# Dealing With Unknown Impedance and Impulsive Noise in the Power-Line Communications Channel

Paulo A. C. Lopes, João M. M. Pinto, and José B. Gerald, *Senior Member, IEEE*

***Abstract*—Power-line communication (PLC) allows establishing digital communications without adding any new wires. It will turn one's house or neighborhood grid into a smart grid. PLC has some issues, namely, high noise at low frequencies and varying characteristic impedance. This paper addresses these issues to improve the signal-to-noise ratio by increasing the signal or reducing the noise. PLC MODEMs are subject to legislations that limit the signals in the line. The radiated signal is proportional to the current, but not to the input current, since the current forms a standing wave along the line. However, better performance can be achieved if the input current is measured. The receiver circuit of the transmitting MODEM can be used to estimate the input impedance. This paper presents a technique to use this new information to achieve better performance and to follow legislation changes in the band above 30 MHz. A study of the viability of using impulsive noise reduction techniques to further increase performance is also presented. The short noise pulses result in high correlation between the noises in different carriers. Impulse position detection should result in an increase in capacity.**

***Index Terms*—Adaptive impedance, adaptive modulation, bit loading, characteristic impedance, electromagnetic compatibility (EMC), impulsive noise, MODEM, non-Gaussian noise, orthogonal frequency-division multiplexing (OFDM), power-line communications (PLCs).**

## I. Introduction

MANY telecommunications channels use wires to establish a physical connection, but most electronic devices already have a pair of wires connected–the power-line wires. These wires can be used to establish digital communications. So, power-line communications (PLC) [1], [2] is an alternative to wireless and other technologies to provide a network to homes and in homes. This is an active area of research with many papers in the field [3]–[5].

However, the power-line channel was not originally meant to be used for communications, and it is a difficult channel for several reasons: it varies widely from one installation to another, namely, its differential- and common-mode characteristic impedance, the attenuations, and the loads; it has high noise at low frequencies.

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The authors are with INESC-ID/IST/UTL, Lisbon 1000-029, Portugal (e-mail: paulo.lopes@ist.utl.pt).

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This paper describes how to increase the signal-to-noise level by increasing the signal level and/or by reducing the noise level. Signal levels must be below the ones established by international regulations. The values are limited by regulations concerning electromagnetic compatibility (EMC) [6]. In Europe, these are regulated by the CISPR 22 standard [7] and connect to the CE marking. In the U.S., FCC part 15 [8] is used. In these standards, there are intentional and unintentional radiators. Considering PLC as an intentional radiator would require a band to be attributed by the legislator. The PLC signal from different homes would compete for bandwidth. Considering them as unintentional radiators would simplify things while still enabling transmission with high bandwidth and coexistence with other technologies. These regulations impose limits on the voltage of the signal in the line for frequencies bellow 30 MHz, and limits on the radiated signal for frequencies above 30 MHz. So, following regulations bellow 30 MHz will be fairly simple, but for frequencies above 30 MHz, estimating the radiated signal from the injected signal will be harder. This will be one of the points of this paper.

The radiated signal can be estimated by noting that it is proportional to the current in the line. However, this is not the input current since the amplitude of the current can vary along the line due to the formation of a standing wave. Nonetheless, calculating the input resistance by using the value for the voltage at the analog-to-digital converter (ADC) of the transmitting MODEM will allow the estimation of the radiated signal. In turn, this will allow compliance with regulations, while still achieving high-speed communications at frequencies above 30 MHz. A number of approaches can be used here. These will be discussed later in this paper.

To increase the signal-to-noise ratio (SNR), this paper also addresses a reduction of the noise level. Impulsive noise in OFDM receivers will appear to have large amplitude in all OFDM carriers although the noise in all of the carriers is highly correlated. Designing a MODEM that uses the knowledge of the high correlation values will result in a better noise model, and should allow a lower error rate and increased capacity. In order to obtain high covariance values, the calculation needs to be conditioned to a given impulse position since the unconditional covariance of impulsive noise is low. Some measurements are presented and analyzed in this paper in order to estimate the improvements that can be obtained from such techniques.

## II. Model for the Power-Line Channel

The power-line channel consists of two or more wires, connected to form the different topologies that can be found in power-line networks. These two wires can be analyzed as a transmission line. Actually, they are more accurately analyzed

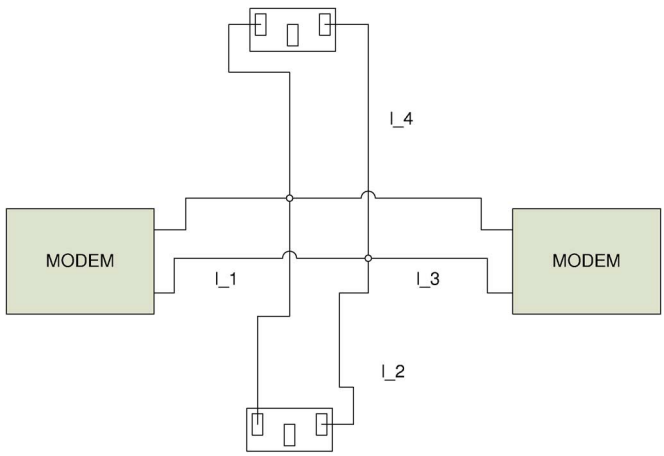


Fig. 1. Simple PLC configuration with four stubs.

