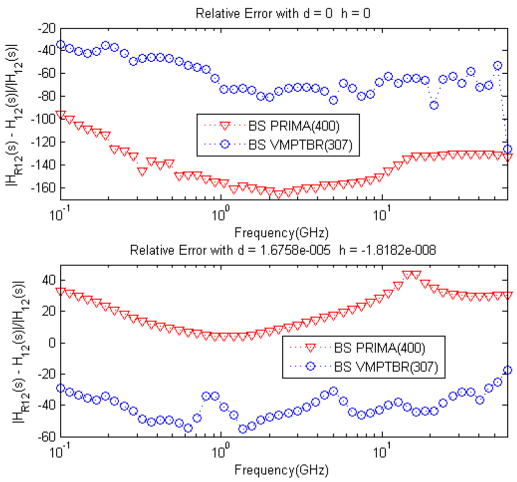

Fig. 9. Double Spiral: Relative Error (dB) in $|H_{12}(s)|$ for (Up) the nominal response, and (Down) the perturbed response at a single parameter set.

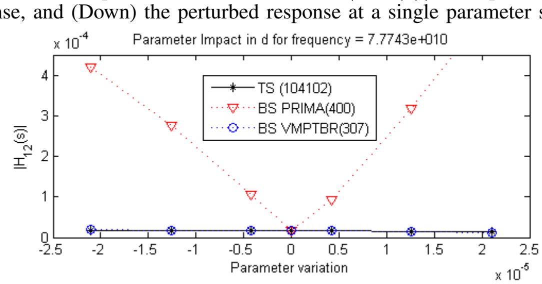

Fig. 10. Double Spiral: $|H_{12}|$ vs. the variation of the parameter d at a frequency point for the original TS and the ROMs: PRIMA, and VPMTBR.

the thickness h of the corresponding metal layer. In this case a single domain is used, but the BSP approach is applied on the inner structure provided by the different variables in the FIT method (electric and magnetic grid). For the reduction, the proposed BSP VPMTBR methodology is benchmarked against a nominal BSP PRIMA (400 dofs) methodology, and compared with the original Taylor Series formulation. The ROM size in this case is 142 and 165 respectively for the blocks. The results are presented in figures 9 and 10. Figure 9 shows the frequency relative error of the ROMs with respect to the original Taylor Series. PRIMA, although accurate for the nominal response, fails to capture the parametric behaviour, whereas the proposed method succeeds in modeling such behaviour. This is also the conclusion that can be drawn from the parameter impact in Figure 10.

V. Conclusions

This paper presents a complete procedure for efficient generation of parametric reduced order models of passive systems. Starting from the electromagnetic description of their behaviour, the modeling techniques and ensuing reduction are detailed, leading to small systems amenable to be included inside simulation environments for coupled analysis with other linear and non-linear devices. The methodology can be combined with any EM modeling technique, including those with parametric descriptions, and takes advantage of the hierarchical information provided by either the topology of the domain or the EM description (in particular, from divide and conquer approaches that lead to sub-domain division and connection by means of hooks).

Noticeable advantages are different compression order for each block based on its relevance in the global behavior, higher degree of sparsification of the nominal matrices, and in particular, of the sensitivities, and the maintenance of the block domain hierarchy and block parameter dependence after reduction, which can lead to simulation advantages.

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