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Author(s):	L.M. Silveira, J. Fernández , D. Ioan, G. Ciuprina, W. Schilders, N.v.d. Meijs, K-J.v.d. Kolk
Affiliation(s):	INESC-ID, PUB, NXP, TUD
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Table of Contents

Abstract

This deliverable is a report on the activities carried out in Task 3.2 of the CHAMELEON-RF project. In this task we seek to develop methods that will allow us to take advantage of the 2-level hierarchy envisioned in CHAMELEON-RF and pursue more efficient reduction of individual models, while taking their coupling into account. That hierarchy depicts the structure of the systems to be analyzed and represents the description of the compact models augmented with connectors representing the interaction with the surrounding environment. In this deliverable the results of the research in the application of Model Order Reduction techniques to Interconnected Sets of Compact Models are presented. These results include a short discussion of existing techniques as well as several extensions over the existing literature, needed for the aims of CHAMELEON-RF. These extensions are related to the ability to work with more complex cases such as multi-input multi-output systems and techniques for the preservation of physical system properties such as passivity. The activities performed include also the development of standards and procedures for the treatment of these interconnected systems by maintaining, as much as possible, the hierarchy and structure of the complete RF block.

1. Introduction

According to the Description of Work (DOW), the main objectives of CHAMELEON-RF's WP3 ("Model Order Reduction for Massively Coupled Systems including parameterisation") are:

1. To develop robust and efficient methods for order reduction of parameterised system descriptions generated in WP2, which can take into account variability induced by lithography or process variations as well as changing operating conditions.
2. To investigate existing order reduction techniques and to develop new algorithms that are applicable to massively coupled system. Such methods must enforce required physical properties and guarantee the generation of realizable simulation models.
3. To investigate techniques aimed at efficient simulation of coupled circuit-interconnect system containing large parasitic descriptions.

The first issue is dealt with in Task 3.1 which is central in the study of algorithmic techniques for reduced systems taking into account the variability, i.e. Parametric Model Order Reduction.

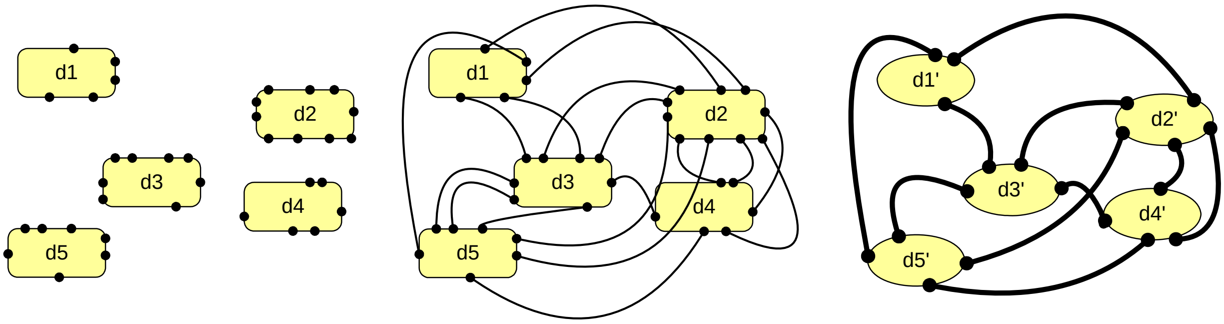
The second issue raised is related to the fact that most of the systems to be analyzed are prone to be affected by a large number of couplings and connections, which will result in a high number of ports. Task 3.3 has been setup to research and improve analysis under such conditions.

The third and last goal is related to the topic of this document and is also the central issue in Task 3.2, namely the study of algorithms for the analysis and order reduction of interconnected sets of compact models, that is the reduction of systems taking into account their interconnections and couplings.

New coupling mechanisms, including EM field coupling, are becoming too strong to be neglected and the behaviour of the devices in the RF regime can be influenced by neighbouring structures. The key to analyzing such components is the recognition that devices, both active and passive, can no longer be treated in isolation and complete RF blocks must be considered as one entity, and be treated as such by the design automation tools. Today, it is not possible to perform such analyses of complete RF blocks. For this reason, we envisage a new kind of hierarchical modelling procedure that will augment compact models of the basic (active and passive) devices with connectors (a kind of "hooks") which will allow interaction with and representation of their environment. If we have the compact models with connectors, we will subsequently determine the coupling among these elements, and after that we will massage the resulting equivalent circuit model into another model, which is still accurate enough but might be much simpler.

But the models generated during the extraction process may prove to be too large so that the computational effort spent in simulating these models may be unacceptable. Model Order Reduction (MOR) techniques have been proposed as potential solutions to this problem. Model order reduction is a well-established methodology for compressing information resulting from the detailed description of system behaviour, thus enabling the characterization of their effect in system behaviour. Such techniques are now routinely used to capture the electromagnetic interference effects of interconnect and packaging structures in system behaviour. In a naive sense, the main idea behind model order reduction is quite simple: one seeks to generate compressed circuit descriptions or system formulations that capture the essence of input-output terminal behaviour in a model of reduced size. Even though this is a simple concept, the actual techniques used to achieve this goal are quite sophisticated. Furthermore, MOR techniques are usually applied to isolated systems, and the models obtained are then linked. This is no longer an acceptable approach due to the above mentioned interactions between interconnected systems. We must now search for effective ways to reduce the complexity of systems that may exhibit strong, relevant connections.

The simplest solution is to reduce the system composed from the various models generated by directly applying the standard MOR techniques to it, i.e. to reduce the global system, sometimes referred to as the flat representation. This is not generally a good approach, because the system obtained by joining the different models could be very large, and it may not be possible to efficiently apply reduction techniques to it. Furthermore, if the global system is obtained and reduced with the existing MOR techniques, some useful and necessary information from the extraction step may be lost. In particular any structural information describing the hierarchy of interconnections is discarded and lost.



(a) (b) (c)

Figure 1- Illustration of our proposed modelling and model order reduction procedures: (a) shows compact models of devices, such as transistors or inductors, which are "equipped" with connectors to account for the interactions, which is added in (b). This overall model is transformed into a simpler but still accurate model using model order reduction procedures in (c).

The objective of Task 3.2, "Order Reduction of Interconnected Sets of Compact Models", is to address this problem and to pinpoint acceptable solutions that allow us to use relevant information received from the extraction steps and prevent the full flattening of the model information. Instead, we seek to take advantage of the 2-level hierarchy previously mentioned, consisting of compact models augmented with connectors representing the interaction with the surrounding environment, and pursue more efficient reduction of individual models, while taking their coupling into account. Pictorially, the idea is to obtain the compact models as shown in Figure 1(a), and some information about the interconnections between them, as shown in Figure 1(b). The reduction is then performed on the individual models by applying techniques that maintain the hierarchy and the structure of the complete system as much as possible, as shown in Figure 1(c), so that the global input-output terminal behaviour is accurately matched.

To this end, techniques for handling multiply interconnected systems have been studied as well as realization issues and potential requirements on the model dictated by the need to make them simulation-friendly. Such techniques must guarantee the relevant model properties are retained, but must also, as much as possible, keep the models sparse and their connectivity small, since that directly affects the number of elements in the realization and ultimately the simulation cost. Fortunately, hierarchical model order reduction is not a novel theme and considerable research effort has been devoted to the topic, which we hope to leverage upon in order to reach our goals.

In this document a complete review of the research carried out so far in pursuing the objectives of T3.2 are presented. First, in Section 3, an overview of the requirements and the motivations of this work will be presented, followed in Section 3 by a review of the most suitable Model Order Reduction techniques. Later, in Sections 4 and 5, the two most promising approaches pursued will be described, along with the formulation and the MOR techniques more suitable for T3.2 and the CHAMELEON-RF objectives. Finally, we will draw some conclusions in Section 6.