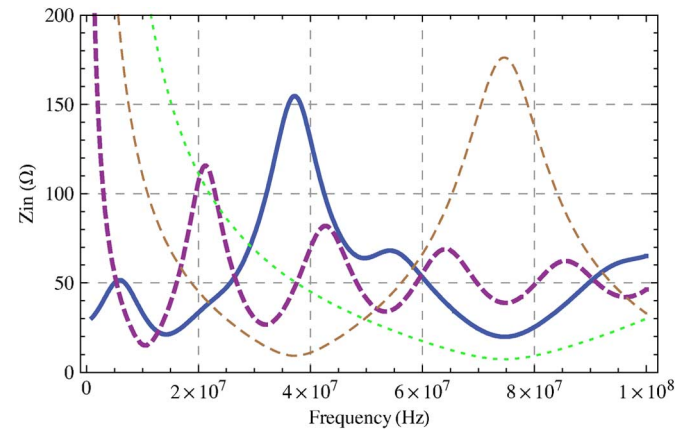


Fig. 2. Input impedance of several stubs. The thick blue line is the 4-stub model described, the thick dashed purple line is the open termination 9-m stub, and the thin dashed brown line is 2 m, and the thin dotted green line is 1 m.

as two transmission lines between each wire and the earth, or as a three-wire line. This can also be modeled as two modes of propagation: differential, between the two wires; and common mode, from both wires to earth, with some degree of crosstalk between the modes. The conversion of a differential mode to common mode is measured by the longitudinal conversion loss (LCL) factor [6]. This is equal to the ratio of the common-mode signal to the differential-mode signal after conversion to decibels. Conversion results from a lack of line symmetry. In a power line, the radiation from the differential mode is usually much lower than that from the common mode, because the radiated signal from each wire will cancel the other out. This means that using the differential-mode signal for PLCs will result in lower radiation. The greatest contribution to radiation will be one resulting from conversion to common mode, so the LCL will be important to determine the radiated signal level.

The signal in a transmission line can be considered as the sum of two waves—the positive and the negative traveling waves—so the voltage and current in the line will be [9]

$$V(x,t) = V_+ e^{\omega t - kx} + V_- e^{\omega t + kx} \tag{1}$$

$$I(x,t) = I_+ e^{\omega t - kx} - I_- e^{\omega t + kx}. \tag{2}$$

The voltage and current of the waves will be related by the characteristic of the line $Z$, namely, $V_+ = ZI_+$ and $V_- = ZI_-$.

As an example, a line with four 50-Ω stubs is considered, as in Fig. 1. Stub 1 connects the transmitting MODEM to a midpoint. The other three stubs leave from the midpoint. Stub 3 connects to the receiving MODEM that has a 25-Ω impedance; and stubs 2 and 4 are open terminated. The stubs have lengths of 2, 2, 7, and 1 m, respectively. The input impedance of this line is presented in Fig. 2, along with the input impedance of the related stubs.

It can be seen that the input impedance varies greatly with frequency. It varies around the characteristic impedance of the line, but, in fact, there is no guarantee that the characteristic impedance can be taken from the average. For instance, for two parallel branches with two minimums at different points, the impedance will be low at both minimums. The result is an average that is lower than the characteristic impedance. Techniques to try and estimate the full topology and impedances of the line from different frequency measurements may be worth considering but mainly with other applications.

TABLE I
CHARACTERISTIC IMPEDANCE OF A TRANSMISSION LINE ($Z$ IN OHMS) AS A FUNCTION OF THE LINE GEOMETRY AND DIELECTRIC

| d/2a | 1 | 1,1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 |
|---|---|---|---|---|---|---|---|---|
| Air | 0 | 53 | 75 | 91 | 104 | 115 | 126 | 135 |
| PVC | 0 | 30 | 42 | 51 | 58 | 65 | 70 | 75 |

The characteristic impedance of the line depends on its geometry, namely, on the distance between the center of the conductors $(d)$ divided by the conductor radius $(a)$. This is presented in Table I.

## III. Radiation From the Power Line

The radiated electric field from a line section in the direction of greater radiation is given by [10]

$$E(t) = \int_0^\infty j \frac{Z_0 \beta_0}{4\pi} \frac{I(x)}{r} e^{j(\omega t - \beta r)} dx \tag{3}$$

where $r$ is the distance from the measuring point to the point of the line at position $x$, $\omega$ is the frequency in radians/s, and $I(x)$ is the current in the line, formed by the positive traveling wave and the negative traveling wave. The air-wave number is $\beta_0$ as opposed to the line-wave number $\beta$.

Since the radiation is always lower than two times the radiation from the transmitted current wave, in the far field, considering a straight line and integrating from zero to infinity, results in the following limit for the radiated signal [11]:

$$|E| < 2 \left| \frac{Z_0 I_+}{4\pi r} \right|. \tag{4}$$

## IV. Legislation Limits

Legislation limits the amplitude of the signal that can be transmitted in power lines. For frequencies below 30 MHz, the value of the common-mode voltage is limited. The common

TABLE II
LIMITS FOR THE SIGNAL VALUES AT THE MAINS PORT

| Frequency (MHz) | Mains port Voltage ($\mathrm{dB}\mu V$) |
|---|---|
| 0.15 - 0.50 | 56-46 |
| 0.50 - 5 | 46 |
| 5-30 | 50 |

TABLE III
SIGNAL MEASUREMENT BANDWIDTHS BY CISPR 16

| | Band A | Band B | Band C | Band D |
|---|---|---|---|---|
| | 0-150 KHz | 0.15-30 MHz | 30-300 MHz | 0.3-1 GHz |
| 6 dB Bandwidth | 200 Hz | 9 KHz | 120 KHz | |

TABLE IV
LIMITS FOR EMISSION

| Frequency of Emission (MHz) | Field Strength ($\mu V/m$) | |
|---|---|---|
| | CISPR (at 10 m) | FCC (at 3 m) |
| 30 - 88 | 31.6 | 100 |
| 88 - 216 | 31.6 | 150 |

mode is related to the differential mode by the LCL as previously discussed. There are some studies concerning the values of LCL used in PLC [12], suggesting values of about 36 dB.

The limits for the rms of the common-mode voltage signal are presented in Table II. These are for class B devices and should be measured using a line-impedance stabilization network (LISN) [13] with 50-Ω impedance.

The measurement bandwidths for these values are presented in Table III [13].

For frequencies above 30 MHz, the limits are for the radiated signal rather than the line signal. These are presented in Table IV.

## V. COMPLYING WITH LEGISLATION LIMITS

Complying with legislation below 30 MHz will consist in limiting the signal injected in the line, since this dictates the voltage measured by LISN. Issues regarding the LCL factor must also be addressed. For frequencies above 30 MHz, the limit is on the radiation level. This section will be about limiting the radiation levels using the values of the signals available in a typical PLC MODEM.

The radiation from a power line is related to the current in the line by (4). Using this formula, one can obtain maximum values for $I_+$, the rms value of the transmitted wave current that will result in a MODEM that complies with legislation.

Let the limit on $I_+$ be $I_{\max}/2$, namely, $I_+ < I_{\max}/2$. Using the limits due to the Federal Communications Commission (FCC) for frequencies above 30 MHz and an LCL of 36 dB, the value of $I_{\max}$ will be $1.82 \times 10^{-6}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \times \sqrt{B}$, where $B$ is the bandwidth. However, in a typical MODEM, $I_+$ is also unknown. This can be dealt with as follows.

The current along the line will not be constant but forms a standing wave. Its complex amplitude will be $I(x)$. Let the complex amplitude of the current at the input of the line be $I$ equal to $I(0)$, resulting in $I = I_+ - I_-$. If we take a relatively short length at the beginning of the line, the attenuation of both waves will be low, and the current magnitude will be given by

$$|I(x)| = I_+ \left(1 + \alpha \sin(2kx + p)\right) \tag{5}$$

where $\alpha$ is the ratio of the reflected to the transmitted wave, and $k$ will be the wave number. The standing-wave ratio (SWR) is related to $\alpha$ by $\mathrm{SWR} = (1+\alpha)/(1-\alpha)$. The input current will be a sample of this function at $x = 0$. The value of $p$ will be the sample phase. For large values of $\alpha$, the transmitted wave current can have high values while the input current is low. This will occur if the line is excited in resonance. If the characteristic impedance of the line $Z$ is known, we can calculate $I_+$ by

$$I_+ = \frac{V + (Z - R)I}{2Z} \tag{6}$$

where $V$ is the amplitude of the voltage signal from the voltage source that drives the line and $R$ is the source-output resistance. Note the difference between $Z$ and the input impedance $Z_{\mathrm{in}}$. The characteristic impedance of the PLC channel varies from line to line, as shown before, but the previous equation can be used to calculate a worst case value for $I_+$, using the worst case for $Z$. Further, in this paper, we use 10 Ω as an example.

This could be one way of limiting the radiation levels. Another way would be to assume a zero worst case input impedance of the line and take the input current to be the access voltage divided by the access resistance. This paper proceeds to discuss several access techniques. One can have several models. The models will differ based on what is known in each case. It is assumed that the MODEM accesses the line with some circuit that is equivalent to the following. A digital-to-analog converter (DAC) will be the voltage source that drives the line with a known output impedance $R$ (50 Ω as an example). An ADC will measure the voltage at the input of the line. The voltage-source amplitude will be $V$, while the voltage at the input of the line, measured by the ADC, will be $V_0$. The input current of the line will be

$$I = \frac{(V - V_0)}{R} \tag{7}$$

The models will differ in what is assumed to be known by the MODEM, as presented in Table V. The expressions are obtained by rearranging (5)–(7) so that only known values and the values $Z$, $\alpha$, and $p$ appear in the equations. The function $\mathrm{prl}(Z_1, Z_2)$ is $Z_1 * Z_2/(Z_1 + Z_2)$. The unknown values will be replaced in the final result by worst case values or removed by performing expectations using statistics for the variables. Note that the values of $\alpha$ and $p$ are not directly related to the value of $Z$, but rather to other factors. These are reflections at the loads, and not at the emitter, line length, and attenuation.

Electric compatibility regulations for unwanted radiators are meant to prevent adding up the radiation from many devices so that they interfere with the working of other devices, namely, telecommunication devices and others. This implies that the average value for the radiation, and not only the worst case value,

TABLE V
KNOWN VALUES IN EACH MODEL AND THE EXPRESSION REQUIRED TO CALCULATE THE TRANSMITTED WAVE CURRENT

| | Known | |
|---|---|---|
| Model 1 | $V$, $V_0$ | $I_+ = V_0/2/\mathrm{prl}(Z_{in}, Z)$ |
| Model 2 | $V$ (unknown $V_0$ results in adding $p$ and $\alpha$ to the model) | $I_+ = \frac{V}{R+Z+(R-Z)\alpha\sin(p)}$ |
| Model 3 | $I$ | $I_+ = I\frac{1}{1-\alpha\sin(p)}$ |

TABLE VI
EXPECTED VALUES ON $p$ FOR THE TRANSMITTED WAVE CURRENT FOR THE DIFFERENT MODELS

| Model | $\mathrm{E}[I_+]$ |
|---|---|
| 1 | $V_0/2/\mathrm{prl}(Z_{\mathrm{in}}, Z)$ |
| 2 | $k(-\alpha\Gamma_E)V/(R+Z)$ |
| 3 | $k(\alpha)I$ |

TABLE VII
VALUES OF $k(\alpha)$

| $\alpha$ | 0.1 | 0.5 | 0.7 | 0.9 | 0.99 |
|---|---|---|---|---|---|
| $\mathrm{E}[I_+\|I]/I$ | 1.005 | 1.155 | 1.400 | 2.294 | 7.089 |

will be of interest. The sample phase $p$ is a random variable with uniform distribution, between $-\pi$ and $\pi$, so this can be used to calculate some values for the average radiation.