2. Requirements for Reduction of Interconnected Sets of Compact Models

CHAMELEON-RF envisages the development of prototyping tools and procedures for the simulation of next generation IC designs. Inside WP3, the main task is to reduce the size of the (passive) compact models resulting from the previous extraction steps, so that these reduced order models accurately encapsulate the original behaviour, and are suitable for inclusion in standard simulation tools. However, this reduction can no longer be applied blindly by direct application of the standard reduction techniques to the individual blocks.

In fact, the performance of each device in the circuit is strongly affected by the environmental situation surrounding it. In other words, the response of each circuit block depends not only on its own physical and electrical characteristics, but to a great extent also on its positioning in the IC, i.e. on the devices to which it is connected. The treatment and modelling of these environmental effects is not a goal of the current task. On the other hand, the reduction of the models by taking into account the physical and electromagnetic connections to the rest of the circuit is exactly the target of the next study.

First of all, we should point out that the aim of the current work is to investigate Model Order Reduction (MOR) techniques that are applicable to interconnected sets of compact **electrical and passive** models. This means that we are not working with generic systems without physical basis, but with systems which are mathematical abstractions that represent models of electrical circuits. For this reason, we cannot perform any reduction without taking into account the fact that the generated reduced order model must maintain some relevant physical properties of the original model beyond the input-output behaviour. This includes important properties such as stability and passivity. Therefore, the techniques to be discussed will have take these restrictions into account.

The main techniques in MOR are geared towards the reduction of state space linear time-invariant systems, generally representing some formulation of a physical system of the result of some previous extraction step. These systems represent the devices as a system where the output is related to the input via some inner "states" in a differential algebraic system:

$$\begin{aligned} C\cdot\dot{x}(t) + G\cdot x(t) &= B\cdot u(t) \\ y(t) &= L\cdot x(t) + D\cdot u(t) \end{aligned} \qquad \textbf{(1)}$$

Within CHAMELEON-RF, the systems to be treated are models with several inputs and outputs, i.e. Multiple Input Multiple Output (MIMO) systems, related to the rest of the circuit models through physical connections (i.e. a wire or metal line) or through electromagnetic couplings (i.e. "hooks"). From the viewpoint of WP3, there is no mathematical difference between these two kinds of connection, since both the terminals as well as the "hooks" are treated as generic network ports. For this reason it is expected that each of the models representing a device will have many inputs and outputs, and this fact means that the techniques used must be able to handle this kind of system representation.

Techniques for handling systems described as in $$\begin{aligned} C\cdot\dot{x}(t) + G\cdot x(t) &= B\cdot u(t) \\ y(t) &= L\cdot x(t) + D\cdot u(t) \end{aligned}$$ (1) are well developed, and there are solid algorithms. The novelty of the CHAMELEON-RF approach is that each system is no longer reduced in isolation. As was previously said, the main target of a generic MOR step is to match the input-output behaviour of the system by maintaining some physical properties. From that standpoint, the easiest approach when handling a set of compact models is to consider the global system resulting from the interconnection of the various sub-systems (i.e. the stand-alone models in system representation), and to perform an order reduction step to the global system. In this setting, a global Reduced Order Model (ROM), which captures in a very accurate way the input-output response of the set of interconnected systems, is obtained. However, as previously mentioned, this is not the approach envisioned in this project mainly for two reasons: First, the global system is usually very large, and to perform a direct MOR over it is very consuming in time and computer resources. Second, if a direct MOR step were performed, the hierarchy and the structure of the interconnection are lost.

The approach pursued in this project, whenever possible is to work with each subsystem in a stand alone mode. This means applying MOR techniques to each subsystem individually, without, as much as possible, taking the rest of the sub-systems into account. The subsystems are smaller than the global interconnected system, and in a MOR scheme usually only the internal states of the system are reduced, so the inputs and outputs remain invariant. For these reasons the reduction techniques applied to the isolated systems do not suffer from the above mentioned problems. However, in spite of the fact that such a procedure allows us to retain the hierarchy and structure of the interconnection unaltered, this is also not the optimal path to achieve our target.

In fact,if we perform MOR separately on each subsystem we capture the subsystem input-output behaviour, but not the global system transfer function. Some specific behaviour of the global system may be not accurately matched, or it is possible that in some cases excessive effort is spent in retaining a good model of some particular behaviour of a particular subsystem whose response does not have an important effect in the global system. In other words, direct reduction of the individual sub-systems, in spite of the fact that it allows one to locally taylor the reduction effort, is not necessarily the best approach when one is really interested only in the behaviour of the global system as a whole. Consider for instance a description of a set of interconnected sub-systems,which represent a transceiver. We may have some sub-systems whose response has considerable energy at certain frequencies beyond the band of interest. If one tries to obtain a model for that sub-system in isolation, considerable effort is required to capture that behaviour which is later lost when filtered out by another sub-system in the interconnect block. This is unnecessary, since we can reduce these sub-systems for capturing the global input-output with much less effort than in the case of reducing each sub-system without taking into account the rest.

Both approaches described have therefore their pros and cons. The target of Task 3.2 is to research, develop and test procedures and methodologies that help us achieve a more efficient reduction of a set of compact models leading to an accurate model of the whole system but without breaking the inherent structure and hierarchy of the interconnection, properties that will provide us with some advantages when simulating the complete circuit. The key idea to achieve this is to reduce each of the subsystems separately with techniques that capture the global input-output behaviour of the interconnected system, but allow us to rebuild the structure of the interconnection after reduction. This kind of technique would also provide additional advantages, for instance when reusing some already reduced sub-systems.

Furthermore, the “hook” effects are not only inside each sub-system, they affect the nearby devices, and may have long range effect. A more efficient treatment of these effects may be performed if we contemplate the whole system.

3. Background in Model Order Reduction

The two most common techniques for Model Order Reduction are based on either truncated balanced realization or projection schemes. In the following we review both, describing some of the particulars of each of these.

3.1 Introduction to Balanced Truncation Techniques

Given a state-space model in descriptor form

$$\begin{aligned} E\cdot\dot{x}(t) &= A\cdot x(t) + B\cdot u(t) \\ y(t) &= L\cdot x(t) + D\cdot u(t) \end{aligned} \qquad \textbf{(2)}$$

where $E \in \Re^{nxn}$, $A \in \Re^{nxn}$, $B \in \Re^{nxm}$, $L \in \Re^{pxn}$, $D \in \Re^{pxm}$ are the state space matrices, $u(t) \in \Re^{m}(t)$ is the vector of inputs, $y(t) \in \Re^{p}(t)$ is the vector of outputs and $x(t) \in \Re^{n}(t)$ is the vector of inner states. For simplicity we assume that $E = I \in \Re^{nxn}$, where I is the identity matrix, as this assumption simplifies the computational procedure without loss of generality.

The Balanced Truncation approach, or Truncated Balanced Realization (TBR) **[2],[3],** is centered around the information obtained from the Controllability Gramian P, which can be obtained by solving the Lyapunov equation

$$A\cdot P + P\cdot A^{T} = -B\cdot B^{T} \qquad \textbf{(3)}$$

The Observability Gramian Q can be obtained by solving the dual Lyapunov equation

$$A^{T}\cdot Q + Q\cdot A = -L^{T}\cdot L \qquad \textbf{(4)}$$

Under a similarity transformation of the state-space model

$$A \to T^{-1}\cdot A\cdot T\text{, } B \to T^{-1}\cdot B\text{, } L \to L\cdot T\text{,} \qquad \textbf{(5)}$$

the input-output properties of the state-space model, such as the transfer function, are invariant. However, the Gramians are not invariant as they vary under the transformation

$$P \to T^{-1}\cdot P\cdot T^{-T}\text{, } Q \to T^{T}\cdot Q\cdot T \qquad \textbf{(6)}$$

One of the key facts of the TBR procedure is that the eigenvalues of the product of the Gramians $P\cdot Q$ do not change. These Hankel singular values contain useful information about the input-output behaviour of the system. In particular, the “small” eigenvalues correspond to internal states that have a weak effect on the input-output response of the system and are, therefore, close to non-observable, non-controllable or both.