Define $k(\alpha)$ as

$$k(\alpha) = E\left[\frac{1}{(1-\alpha\sin(p))}\right] \tag{8}$$

where the expectation is on $p$. Using this result and performing the expectation on $p$ on the previous models results in the values in Table VI.

Let $\Gamma_E = (R-Z)/(R+Z)$. The values of $k(\alpha)$ can be calculated numerically and are represented in Table VII.

Note that the expected value of $I$ given $I_+$ is equal to $I_+$, but the expected value of $I_+$ given $I$ is not $I$. It will always be greater than I, as seen in Table VII.

## VI. IMPROVEMENT ACHIEVED BY USING THE INPUT IMPEDANCE ESTIMATE

To compare the different cases, the expected value for the voltage signal at the receiver will be used. The voltage amplitude at the receiver will be equal to the transmitted wave voltage amplitude at the receiver plus the reflection wave amplitude

$$V_R = I_+ Z A_V (1+\Gamma_R). \tag{9}$$

So the voltage at the receiver will be proportional to $I_+$. In a typical scenario, the line will have typical parameters, but since some are unknown, worst case parameters must be used by the MODEM. There are two maximums of $I_+$. The maximum value that could be used if all of the line characteristics were known will be $I_{+M\mathrm{re}}$. The maximum chosen by the MODEM will be $I_{+M\mathrm{md}}$. The ratio of the two $I_{+M\mathrm{re}}/I_{+M\mathrm{md}}$, will be refereed as the signal gap $S_\Gamma$. As an example, for the case of model 1, the signal gap will be

TABLE VIII
VALUES FOR THE SIGNAL GAP FOR DIFFERENT MODELS

| Model | Known | $S_\Gamma$ | |
|---|---|---|---|
| | | Assemble average | Worst for p |
| 1 | $V$, $V_0$ | — | 3.0 |
| 2 | $V$ | 2.1 | 4.2 |
| 3 | $I$ | 2.3 | 10 |

$$S_\Gamma = \frac{I_+}{I_{+M}} = \frac{((Z+Z_{in})Z_m)}{(Z(Z_{in}+Z_m))} \tag{10}$$

where $Z_m$ is the worst case (minimum) value for the line characteristic impedance. Table VIII represents the values of the signal gap for different models. This table has two columns. In the first column, the limits were calculated using expectation on $p$, as previously discussed, and worst case for the remaining parameters. In the second column, the limits are calculated using worst case on all parameters. The values should only be taken as examples and were calculated using typical values $Z = 50\,\Omega$, $\alpha = 0$, $\Gamma_{\mathrm{E}} = 0$, and worst case values of $Z_m = 10\,\Omega$, $\alpha_M = 0.9$, $\Gamma_{\mathrm{EM}} = 2/3$, and a value of $R$ of 50 Ω.

As can be seen in the table, using measurements of $V_0$ will result in a significant decrease in the signal gap compared to, simply, accessing an unknown characteristic impedance line with a resistor. More information will reduce the uncertainty. The cases that use expectation only give compliance with legislation in the average, and may require special legislation adapted to PLC.

## VII. POWER-LINE NOISE MEASUREMENTS

After having discussed how to increase the signal power, techniques to reduce the noise are discussed. The measurement of the power lines' noise signal was made in the frequency band from 1 to 500 MHz using a 50-Ω pickup circuit with two decoupling capacitors, a wideband transformer, and an amplifier. The power spectral density of the noise was obtained with a spectrum analyzer, and the time-domain signal in the band from 1 to 100 MHz was stored digitally and analyzed. The measurements were performed at a few locations of an office area, and the fundamental features of the noise were similar. A typical power spectral density (PSD) of the noise is presented in Figs. 3 and 4. These show the PSD in two laboratories of the university, along with the measurement device noise floor. A high noise level can be seen in frequencies up to 50 MHz (mostly impulsive), and lower noise is seen in higher frequencies, with narrowband interference from radio and television stations.

Fig. 5 presents the time-domain signal. This shows the impulsive nature of low-frequency noise, with short impulses with a duration of some microseconds [14]. Impulsive noise results in high covariance between the noise signals in OFDM carriers,

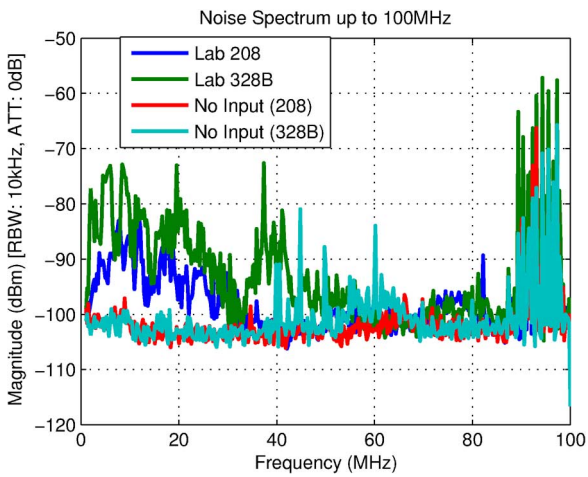


Fig. 3. Power spectral density of the PLC noise up to 100 MHz. The chart shows the values in two labs in the university, together with the spectrum-analyzer noise floor.

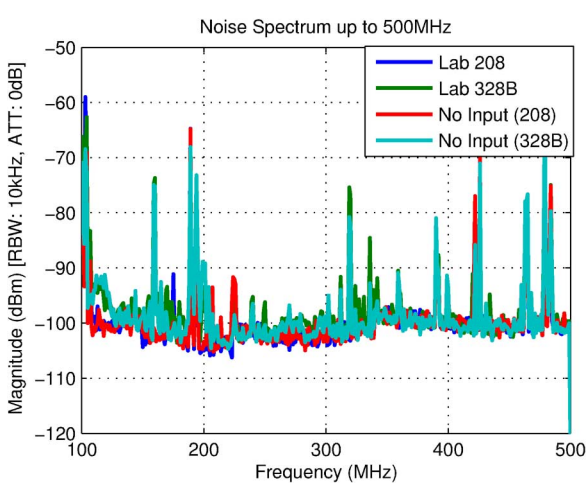


Fig. 4. Power spectral density of the PLC noise from 100 to 500 MHz. The chart shows the values in two labs in the university, together with the spectrum-analyzer noise floor.

in each OFDM symbol, with a phase related to the impulse position.

## VIII. Modeling the Power-Line Noise

Impulsive noise may be considered as a non-Gaussian stationary process. The probability density function (PDF) of the signal remains stable with time. This is regardless of the fact that impulse appears only in particular time instants. However, the convolution with the channel impulse response will result in each output sample being the sum of many non-Gaussian variables and the result will be close to Gaussian. This will make it hard to work directly with the non-Gaussian stationary model.

### *A. Impulsive Noise as a Non-Gaussian Stationary Process*

A model for impulsive noise will be presented for a simple system with no memory. It is a random variable with a non-Gaussian PDF, given by the sum of two Gaussians. At each time instant, an impulse is generated with probability $P$, resulting

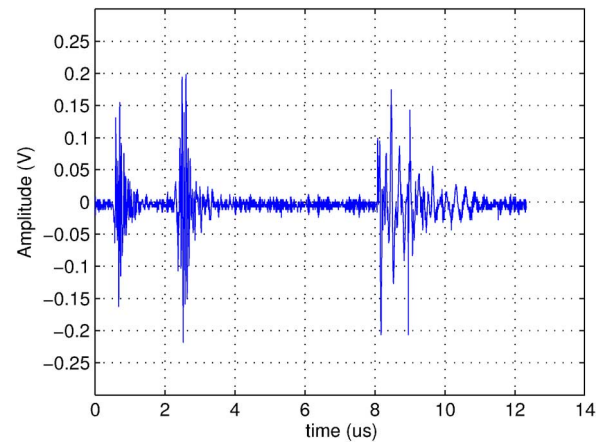


Fig. 5. Sample of PLC noise in the time domain with a bandwidth up to 100 MHz (sampling at 200 MHz), showing the impulses.

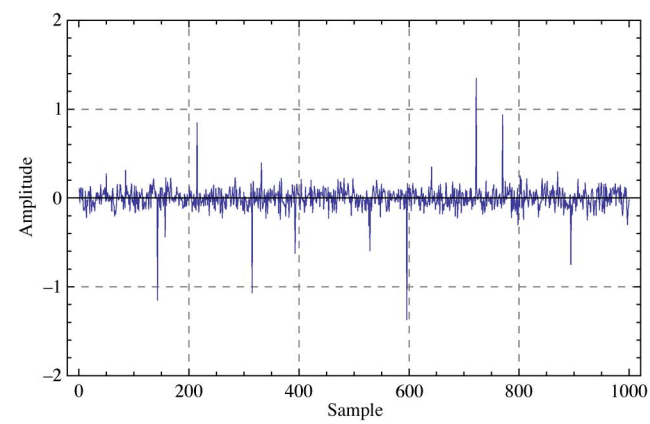


Fig. 6. Impulsive noise generated by a stationary non-Gaussian process.

in a Gaussian with a high variance, or no impulse, a Gaussian with small variance. The resulting PDF is the sum of the two Gaussians, as presented in (11). Let $x$ be a Gaussian variable with zero mean and a variance of one, $\sigma_n^2$ be the background noise variance, and $\sigma_p^2$ be the impulse amplitude variance, then the PDF of the noise signal $v$ will be

$$\mathrm{PDF}_v(z) = (1-P) \times \mathrm{PDF}_x\left(\frac{z}{\sigma_n}\right) + P \times \mathrm{PDF}_x\left(\frac{z}{\sigma_p}\right). \tag{11}$$

The resulting time-domain signal is presented in Fig. 6.

### *B. Impulsive Noise as a Nonstationary Process With High-Frequency Domain Correlation*

An impulsive noise process is stationary as previously discussed. If the impulse position is shifted to the origin, then it becomes nonstationary. This is the same as saying that the joint PDF will become time varying for a known impulse position. The noise will be higher close to the impulse center and lower far from the center.