The second key fact is that the Gramians are transformed under congruence, and any two symmetric matrices can be simultaneously diagonalized by an appropriate congruence transformation. So it is possible to find a similarity transformation T that leaves the state-space system dynamics unchanged, but transforms the Gramians into $\tilde{P}$ and $\tilde{Q}$ equal and diagonal. So in these new coordinates we may apply the partition

$$\tilde{P} = \tilde{Q} = \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \qquad \textbf{(7)}$$

where Σ_1 is related to the "strong" states which have relevant effect on the input-output behaviour and Σ_2 is related to the "weak" ones with small effect on the input-output response. The transformed matrices can be partitioned in the same way

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \quad \tilde{L} = \begin{bmatrix} \tilde{L}_1 & \tilde{L}_2 \end{bmatrix}, \qquad \textbf{(8)}$$

and we can truncate the system by retaining the part that has a "stronger" effect on the input-output behaviour, and deleting the related to "weaker" states. This way we obtain a reduced system of size $q << n$

$$\begin{aligned} \tilde{E} \cdot \dot{x}(t) &= \tilde{A} \cdot \tilde{x}(t) + \tilde{B} \cdot u(t) \\ y(t) &= \tilde{L} \cdot \tilde{x}(t) + \tilde{D} \cdot u(t) \end{aligned} \qquad \textbf{(9)}$$

where $\tilde{E} \in \Re^{qxq}$, $\tilde{A} \in \Re^{qxq}$, $\tilde{B} \in \Re^{qxm}$, $\tilde{L} \in \Re^{pxq}$, $\tilde{D} \in \Re^{pxm}$ are the state space matrices and $u(t) \in \Re^m(t)$ is the vector of inputs, $y(t) \in \Re^p(t)$ is the vector of outputs and $\tilde{x}(t) \in \Re^q(t)$ is the vector of inner states. Such a model is a reduced order representation of the original system, retaining by construction the most relevant input-output behaviour. One of the main advantages of the TBR procedure is that the procedure provides *a-posteriori* error bounds on the truncation **[2]**, **[3].** The existence of such bounds is quite relevant as it provides a clear way to exchange accuracy for simplicity in the representation. Unfortunately, the balancing procedure is expensive, requiring the solution of the two Lyapunov equations and the eigenvalue decomposition of the transformed Gramians. For these reasons, TBR is usually not used for large scale systems.

3.2 Introduction to Krylov Projection Techniques

Given a state-space model in descriptor form

$$\begin{aligned} C \cdot \dot{x}(t) + G \cdot x(t) &= B \cdot u(t) \\ y(t) &= L \cdot x(t) + D \cdot u(t) \end{aligned} \qquad \textbf{(10)}$$

where $C \in \Re^{nxn}$, $G \in \Re^{nxn}$, $B \in \Re^{nxm}$, $L \in \Re^{pxn}$, $D \in \Re^{pxm}$ are the state space matrices, $u(t) \in \Re^m(t)$ is the vector of inputs, $y(t) \in \Re^p(t)$ is the vector of outputs and $x(t) \in \Re^n(t)$ is the vector of inner states. We can suppose $D = [0]$ without loss of generality.

Krylov techniques are implicit Moment Matching approaches, i.e. are based on the construction of a projection matrix whose columns are obtained from the block moments of the Transfer Function in the frequency domain. Let us define the matrices

$$\begin{aligned} A &= -G^{-1} \cdot C \\ R &= G^{-1} \cdot B \end{aligned} \qquad \textbf{(11)}$$

From the expression in **(10)** with unit impulses at the ports, the Laplace Transform can be applied to yield the Transfer Function

$$H(s) = L \cdot (G + s \cdot C)^{-1} \cdot B \tag{12}$$

Using **(11)**, the transfer function can be reformulated as

$$H(s) = L \cdot (I - s \cdot A)^{-1} \cdot R \tag{13}$$

where $I \in \Re^{nxn}$ is the identity matrix.

The Block Moments of the transfer function can be defined as the coefficients of the Taylor Series expansion around frequency $s = 0$

$$H(s) = M_0 + M_1 \cdot s + M_2 \cdot s^2 + \ldots \tag{14}$$

where $M_i \in \Re^{nxn}$ are the block moments that can be computed by

$$M_i = L \cdot A^i \cdot R \tag{15}$$

The Block Krylov Subspace generated by matrices $A \in \Re^{nxn}$ and $R = \begin{bmatrix} r_1 & r_2 & \ldots & r_m \end{bmatrix} \in \Re^{nxm}$ is defined as

$$\begin{aligned} &Kr(A, R, q) = colsp\left[R, A \cdot R, A^2 \cdot R, \ldots, A^{k-1} \cdot R, A^k \cdot r_1, A^k \cdot r_2, \ldots A^k \cdot r_l\right] \\ &k = \left\lfloor \frac{q}{m} \right\rfloor, \quad l = q - k \cdot m. \end{aligned} \tag{16}$$

This means that the Krylov Subspace spans the combination of moment vectors generated by the different sources in the circuit. So any basis of this subspace can be used to project the system matrices onto it, so the k moments of the original transfer function are matched by the projected system, i.e. the reduced system.

The Krylov subspace can be efficiently generated via robust and well developed algorithms such as the Block Arnoldi Algorithm or the Lanczos Process **[6],[7],[8].** The most popular Krylov projection algorithm is the PRIMA algorithm **[6]**, where an orthonormal matrix V that spans the Krylov subspace of order q is built and applied via a congruence transformation over the system matrices for obtaining a Reduced Order Model (ROM) of size q that matches the first k block moments of the original transfer function. This projection is performed in the following way

$$\begin{aligned} \hat{C} &= V^T \cdot C \cdot V \\ \hat{G} &= V^T \cdot G \cdot V \\ \hat{B} &= V^T \cdot B \\ \hat{L} &= L \cdot V \end{aligned} \tag{17}$$

where $\hat{C} \in \Re^{qxq}$, $\hat{G} \in \Re^{qxq}$, $\hat{B} \in \Re^{qxm}$, $\hat{L} \in \Re^{pxq}$ are the reduced order system matrices and $V \in \Re^{nxq}$ is the projector, i.e. the orthonormal basis for the Krylov subspace. Finally, the reduced system can be expressed via its transfer function:

$$\hat{H}(s) = \hat{L} \cdot (\hat{G} + s \cdot \hat{C})^{-1} \cdot \hat{B} \tag{18}$$

The PRIMA algorithm (and in general any algorithm that applies congruence projection) shows another advantage. Necessary and sufficient conditions for the system transfer function in **(18)** to be passive are:

- $\hat{H}(s^*) = \hat{H}^*(s)$ for all complex s, where * is the complex conjugate operator.

- $\hat{H}(s)$ is a positive matrix, that is, $z^{*T} \cdot \left(\hat{H}(s) + \hat{H}^T(s^*)\right) z \geq 0$ for all complex values of s satisfying $\mathrm{Re}(s) > 0$ for any complex vector z.

Any congruence transformation applied to the system matrices satisfies the previous conditions if the original system satisfies them, and so preserves the passivity of the system if the following conditions are true:

- The system matrices are positive definite, $C, G \geq 0$

- $B = L^T$

These conditions are sufficient, but not necessary. They are usually satisfied in the case of electrical circuits, which makes congruence-based projection methods very popular in circuit domains.. For further details on passivity see Section 0**.**

A refinement in the previous approach yields the so called Rational, or Multipoint, Krylov Methods **[9]**. This approach consists in obtaining several Krylov subspaces at several frequency points $s = s_j$ and building a projection matrix from the orthonormal basis of the join of all those subspaces. The Krylov subspace at the chosen frequency $s = s_j$ is obtained by spanning the block moments at that frequency, i.e. the coefficients of the Taylor Series expansion of the Transfer Function around frequency $s = s_j$. Let us define:

$$\begin{aligned} A_j &= -(G + s_j \cdot C)^{-1} \cdot C \\ R_j &= (G + s_j \cdot C)^{-1} \cdot B \end{aligned} \qquad \textbf{(19)}$$

so the Krylov Subspace of the first k block moments of the transfer function at frequency $s = s_j$ is:

$$Kr\left(A_j, R_j, q\right) = colsp\left[R_j, A_j \cdot R_j, A_j^{\,2} \cdot R_j, \ldots, A_j^{\,k-1} \cdot R_j, A_j^{\,k} \cdot r_{j_1}, A_j^{\,k} \cdot r_{j_2}, \ldots A_j^{\,k} \cdot r_{j_l}\right]$$

$$k = \left\lfloor \frac{q}{m} \right\rfloor, \quad l = q - k \cdot m.$$

A drawback of Krylov-based projection technique is the lack of an efficient technique for error control. Error estimators do exist but are seldom used in practice as they are expensive and cumbersome to use. However the techniques are very efficient and generally produce very good results, which has led to their widespread usage in VLSI settings where reduction of large passive interconnect systems is often required.