The time-domain covariance matrix will be

$$\Sigma_{\mathrm{tm}} = \mathrm{E}\left[\mathbf{u}_2\mathbf{u}_2^H\right] \tag{12}$$

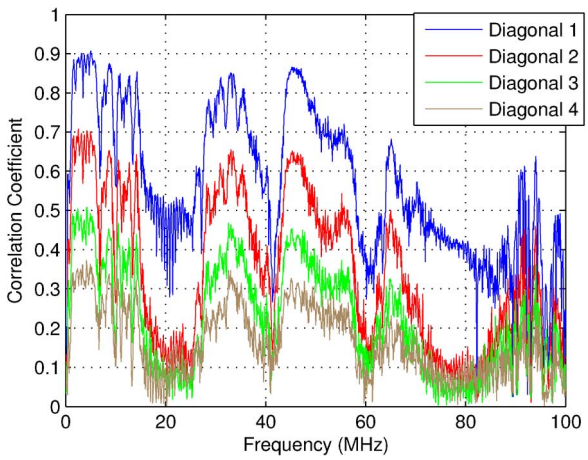


Fig. 7. Values for correlation coefficient of adjacent OFDM carriers, with a distance of one (diagonal 1), carriers with a distance of two (diagonal 2), three, and four. The values are taken from the respective diagonal of the correlation coefficient matrix. The correlation decreases with increasing distance. The length of the OFDM symbols was 2467 samples at 200 MHz.

where $\mathbf{u}_2$ is the time-domain OFDM symbol with the impulse shifted to the origin. For the nonstationary signal, this will no longer be Toeplitz $\Sigma_{\mathrm{tm}i,i} \neq \Sigma_{\mathrm{tm}j,j}$, because the power at different times will not be the same. This will also imply that the correlation coefficients of the noise between OFDM carriers will have a high magnitude while having a complex value. This is because a narrow time signal at the origin will vary slowly in frequency. The frequency-domain covariance matrix will be

$$\Sigma = \mathrm{E}\left[U_2 U_2^H\right] \tag{13}$$

where $U_2$ is the DFT $\mathbf{u}_2$. The matrix of the correlation coefficients $C$ can be calculated from

$$C_{i,j} = \frac{\Sigma_{i,j}}{\sqrt{\Sigma_{i,i}\Sigma_{j,j}}}. \tag{14}$$

In the case of stationary noise, the correlation coefficients are typically low, because the discrete Fourier transform (DFT) approximately diagonalizes the time-domain Toeplitz covariance matrix. The estimation of the covariance matrix and a more concrete model for the noise are discussed later in this paper. In the case of impulsive noise, the correlation coefficients are typically high. The case of white but nonstationary noise will be considered as an example. For this type of noise, $\mathrm{E}[u[i]u[j]] = \sigma_i^2\delta[i-j]$. The covariance matrix will be given by

$$\Sigma_{k,k+l} = \mathrm{E}\left[\sum_{i=0,j=0}^{N-1} u[i]u[j]e^{2\pi(k(j-i)+lj)/Ni}\right] \tag{15}$$

or

$$\Sigma_{k,k+l} = \sum_{j=0}^{N-1} \sigma_j^2 e^{2\pi(lj)/Ni}. \tag{16}$$

The values of $\Sigma_{k,k+l}$ will be the Fourier transform of $\sigma_j^2$ as shown in (16). These vary slowly for short impulses at the origin. This will, in turn, imply that the correlation coefficients $C_{k,k+l}$ will be close to one for low values of $l$. The presence of background, nonimpulsive noise will make the correlation drop.

*1) Estimating Impulse Position:* The phase of the correlation coefficients will depend on the impulse position. If the calculation is performed ignoring the impulse position, it will average for all of the possible positions and the result will be close to zero. The calculation of the correlation coefficient matrix for the measured noise signal must be made after circular time shifting the impulses to the OFDM symbols to the origin. Note that this corresponds to multiplying by a linear phase signal in the DFT domain. In order to do this, the impulse position must be determined. Let $U[k]$ be the noise signal in the DFT domain and $U_2[k]$ be the signal time-shifted by $d$ samples, then we have

$$U_2[k] = U[k]e^{2\pi(kd)/Ni} \tag{17}$$

for all $k$. We are going to choose the value of $d$ that minimizes the distance between the complex amplitude of successive carriers $U_2[k]$ and $U_2[k+1]$, as in (18). This gives simple results and is equivalent to maximizing the correlation, as shown. The distance is

$$\sum_{k}^{N-2} \left|U_2[k] - U_2[k+1]\right|^2. \tag{18}$$

Minimizing this expression is equivalent to calculating

$$\min_d \sum_{k}^{N-2} \left(-2Re(U[k]U[k+1]^* e^{-2\pi d/Ni}\right) \tag{19}$$

that is given by

$$\frac{2\pi d_i}{N} = \angle\left(\sum_{k=0}^{N-2}(U_i[k]U_i[k+1]^H\right) \tag{20}$$

where we must choose the angle so that it is always a positive number (add $2\pi$ if it is below zero). The angle should be calculated by using a function that gives the angle given the coordinates of the vectors $x$ and $y$, usually known as atan2(x, y).

This technique will not give an accurate value for the position if there is more than one impulse per OFDM symbol, but it will accurately maximize the correlation which is the actual goal. However, it is still desirable to have zero or one impulse per OFDM symbol. More impulses per symbol can be dealt with more elaborated models, but this may not be required.

*2) Correlation Coefficients Measurements:* Using previously discussed techniques, the correlation coefficients of the time-shifted OFDM carrier noise vector was determined based on experimental values. This is shown in Fig. 7. The correlation coefficient of adjacent carriers has very high values at some frequencies, close to 0.9, showing that these techniques may be used to lower bit-error rates (BERs) and increase capacity of PLC Modems. However, note that the values for the correlation coefficients decrease with decreasing OFDM symbol length. Other techniques may use shorter symbols to achieve similar goals.

## IX. Capacity of an Impulsive Noise Channel

It has been shown that the covariance of the noise between OFDM carriers will be high given the impulse position. A high

covariance suggests that the noise in each carrier can be reduced by using information from the noise in the other carriers. In more general terms, the noise is lower than the value obtained if each carrier is taken individually. This can easily be seen by looking at the formula for the capacity for multicarrier systems. As shown later in this paper, this is

$$C = \log_2 \left( \frac{\det(S+N)}{\det(N)} \right) \tag{21}$$

where $S$ and $N$ are the covariance matrices of the signal and noise, respectively. Note that this is the capacity per sample in bits and not in bits per second. Calculating the latter would require multiplication by the sample rate. Using this formula, one can define the concept of noise volume $\det(N)$ since the determinant is the volume of the parallelepiped formed by vectors of the columns of a Hermitian matrix, and this will be related to the volume occupied by samples of the noise in the receiver vector space. High correlation in the noise will result in lower noise volume and higher capacity. For instance, in two dimensions, a narrow ellipse can be taken as a circle with much greater area if each carrier is taken separately.

The effect of a high correlation coefficient on capacity can be better understood by calculating the determinant of a simple two by two matrix, namely

$$\det \begin{pmatrix} u & c \\ c & u \end{pmatrix} = u^2 - c^2 \tag{22}$$

showing that the increase in covariance will result in lower noise volume.

The formula for capacity can be generalized to the case of non-Gaussian noise. The capacity is the maximum of the mutual information between the input and output of a channel for all of the possible input distributions [15], [16]. In the case of an additive linear Gaussian noise signal, the distribution that maximizes the mutual information is also Gaussian. The mutual information is given by

$$I(Y;X) = H(Y) - H(Y|X) \tag{23}$$

with

$$H(Y|X) = \int_{-\infty}^{\infty} H(Y|X=x)\mathrm{PDF}_X(x)dx \tag{24}$$

where $X$ is the input random variable, $Y$ is the output random variable, and $H(Z)$ is the entropy (differential) of the random variable $Z$. The entropy of a $n$-dimensions real Gaussian variable distribution is given by

$$H(Z) = \log_2 \sqrt{(2\pi e)^n \det(\Sigma)}. \tag{25}$$

The expression for capacity can be easily obtained by noting that: $n$ complex variables are $2n$ real variables; that $H(Y|X = x)$ is constant, independent of $X$ and equal to the entropy of the noise. The expression for capacity can be interpreted as a ratio of volumes. There is also a connection between coding and sphere packing.

### A. Impulsive Noise MODEM

The high covariance between the noises in different carriers could easily be used to increase the capacity if the covariance matrix were known by the transmitter *a priori*. A transformation matrix could be applied at the input and output that would diagonalize the noise covariance matrix. In case of impulsive noise, this technique cannot be applied, because the covariance matrix depends on the position of the impulse, which is not known. Looking at it in another way, the noise ellipsoids could be densely packed in the transmitter, but this would require knowledge of the orientation of the ellipsoids, namely, the impulse position.

However, at the receiver, a much better noise model will result in better estimation and a lower error rate and this, in turn, will allow the transmitter to place more bits in the channel. This will require the receiver to perform joint estimation of the signal $x$, impulse position estimation $p$, and correlation coefficient magnitudes. Least mean squares joint estimation can be solved analytically by: considering each impulse position and correlation as a possible model for the noise, conducting a signal estimate using each model, and calculating the global estimate as a weighted average. The weights are the likelihood of each model given the measurements $y$. This is represented in

$$\mathrm{E}[x|y] = \sum_i \mathrm{E}\left[x|(y \wedge m_i)\right] p(m_i|y) \tag{26}$$

where $m_i$ represents the model $i$ for the noise from the finite set of models. This can be demonstrated by using Bayes' rule. But the calculation in (26) may be very computationally expensive. A technique to reduce the computational complexity is iterative decoding. Estimates of the signals (hard or soft output) could be obtained by using a simple model for the noise. This would give a few signal estimation errors. Then, an estimate for the noise signal would be obtained and the model for the noise refined. The signal would be re-estimated and few errors removed. The new noise model would help the demodulator in making the right decisions. The new error probability can be calculated considering a known noise model, and the integration of the multivariable Gaussian PDF outside the decision surface. The decision surface can be taken as a sphere and not an ellipsoid, since the constellation points are defined at the transmitter, once more without the knowledge of the impulse position.

Some care must be taken in adding extra parameters to the noise model. Otherwise, the new model could be worse than the simpler one because of estimation errors. This can be done by using caution in the estimation: selecting lower values for the covariance parameter than the direct estimate.

*1) Noise Variance Estimation Margin:* The fact that there are errors in the model used for the channel and the noise needs to be taken into account. This can be done by using values for the noise variance that are greater than the measured ones. This will be referred to in this paper as the noise variance estimation margin, and it will subtract from capacity. This section serves as a short review of this topic in order to gain some insight into the same problem when applied to covariance.