3.3 Introduction to PMTBR Model Order Reduction

Poor's Man TBR (PMTBR) **[4]**,**[5]** is a projection MOR technique that exploits the direct relation between the multipoint rational projection framework and the Truncated Balanced Realizations (TBR) **[2]**. This new approach can take advantage of some a priori knowledge of the system properties, and it is based in a statistical interpretation of the system Gramians.

Given a state-space model in descriptor form

$$\begin{aligned} E\cdot\dot{x}(t) &= A\cdot x(t) + B\cdot u(t) \\ y(t) &= L\cdot x(t) + D\cdot u(t) \end{aligned} \quad \textbf{(20)}$$

where $E \in \Re^{nxn}$, $A \in \Re^{nxn}$, $B \in \Re^{nxm}$, $L \in \Re^{pxn}$, $D \in \Re^{pxm}$ are the state space matrices and $u(t) \in \Re^{m}(t)$ is the vector of inputs, $y(t) \in \Re^{p}(t)$ is the vector of outputs and $x(t) \in \Re^{n}(t)$ is the vector of inner states. For simplicity we assume $E = I \in \Re^{nxn}$ where I is the identity matrix, $B = L^T$ and $D = [0]$.

The Gramian X can be computed in the time domain following the expression

$$X = \int_0^\infty e^{A\cdot t}\cdot B\cdot B^T\cdot e^{A^T\cdot t}\cdot dt \quad \textbf{(21)}$$

Applying Parseval’s theorem we can obtain an equivalent frequency based expression for the Gramian

$$X = \int_\infty^\infty (j\cdot\omega\cdot I - A)^{-1}\cdot B\cdot B^T\cdot(j\cdot\omega\cdot I - A)^{-H}\cdot d\omega \quad \textbf{(22)}$$

where superscript H denotes Hermitian Transpose.

The integral in **(22)** can be evaluated via numerical quadrature with a scheme of nodes ω_k and weights w_k by defining

$$z_k = (j\cdot\omega_k\cdot I - A)^{-1}\cdot B \quad \textbf{(23)}$$

An approximation $\breve{X}$ to the Gramian X can be computed as:

$$\breve{X} = \sum_k w_k\cdot z_k\cdot z_k{}^H \quad \textbf{(24)}$$

We can build a matrix Z such that its columns are z_k and another diagonal matrix W with diagonal entries $W_{kk} = \sqrt{w_k}$. From these matrices we can rewrite Equation **(24)** as:

$$\breve{X} = Z\cdot W\cdot Z^H \quad \textbf{(25)}$$

If the quadrature rule is accurate, $\breve{X}$ will converge to X, which implies that the dominant eigenspace of $\breve{X}$ converges to the dominant eigenspace of X. A Singular Value Decomposition (SVD) can be applied over the product $Z\cdot W$

$$Z\cdot W = V_Z\cdot S_Z\cdot U_Z \quad \textbf{(26)}$$

where S_Z is real diagonal, and V_Z and U_Z are unitary matrices. The estimated Gramian can be obtained from

$$\breve{X} = V_Z\cdot S_Z{}^2\cdot V_Z{}^T \quad \textbf{(27)}$$

Hence, the singular vectors in V_Z yield the eigenvectors of $\breve{X}$. If a good quadrature rule has been chosen, V_Z converges to the eigenspace of X and the Hankel Singular Values are obtained directly from the diagonal entries of S_Z. The dominant eigenvectors of V_Z corresponding to the dominant eigenvalues of S_Z can be used as a projection matrix in a congruence transformation over the system matrices for model order reduction. The eigenvalues of S_Z can be used in an a priori error estimation in the way the Hankel Singular Values were used in the TBR procedures for error control.

3.4 Passivity Issues

Here some brief and basic notions of passivity and its importance are given, and will be further related to the projection techniques meant to be applied in the MOR schemes.

A passive system does not deliver energy and a strictly passive system only consumes energy. A system composed by interconnected passive systems is always passive (this closure property does not hold for stable systems). In the case of electrical circuits, described by linear time-invariant systems (as the ones considered here), passivity is equivalent to the positive realness of the system's impedance and admittance functions, and the scattering parameter matrix of a passive system must have singular values no greater than one.

The passivity itself is an inherent property of the physical interconnect network, but when these networks are approximated with compact models, to guarantee a passive model in the simulation is a must. The main reason is that a non-passive system, when excited with appropriate stimuli, may present and unbounded response during simulation, thus violating conservation of energy. Physical systems obviously do not behave this way and therefore care must be taken such that all models of such systems show appropriate behaviour. Ensure that the models are themselves passives is a good step in avoinding such pitfalls. This is applicable also to the MOR techniques, since the reduced order model must maintain the properties of the physical network.

In the projection techniques the application of a congruence transformation on the system matrices guarantees the positive realness of the transfer function given the mentioned properties of such matrices and, hence, the passivity of the reduced system.

4. Block Hierarchical Systems

In this section and the following, we discuss two approaches aimed at generating reduced order models of interconnected sets of systems. The first approach pursued directly explores the global system formulation obtained via the two level hierarchical extraction. This means that we assume a global system representation composed of several sub-systems and a set of connections and couplings between them, entitled "hooks". We assume here that the sub-systems are easily recognized since in our context, the hook-based representation is constructed to show that. In other settings, the individual sub-systems have to be determined or defined (see **[11]** for a discussion of algorithms that accomplish this task) The idea here is to use the structural information that the extraction steps have provided in order to perform a more accurate reduction and at the same time maintain the hierarchy that already exists in the description. Presumably the system representation will consist of dense sub-blocks, corresponding to the tight sub-systems, and some sparser off-diagonal portions related to the hooks or connections. It should be noticed that while the number of "hooks" can be very high, their number is usually much smaller than the number of elements in a typical subsystem. In a mathematical sense, the matrices can be easily split in several blocks, some of them related to the subsystems, and the rest related to the "hooks" between them, as shown in Figure .

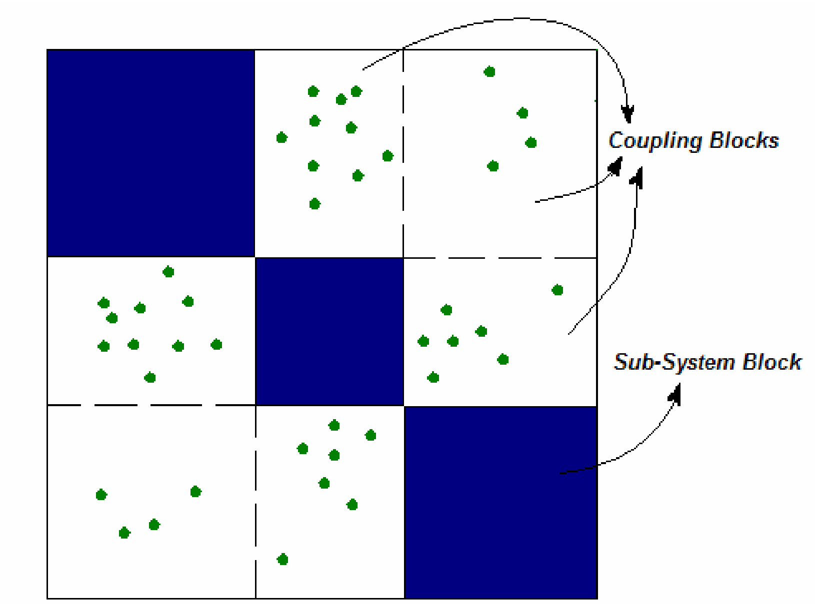


Figure 2. Example of Block Hierarchical System

In the presence of such a formulation and hierarchical information, to apply brute force MOR means to reject precious chances to guide our reduction step. The application of any MOR technique over a system like this will result in a full matrix system, where all the hierarchy would have been lost. In the literature there are some methods **[11],[12]** that preserve the structure of the matrices during the reduction step. These methods were traditionally applied to grid problems, where the maintenance of the relation between the physical system and the model was a must. In the next section some approaches inside this context and oriented towards the aims of CHAMELEON-RF will be discussed.

4.1 Model Order Reduction of Block Hierarchical Systems via Krylov

The most relevant work for our purposes was the one developed in **[11]**. The key idea of the Block Structure Preserving Model Reduction (BSMOR) is to obtain a structured Krylov subspace. The Krylov MOR approaches are based on the computation of the subspace of moments of the transfer function, on which the system matrices are projected under a congruence transformation (usually with the orthonormal basis of such a subspace), so that the obtained matrices are reduced as was previously shown. The BSMOR algorithm states that any base that spans the subspace is a suitable projector for the reduction. The main problem in the BSMOR schema is to find a suitable partition of the complete system into blocks. In the CHAMELEON-RF scheme this is no longer an issue, since the two level hierarchical extraction already provides us with this information. So given a state-space model in descriptor form

$$\begin{aligned} C\cdot\dot{x}(t) + G\cdot x(t) &= B\cdot u(t) \\ y(t) &= L\cdot x(t) + D\cdot u(t) \end{aligned} \qquad \textbf{(28)}$$

where $C \in \Re^{nxn}$, $G \in \Re^{nxn}$, $B \in \Re^{nxm}$, $L \in \Re^{pxn}$, $D \in \Re^{pxm}$ are the state space matrices, $u(t) \in \Re^{m}(t)$ is the vector of inputs, $y(t) \in \Re^{p}(t)$ is the vector of outputs and $x(t) \in \Re^{n}(t)$ is the vector of inner states. We can suppose $D = [0]$ without loss of generality. The system is supposed to have some hierarchical structure so that

$$G = \begin{bmatrix} G_{1,1} & \cdots & G_{1,N} \\ \vdots & \ddots & \vdots \\ G_{N,1} & \cdots & G_{N,N} \end{bmatrix} \quad B = \begin{bmatrix} B_1^{\,T} & \ldots & B_N^{\,T} \end{bmatrix}^T$$

$$C = \begin{bmatrix} C_{1,1} & \cdots & C_{1,N} \\ \vdots & \ddots & \vdots \\ C_{N,1} & \cdots & C_{N,N} \end{bmatrix} \quad L = \begin{bmatrix} L_1 & \ldots & L_N \end{bmatrix} \qquad \textbf{(29)}$$

where $C_{i,j} \in \Re^{n_i x n_j}$, $G_{i,j} \in \Re^{n_i x n_j}$, $B_i \in \Re^{n_i x m}$, $L_i \in \Re^{p x n_i}$

In this way we can obtain the Krylov subspace that spans the combination of k block moment vectors generated by different sources in the circuit

$$Kr(A,R,q) = colsp\left[R, A\cdot R, A^2\cdot R,..., A^{k-1}\cdot R, A^k\cdot r_1, A^k\cdot r_2,...A^k\cdot r_l\right]$$
$$k = \left\lfloor \frac{q}{m} \right\rfloor, \quad l = q - k\cdot m. \tag{30}$$

Any basis that spans this subspace is a suitable projector. Let us denote by $V \in \Re^{nxq}$ the usual orthonormal basis that spans the Krylov Subspace. We can split it as follows in the system structure

$$V = \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = colsp[Kr(A,R,q)] \tag{31}$$

where $V \in \Re^{nxq}$, and $V_i \in \Re^{n_i xq}$

This projector can be restructured so that

$$\tilde{V} = \begin{bmatrix} V_1 & & 0 \\ & \ddots & \\ 0 & & V_N \end{bmatrix} = colsp[Kr(A,R,q)] \tag{32}$$

where $\tilde{V} \in \Re^{nxN\cdot q}$, and a projection on the system matrices is performed via a congruence transformation

$$\begin{aligned} \tilde{C} &= \tilde{V}^T \cdot C \cdot \tilde{V} \\ \tilde{G} &= \tilde{V}^T \cdot G \cdot \tilde{V} \\ \tilde{B} &= \tilde{V}^T \cdot B \\ \tilde{L} &= L \cdot \tilde{V} \end{aligned} \tag{33}$$

where $\tilde{C} \in \Re^{N\cdot qxN\cdot q}$, $\tilde{G} \in \Re^{N\cdot qxN\cdot q}$, $\tilde{B} \in \Re^{N\cdot qxm}$, $\tilde{L} \in \Re^{pxN\cdot q}$ are the reduced order system matrices. Element wise, the reduction can be performed in the following way

$$\begin{aligned} \tilde{C}_{i,j} &= V_i^T \cdot C_{i,j} \cdot V_j \\ \tilde{G}_{i,j} &= V_i^T \cdot G_{i,j} \cdot V_j \\ \hat{B}_i &= V_i^T \cdot B_i \\ \hat{L}_i &= L_i \cdot V_i \end{aligned} \tag{34}$$

It must be noticed that the projection matrix $\tilde{V} \in \Re^{nxN\cdot q}$ used in this reduction context has N times more columns than the original matrix $V \in \Re^{nxq}$. This leads to an N times larger reduced system. However, it should be noticed that for this reason the transfer function

$$\tilde{H}(s) = \tilde{L} \cdot (\tilde{G} + s\cdot\tilde{C})^{-1} \cdot \tilde{B} \tag{35}$$

should match N times more moments of the original one. This is only true in the case of very weak couplings, i.e. very weak and sparse entries in the related matrix block.

On the other hand, this technique maintains the structure of the original system and gives us some flexibility when choosing the size of the reduced model depending on the kind and strength of the "hooks". The projector block V_i is only used in the reduction of the blocks $C_{i,x}$ and $C_{x,i}$, so each reduced block will match k block moments of the original one, and the complete system will be able to match up to $N \cdot k$ block moments of the original complete transfer function in the above mentioned conditions (in the worst conditions, only k block moments are matched).

These methods, here presented for the PRIMA algorithm, are extendable to any projection based technique, and specifically it seems feasible to extend it to the PMTBR framework.

4.2 Passivity in Block Hierarchical Systems MOR

For matrices derived from circuits and in general for most matrices of interest for CHAMELEON-RF, the required sufficient properties for passivity preserving in a congruence transformation scheme as described in Section 4.4 are usually guaranteed, thus in the present study case, if those conditions are met it can be said that the passivity of the complete system is preserved. For the hierarchical reduction schemes below, one should recall that any interconnection of passive systems is also passive. Therefore, if reduction algorithms are applied to individual sub-systems that guarantee passivity of the resulting models, the final interconnection of reduced sub-systems is also guaranteed passive. We will later on in our development make use of this simple property.

5. Interconnected Systems

In the interconnected system approach presented in **]** a complete formalism of the interconnection of several sub-systems to yield a global large scale linear system is presented. In that work, the formalism is presented for standard state-space descriptors in their **ABCD** form, but inside CHAMELEON-RF context it is more convenient to derive such formalism for the **CGBLD** state-space formulation (in other words a descriptor form of the system). This derivation is straightforward and the ensuingdiscussion will be done using this formalism. We can define the Global Transfer Function TF(s) as a Multiple Input Multiple Output (MIMO) system of 'm' inputs and 'p' outputs.