Considering a MODEM with a probability of error Pe (uncoded average error probability) with a $m = 3$-b error-correction code, which needs $2m + 1 = 7$ bits of redundancy, with a word of length $n = 10$ and with packets of $l = 10000$ b. The normal retransmission rate (due to cyclic redundancy check (CRC) errors) will be one packet in $1/\mathrm{RTR} = 100$, and the channel re-estimation rate will be one out of $1/\mathrm{CER} = 10$ channel estimates. If the retransmission rate is too high, the MAC layer should detect it and estimate the channel again. The probability of packet error [15] will be lower than

$$\mathrm{Pe}_{\mathrm{pkt}} < \frac{(\mathrm{Pe}n)^{m+1}l}{n}. \tag{27}$$

Our goal will be to have

$$P(P_{\mathrm{pkt}} > 0.01) < 0.1. \tag{28}$$

The maximum packet error rate $\mathrm{Pe}_{\mathrm{pkt}}$ can be converted to the maximum bit-error rate (BER) $\mathrm{Pe}_{\max}$, using (27). Assuming that the distance between constellation points is chosen proportionally to the estimated noise standard deviation (high signal-to-noise case) and that the standard deviation is estimated, simply, by the square root of average of the squares of $N$ samples of the measured signal, one can write

$$P(\mathrm{Pe} > \mathrm{Pe}_{\max}) = P\left(\frac{x}{N} < \frac{Q^{-1}(\mathrm{Pe}\,\mathrm{n}_{\max})^2}{Q^{-1}(\mathrm{Pe}_{\mathrm{prj}})^2}\right) \tag{29}$$

where $x$ is a Chi-Square distribution with $N$ degrees of freedom and $Q(x)$ is the complement cumulative Gaussian distribution function and $Q^{-1}(x)$ is its inverse. The variable $\mathrm{Pe}_{\mathrm{prj}}$ is the error probability used in projecting the MODEM, noting that this is performed by assuming perfect knowledge of the noise variance. Using this formula, one can derive $\mathrm{Pe}_{\mathrm{prj}}$ from $\mathrm{Pe}_{\max}$ and the CER.

To obtain a feeling of the numbers, here are the results for the values already presented. The project bit uncoded error probability, Pe will be equal to 0.000134 and the maximum bit uncoded error probability is $\mathrm{Pe}_{\max} = 0.0056$. The requirement is to aim at a lower error probability to fit the specification.

*2) Noise Covariance Estimation:* The same problems that arise in the noise variance estimate also appear in the covariance estimate. In this case, the values used for covariance should be lower than the estimated values. One has to make sure that the new model is more accurate than the previous one. This will raise the issue of what low-order model with covariance should be used. The covariance varies from carrier to carrier. One solution could be to use a fixed correlation coefficient, but this may not be the best model. In each carrier, $k$, the noise will have two contributions—one from background nonimpulsive noise, with variance $\sigma_B^2[k]$ and one from impulsive noise $\sigma_I^2[k]$, resulting in a total noise power of $\sigma^2[k] = \sigma_B^2[k] + \sigma_I^2[k]$. The covariance between carriers will be mostly due to impulsive noise, so this will be close to $\sigma_I^2[k]$. As such, one can estimate the covariance from the variance by subtracting $\sigma_B^2[k]$, taken as constant. In order to be sure that the new model is more accurate than previous ones, the worst case for the background component in each carrier should be used. This will result in the following noise model, defined by the covariance matrix of the noise signal $\Sigma$

$$\Sigma = \mathrm{E}\left[U_2 U_2^H\right] \tag{30}$$

$$\Sigma_{i,i} = \sigma^2[i] \tag{31}$$

$$\Sigma_{i,i+1} = \Sigma_{i+1,i}^* = \max\left(\sigma^2[i] - \sigma_B^2, 0\right) \tag{32}$$

$$\Sigma_{i,j} = 0 \text{ for } |i-j| > 1. \tag{33}$$

The goal will be to estimate $\sigma_B^2$ and the impulse position $p$. This may be done using a maximum a posteriori estimator (MAP) or other, but care must be taken that $\sigma_B^2$ should be the worst case value from the values of each carrier, so probably the estimated value should be increased by some margin as in the noise variance estimate.

## X. Conclusion

The signal in power-line channels is limited by legislation of EMC. The radiation level is limited above 30 MHz. Radiation from the line is related to the transmitted wave current in the line, and not directly to the input current. The transmitted wave current in the line is not actually known, so it has to be estimated, but there will always be some uncertainty in the estimate. This paper presents techniques to reduce this uncertainty, using additional measurement from the AD in the MODEM, allowing an increase of the amplitude of the transmitted signal.

Measurements from PLC noise are presented, showing the impulsive nature of the noise. These measurements were analyzed. After the determination of the impulse position, the impulsive noise can be modeled by a nonstationary process. The covariance matrix of the signals in the OFDM carriers will have high-valued nondiagonal entries, or high covariance. Note that safety margins are required in the estimation of the covariance. The high values of the covariance may be used by advanced demodulators with impulse detection to reduce the error rate and increase bit rate.

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**Paulo A. C. Lopes** was born in Lisbon, Portugal, in 1974. He received the Ph.D. degree in electrical engenering from Instituto Superior Técnico (IST), Lisbon, in 2003.

Currently, he is an Assistante Professor at IST and a Researcher at Instituto de Engenharia de Sistemas e Computadores Investigação e Desenvolvimento (INESC-ID), Lisbon.

**João M. M. Pinto** received the M.Eng degree in electrical and computer engineering from Instituto Superior Técnico, Lisbon, Portugal, in 2008.

He has been performing functions in the telecommunications area since 2011.

**José B. Gerald** (SM'12) was born in Lisbon, Portugal in December 1956. He received the Ph.D. degree in electrical and computer engineering from Instituto Superior Técnico (IST), Lisbon, in 1992.

Currently, he is a Professor in the Electrical and Computer Engineering Department, IST. Since 1988, he has been a Research Engineer at the Institute for Systems and Computer Engineering: Research and Development (INESC-ID), Lisbon, where he has worked on wireless systems for bioimplants and on power-line communication systems. Recently, he has been working on wireless orthogonal frequency-division multiplexing and ultrawideband systems.