We can formulate the transfer function as the relation between the outputs and the inputs in the frequency:

$$y(s) = TF(s) \cdot u(s) \quad \textbf{(36)}$$

where $y(s)$ and $u(s)$ are respectively the output and the input vectors, with p and m entries respectively, so $TF(s) \in \Re^{pxm}(s)$

A MIMO sub-system is defined in the same way as a transfer function:

$$b_i(s) = T_i(s) \cdot a_i(s) \quad \textbf{(37)}$$

where $T_i(s)$ is the i-th sub-system, $a_i(s)$ is the vector of α_i inputs and $b_i(s)$ is the vector of β_i outputs, so $T_i(s) \in \Re^{\beta_i x \alpha_i}(s)$

$$T_i(s) = L_i \cdot (C_i \cdot s + G_i)^{-1} \cdot B_i + D_i \quad \textbf{(38)}$$

This transfer function corresponds to the state space realization:

$$\begin{aligned} C_i \cdot \dot{x}_i(t) + G_i \cdot x_i(t) &= B \cdot a_i(t) \\ b_i(t) &= L_i \cdot x_i(t) + D_i \cdot a_i(t) \end{aligned} \quad \textbf{(39)}$$

where $C_i \in \Re^{n_i x n_i}$, $G_i \in \Re^{n_i x n_i}$, $B_i \in \Re^{n_i x \alpha_i}$, $L_i \in \Re^{\beta_i x n_i}$, $D_i \in \Re^{\beta_i x \alpha_i}$ are the state space matrices and $a_i(t) \in \Re^{\alpha_i}(t)$ is the vector of inputs, $b_i(t) \in \Re^{\beta_i}(t)$ is the vector of outputs and $x_i(t) \in \Re^{n_i}(t)$ is the vector of inner states.

Figure 3 gives an example of an interconnected system composed of three sub-systems.

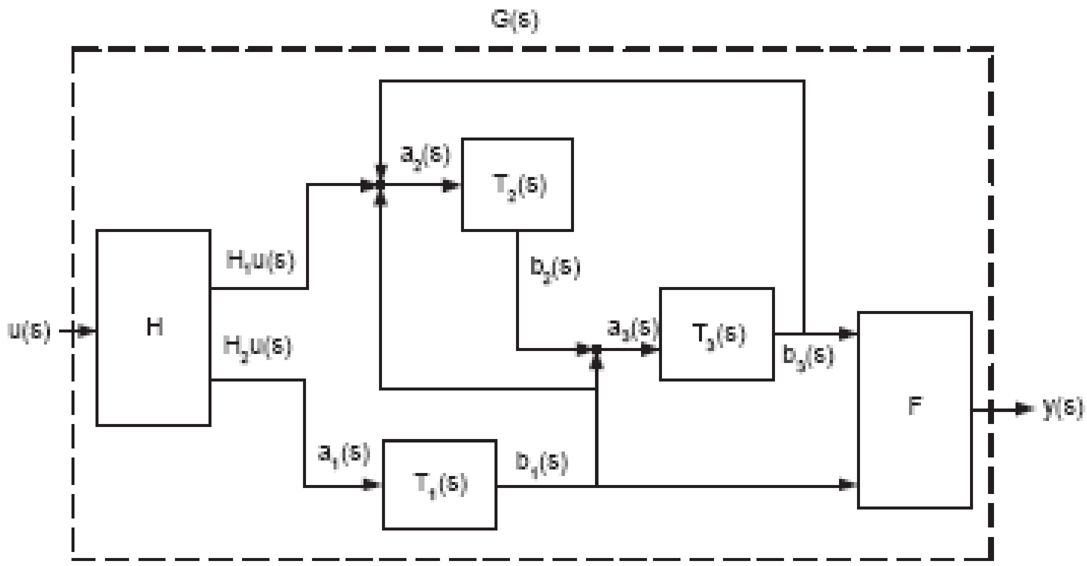


Figure 3. Example of Interconnected System

If there is a set of 'N' interconnected sub-systems, there is a set of matrices, called Interconnected Matrices, which define the relations between the ports (i.e. inputs $a_i(s)$ and outputs $b_i(s)$) of the several interconnected sub-systems to the ports of the rest of sub-systems (i.e. inputs $a_j(s)$ and outputs $b_j(s)$) and the external ports (i.e. inputs $u(s)$ and outputs $y(s)$) of the global system. The generic relation between the set of inputs and outputs is the one that follows:

$$
\begin{aligned}
a_i(s) &= u_i(s) + \sum_{j=1}^{N} K_{i,j} \cdot b_j(s) \\
u_i(s) &= H_i \cdot u(s) \\
y(s) &= \sum_{i=1}^{N} F_i \cdot b_i(s)
\end{aligned} \qquad \textbf{(40)}
$$

where $H_i \in \Re^{\alpha_i xm}$ is the matrix that relates the global input ports, i.e. $u(s)$ to the input ports, i.e. $a_i(s)$, of the i-th sub-system, $F_i \in \Re^{px\beta_i}$ relates output ports of the i-th sub-system, i.e. $b_i(s)$, to the global outputs, i.e. $y(s)$ and $K_{i,j} \in \Re^{\alpha_i x \beta_j}$ relates the j-th sub-system output ports, i.e. $b_j(s)$, to the i-th sub-system input ports, i.e. $a_i(s)$.

In an interconnection of N sub-systems, $T(s)$ can be defined as a diagonal transfer function of N MIMO sub-systems, of size $n = \sum_{1}^{N} n_i$ (the size the number of inner states, i.e. the size of the state vector x(s)), with a set of inputs defined by the vector

$a(s) = \left[a_1(s)^T \quad ... \quad a_N(s)^T \right]^T$, $a(s) \in \Re^{\alpha}(s)$, $\alpha = \sum_1^N \alpha_i$, and a set of outputs defined by the vector $b(s) = \left[b_1(s)^T \quad ... \quad b_N(s)^T \right]^T$, $b(s) \in \Re^{\beta}(s)$, $\beta = \sum_1^N \beta_i$. Rewritting,

$$\begin{aligned} b(s) &= T(s) \cdot a(s) \\ T(s) &= L \cdot (C \cdot s + G)^{-1} \cdot B + D \end{aligned} \quad \textbf{(41)}$$

This transfer function corresponds to the state space realization with a inner state vector $x(t) \in \Re^n(t)$ and $n = \sum_1^N n_i$:

$$\begin{aligned} C \cdot \dot{x}(t) + G \cdot x(t) &= B \cdot a(t) \\ b(t) &= L \cdot x(t) + D \cdot a(t) \end{aligned} \quad \textbf{(42)}$$

Where the state space matrices are

$$C = diag\{C_1 \quad ... \quad C_N\} \in \Re^{nxn},$$
$$G = diag\{G_1 \quad ... \quad G_N\} \in \Re^{nxn},$$
$$B = diag\{B_1 \quad ... \quad B_N\} \in \Re^{nx\alpha},$$
$$L = diag\{L_1 \quad ... \quad L_N\} \in \Re^{\beta xn},$$
$$D = diag\{D_1 \quad ... \quad D_N\} \in \Re^{\beta x\alpha}$$

The global interconnected matrices for the 'N' sub-systems can be defined as:

$$H = \left[H_1^{\ T} \quad ... \quad H_N^{\ T} \right]^T$$

$$F = \left[F_1 \quad ... \quad F_N \right] \quad \textbf{(43)}$$

$$K = \begin{bmatrix} K_{1,1} & \cdots & K_{1,N} \\ \vdots & \ddots & \vdots \\ K_{N,1} & \cdots & K_{N,N} \end{bmatrix}$$

where $H \in \Re^{\alpha xm}$, is the matrix that relates the global input ports, i.e. $u(s)$ to the input ports, i.e. $a(s)$, of the N sub-systems, $F \in \Re^{px\beta}$ relates output ports of the 'N' sub-systems, i.e. $b(s)$, to the global outputs, i.e. $y(s)$ and $K \in \Re^{\alpha x\beta}$ relates the output ports, i.e. $b(s)$, to the input ports, i.e. $a(s)$, of the N sub-systems.

If the relations given in **(50)** for the inputs and outputs are taken into account, a relation between the global outputs $y(s)$ and the global inputs $u(s)$ can be obtained:

$$\begin{aligned} a(s) &= H \cdot u(s) + K \cdot b(s) \\ b(s) &= T(s) \cdot a(s) \\ y(s) &= F \cdot b(s) \end{aligned} \quad \textbf{(44)}$$

These relations result in a formulation for the global transfer function TF(s):

$$
\begin{aligned}
y(s) &= F\cdot\left(I - T(s)\cdot K\right)^{-1}\cdot T(s)\cdot H\cdot u(s) \\
TF(s) &= F\cdot\left(I - T(s)\cdot K\right)^{-1}\cdot T(s)\cdot H
\end{aligned} \qquad \textbf{(45)}
$$

This transfer function corresponds to the next state space realization:

$$
\begin{aligned}
C_G\cdot\dot{x}(t) + G_G\cdot x(t) &= B_G\cdot u(t) \\
y(t) &= L_G\cdot x(t) + D_G\cdot u(t)
\end{aligned} \qquad \textbf{(46)}
$$

Where the state space matrices are $C_G = C$, with $C_G \in \Re^{nxn}$

$G_G = G - B\cdot K\cdot\left(I - D\cdot K\right)^{-1}\cdot L$, with $G_G \in \Re^{nxn}$

$B_G = B\cdot\left(I - K\cdot D\right)^{-1}\cdot H$, with $B_G \in \Re^{nxm}$

$L_G = F\cdot\left(I - D\cdot K\right)^{-1}\cdot L$, with $L_G \in \Re^{pxn}$

$D_G = F\cdot D\cdot\left(I - K\cdot D\right)^{-1}\cdot H$, with $D_G \in \Re^{pxm}$

$u(s) \in \Re^{m}(s)$

$y(s) \in \Re^{p}(s)$

It is possible to show that this formulation is identical to the one introduced in Section 4, if appropriate restrictions are placed on the coupling blocks between sub-circuits, namely bidirectionality is imposed. We will however keep the two formulations separate since it is both instructive and the additional restriction opens new processing possibilities.

5.1 MOR on Interconnected Systems: Balanced Truncation Approaches

Balanced Truncation techniques for MOR have been introduced in the case of single systems. Let us consider now an Interconnected System and the possible application of those techniques to its reduction. Let us define the Interconnected Global System obtained from the interconnection of N sub-systems $T_i(s)$ as given in **(56)**. One can obtain the controllability and observability Gramians of $TF(s)$ by solving the Lyapunov equations:

$$
\begin{aligned}
&A_G\cdot P_G + P_G\cdot A_G^{\,T} + B'_G\cdot B'^{\,T}_G = 0 \quad A_G\cdot P_G + P_G\cdot A_G^{\,T} + B'_G\cdot B'^{\,T}_G = 0 \\
&A_G^{\,T}\cdot Q_G + Q_G\cdot A_G + L'^{\,T}_G\cdot L'_G = 0 \quad A_G^{\,T}\cdot Q_G + Q_G\cdot A_G + L'^{\,T}_G\cdot L'_G = 0
\end{aligned} \qquad \textbf{(47)}
$$

where: $A_G = -C_G^{\,-1}\cdot G_G$

$B'_G = C_G^{\,-1}\cdot B_G$

$$L'_G = L_G$$

The Gramians can be decomposed in the following way:

$$P_G = \begin{bmatrix} P_{1,1} & \cdots & P_{1,N} \\ \vdots & \ddots & \vdots \\ P_{N,1} & \cdots & P_{N,N} \end{bmatrix}$$

(48)

$$Q_G = \begin{bmatrix} Q_{1,1} & \cdots & Q_{1,N} \\ \vdots & \ddots & \vdots \\ Q_{N,1} & \cdots & Q_{N,N} \end{bmatrix}$$

with $P_{i,j} \in \Re^{n_i x n_j}$ and $Q_{i,j} \in \Re^{n_i x n_j}$

We can perform a state space transformation to the realization of $T(s) = diag\{T_1(s) \quad ... \quad T_N(s)\}$:

$$\left(\overline{A}, \overline{B}', \overline{L}'\right) = \left(\Phi \cdot A \cdot \Phi^{-1}, \Phi \cdot B', L' \cdot \Phi^{-1}\right) \quad \textbf{(49)}$$

which yields

$$\left(\overline{A}_G, \overline{B}'_G, \overline{L}'_G\right) = \left(\Phi \cdot A_G \cdot \Phi^{-1}, \Phi \cdot B'_G, L'_G \cdot \Phi^{-1}\right)$$

(50)

$$\left(\overline{P}_G, \overline{Q}_G\right) = \left(\Phi \cdot P_G \cdot \Phi^T, \Phi^{-T} \cdot Q_G \cdot \Phi^{-1}\right)$$

where: $\overline{x}(t) = \Phi \cdot x(t)$

This implies that:

$$\left(\overline{P}_{i,i}, \overline{Q}_{i,i}\right) = \left(\Phi_i \cdot P_{i,i} \cdot \Phi_i^{\ T}, \Phi_i^{\ -T} \cdot Q_{i,i} \cdot \Phi_i^{\ -1}\right) \quad \textbf{(51)}$$

where: $\overline{x}_i(t) = \Phi_i \cdot x_i(t)$

Equation **(61)** is a transformation that only depends on the state space transformation $x_i(t)$, i.e. on the state space associated to $T_i(s)$. From this statement, it is evident that following a Balanced Truncation procedure with block Gramians $P_{i,i}$ and $Q_{i,i}$ (block diagonals of P and Q) on each sub-system $T_i(s)$ we can obtain a reduced model $\hat{T}_i(s)$ by truncating the states corresponding to the smallest eigenvalues. Since the Gramians are obtained from the global system Gramians, it is expected that the truncated states of each sub-system $T_i(s)$ correspond to the less relevant states w.r.t. the global transfer function. This yields the Interconnected Systems Balance Truncation (ISBT), where each sub-system is reduced by applying a TBR procedure with the block diagonals of P and Q (block Gramians $P_{i,i}$ and $Q_{i,i}$) to obtain a reduced sub-system $\hat{T}_i(s)$. These sub-systems are later interconnected with the same interconnection matrices to obtain a reduced Interconnected System.

The Balanced Truncation algorithms are known to guarantee quasi-optimal reduced sizes given that reduction order can be chosen *a-posteriori* from the error bound available. On the other hand, they are very expensive for large state space systems. The interconnection of several sub-systems yields into an even larger system, so to perform this MOR approach falls into a high inefficiency.

5.2 MOR via Krylov Projection Approaches

Krylov Projection techniques have been introduced in a previous section. The difficulty here is how to apply those techniques in the context of the structure and hierarchy preserving MOR of Interconnected Systems. Let us define the Interconnected Global System obtained from the interconnection of N sub-systems $T_i(s)$ as shown in **(56)**. The following matrices can be defined

$$\begin{aligned} A_G &= -(G_G + s_j \cdot C_G)^{-1} \cdot C_G \\ R_G &= (G_G + s_j \cdot C_G)^{-1} \cdot B_G \end{aligned} \qquad \textbf{(52)}$$

and

$$\begin{aligned} A &= -(G + s_j \cdot C)^{-1} \cdot C \\ R &= (G + s_j \cdot C)^{-1} \cdot B \end{aligned} \qquad \textbf{(53)}$$

where $s = s_j$ is the frequency point of expansion where the block moments are going to be computed in. If $s_j \in \Im$ is neither an eigenvalue of A nor an eigenvalue of A_G, it can be shown that:

$$Kr(A_G, R_G, q) \subseteq Kr(A, R, q) \qquad \textbf{(54)}$$

Consider then that an orthonormal basis V for those Krylov Subspaces is obtained, so that

$$Kr(A_G, R_G, q) \subseteq Kr(A, R, q) \subseteq colsp[V] \qquad \textbf{(55)}$$

with

$$V = \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} \qquad \textbf{(56)}$$

where $V \in \Re^{nxq}$, and $V_i \in \Re^{n_i xq}$, and a projection on each sub-system is performed via a congruence transformation

$$\begin{aligned} \hat{C}_i &= V_i^T \cdot C_i \cdot V_i \\ \hat{G}_i &= V_i^T \cdot G_i \cdot V_i \\ \hat{B}_i &= V_i^T \cdot B_i \\ \hat{L}_i &= L_i \cdot V_i \end{aligned} \qquad \textbf{(57)}$$

Then $T\hat{F}(s):\{H,F,K,\hat{T}(s)\}$ interpolates $TF(s):\{H,F,K,T(s)\}$ up to the $k=\min\left\lfloor \frac{q}{m} \right\rfloor$ derivative at the point $s=s_j$, m being the number of inputs of the global system.

On the other hand, if we reduce a sub-system via Krylov in the following way

$$\begin{aligned} &Kr(A_i, R_i, q) \subseteq colsp[V_i] \\ &A_i = -(G_i + s_j \cdot C_i)^{-1} \cdot C_i \\ &R_i = (G_i + s_j \cdot C_i)^{-1} \cdot B_i \end{aligned} \quad \textbf{(58)}$$

and

$$\begin{aligned} &\hat{C}_i = V_i^T \cdot C_i \cdot V_i \\ &\hat{G}_i = V_i^T \cdot G_i \cdot V_i \\ &\hat{B}_i = V_i^T \cdot B_i \\ &\hat{L}_i = L_i \cdot V_i \end{aligned} \quad \textbf{(59)}$$

so that $T_i(s):\{C_i G_i B_i L_i\}$ and $\hat{T}_i(s):\{\hat{C}_i \hat{G}_i \hat{B}_i \hat{L}_i\}$, the reduced global system

$$T\hat{F}(s):\{H,F,K,diag\{T_1(s) \quad ... \quad \hat{T}_i(s) \quad ... \quad T_N(s)\}\}$$

matches $k=\left\lfloor \frac{q}{\alpha_i} \right\rfloor$ block moments of the original global system:

$$TF(s):\{H,F,K,diag\{T_1(s) \quad ... \quad T_i(s) \quad ... \quad T_N(s)\}\}$$

These two properties of the Interconnected Systems provide us with two new tools for their structured and hierarchical model order reduction.

The first approach is to perform Krylov projection schemes over each sub-system independently, obtaining different reduced orders for each sub-system, and consequently matching different block moments in each reduction. When interconnecting these sub-systems, the (reduced) global transfer function obtained is going to match as many block moments of the original transfer function as the minimum of the block moments matched in each reduction of each sub-system. It must be noticed that there is no restriction in obtaining the Krylov subspaces at different frequency points, but in that case nothing can be stated about the interconnected system. This approach, despite describing a moment matching procedure in interconnected systems, is not very useful for our goals, because again we perform MOR on each sub-system in a stand-alone mode, and for this reason some global behaviour/information may be lost.

The second approach, more coherent with our goals, is to obtain the Krylov subspace of a given order q of the global system. This subspace spans the $k=\left\lfloor \frac{q}{m} \right\rfloor$ moments of the global transfer function. Then, we can split the projection matrix V into N matrices and use each of those to project each one of the N sub-systems independently. This approach allows us to maintain the structure of the interconnection and at the same time to control the reduced order in each of the sub-systems.

6. Conclusions

In this deliverable we reported on the activities carried out in Task 3.2 of the CHAMELEON-RF project. In this task we seek to develop methods that will allow us to take advantage of the 2-level hierarchy envisioned in CHAMELEON-RF and pursue more efficient reduction of individual models, while taking their coupling into account. That hierarchy depicts the structure of the systems to be analyzed and represents the description of the compact models augmented with connectors representing the interaction with the surrounding environment.

New coupling mechanisms, including EM field coupling, are becoming too strong to be neglected and the behaviour of the devices in the RF regime can be influenced by neighbouring structures. The key to analyzing such components is the recognition that devices, both active and passive, can no longer be treated in isolation and complete RF blocks must be considered as one entity, and be treated as such by the design automation tools. Today, it is not possible to perform such analyses of complete RF blocks. For this reason, we envisage a new kind of hierarchical modelling procedure that will augment compact models of the basic (active and passive) devices with connectors (a kind of “hooks”) which will allow interaction with and representation of their environment. If we have the compact models with connectors, we will subsequently determine the coupling among these elements, and after that we will massage the resulting equivalent circuit model into another model, which is still accurate enough but might be much simpler. Unfortunately the models generated during the extraction process may prove to be too large so that the computational effort spent in simulating these models may be unacceptable. Model Order Reduction (MOR) techniques, a well-established methodology for compressing information resulting from the detailed description of system behaviour, have been proposed as potential solutions to this problem. In the current context, this requires the development of procedures that take advantage of the 2-level hierarchy previously mentioned thus preserving the structure and hierarchy information available. Several model order reduction techniques have been investigated and two, in particular, have been targeted as potentially useful in this context. These techniques require additional extensions in order to be useful in the context of CHAMELEON-RF, regarding the type of models envisioned.

References

[1]. A. Vandendorpe and P.Van Dooren, "Model Reduction of Interconnected Systems", in Proc. of 16th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2004), Leuven, Belgium, pp. paper THP3-4, July, 2004.

[2]. B. Moore, "Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction", *IEEE Transactions on Automatic Control*, pp. 17-32, Vol AC-26, No. 1, Feb. 1981.

[3]. J.R. Phillips, L. Daniel and L.M. Silveira, "Guaranteed Passive Balancing Transformations for Model Order Reduction", *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, pp. 1027-1041,Vol 22, No. 8, Aug. 2003.

[4].] J.R. Phillips and L.M. Silveira, "Poor man's TBR: A simple model reduction scheme*", In Proc. Design, Automation and Test in Europe Conference and Exhibition, DATE 2004*, pp. 938-943, Paris, France, Feb. 2004.

[5]. J.R. Phillips and L.M. Silveira, "Poor man's TBR: A simple model reduction scheme", *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, pp. 43-55,Vol 24, No. 1, Jan. 2005.

[6]. A. Odabasioglu, M. Celik and L.T. Pileggi, "PRIMA: passive reduced-order interconnect macromodeling algorithm", *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, pp. 645-654,Vol 17, No. 8, Aug. 1998.

[7]. P. Feldmann and R.W. Freund, "Efficient Linear Circuit Analysis by Padé Approximation via the Lanczos Process", *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, pp. 639-649,Vol 14, No. 5, May. 1995.

[8]. L.M. Silveira, M. Kamon, I. Elfadel and J. White, "A Coordinate-Transformed Arnoldi Algorithm for Generating Guaranteed Stable Reduced-Order Models of RLC Circuits", *In International Conference on Computer Aided-Design*, San Jose , CA., USA, pp. 288-294, Nov. 1996

[9]. I.M. Elfadel and D.L. Ling, "A Block Rational Arnoldi Algorithm for Multipoint Passive Model-Order Reduction of Multiport RLC Networks", *In International Conference on Computer Aided-Design*, San Jose, CA., USA, pp. 66-71, Nov. 1997.

[10]. R. Achar and M.S. Nakhla, "Simulation of High-Speed Interconnects", *Inv. Paper in Proc. of the IEEE*, pp. 693-728, Vol. 89, No. 5, May 2001.

[11]. H. Yu, L. He and S.X.D. Tan, "Block Structure Preserving Model Order Reduction", *In IEEE Behavioral Modeling and Simulation Wokshop* pp. 1-6, Sept. 2005.

[12]. R.W. Freund, "SPRIM: Structure-Preserving Reduced-Order Interconnect Macro-Modeling", *In IEEE Proc. International Conference on Computer Aided-Design, San Jose, CA., USA*, pp. 80-87 , Nov. 2004.