Fast Statistical Analysis of RC Nets Subject to Manufacturing Variabilities

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***Abstract*—This paper proposes a highly efficient methodology for the statistical analysis of RC nets subject to manufacturing variabilities, based on the combination of parameterized RC extraction and structure-preserving parameterized model order reduction methods. The sensitivity-based layout-to-circuit extraction generates first-order Taylor series approximations of resistances and capacitances with respect to multiple geometric parameter variations. This formulation becomes the input of the parameterized model order reduction, which exploits the explicit parameter dependence to produce a linear combination of multiple non-parameterized transfer functions weighted by the parameter variations. Such a formulation enables a fast computation of statistical properties such as the standard deviation of the transfer function given the process spreads of the technology. Both the extraction and the reduction techniques avoid any parameter sampling. Therefore, the proposed method achieves a significant speed up compared to the Monte Carlo approaches.**

I. Introduction

Much work has been done aimed at capturing the effects of process variations using parameter-aware techniques. The research focuses mainly on two aspects. One is parameterized Layout Parasitic Extraction (LPE), which models the effect of physical variations by generating linear or quadratic models of capacitances and resistances as a function of process parameters. Most work has concentrated on the capacitances. Approaches for calculating the first-order capacitance sensitivities vary from the enhanced look-up table technique based on analytical models [1], to adjoint methods [2], [3], as well as the domain-decomposition technique [4]. Quadratic models generated by methods based on the Hermite polynomial chaos technique [5], [6], [7] are usually more accurate and yet more computationally expensive compared to the linear models.

On the other hand, a relatively modest amount of work has been done for parameterized resistance extraction [8], [9], because it is relatively simple. More importantly, it is the capacitance computation rather than the resistance computation which dominates the complexity of the overall LPE procedure.

The capacitance and the resistance models generated by the parameterized LPE tools may be used directly for SPICE simulations, or more often, they are fed to a Parameterized Model Order Reduction (pMOR) procedure to achieve an essential speedup. This pMOR procedure is the other aspect of using parameter-aware techniques for capturing variablities.

Many pMOR methods are based on multi-dimensional moment matching. They rely on matching the moments of the parameterized system transfer function, which depends on both the frequency and the parameters [10], [11]. A sampling based approach [12] extended from the PMTBR algorithm [13] proposes to use the statistical information of the parameters as a guidance for a multidimensional sampling of the joint frequency plus parameter space. Recently, a structure-preserving pMOR technique [14] proposes a reformulation of the system to maintain an explicit parameter dependence of the transfer function. This property is very convenient for variational and statistical analysis, as will be demonstrated in this paper.

In order to understand the impact of the physical process variations on the performance of a circuit, the above two aspects have to be fully and well integrated. This has seldom been studied yet, and therefore to draw the overall picture is an important contribution of this paper.

However, a simple combination of the above two aspects does not solve the real problem. The final goal of modeling manufacturing variabilities is to obtain the statistical properties of the system response, given the process spreads of the technology. The traditional Monte Carlo approach has a fatal drawback: the parameter sampling implies a huge computational burden for both the extraction and the reduction procedures, which is least favorable in practice.

In this paper, a fast statistical analysis methodology for RC nets subject to variabilities is proposed. A complete design flow will be demonstrated: from the layout and the process spreads to the statistical properties of the system response. The proposed method avoids parameter samplings by using the parameterized LPE and the pMOR methods. In particular, the parameterized extraction applies the linear model of capacitances presented in [2], [4], and the linear model of resistances which will be presented in Section III. As mentioned, the structure-preserving pMOR technique [14] is used for the reduction procedure.

The rest of the paper is organized as follows. Section II and III present the parameterized extraction methods for the capacitances and the conductances respectively. Section IV introduces the reduction methods for the parameterized system generated by the previous extraction, focusing on the structure-preserving pMOR technique. In Section V, the proposed statistical analysis methodology for RC nets is presented. Finally, Section VI concludes the paper.

II. Parameterized Capacitance Extraction

A system subject to manufacturing variabilities is often described using a parameterized representation of the conductance and the capacitance matrices $\mathbf{G}(\lambda) \in \mathbb{R}^{n\times n}$ and

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$\mathbf{C}(\lambda) \in \mathbb{R}^{n\times n}$. These matrices can be approximated by a Taylor series w.r.t. multiple parameter variations. For instance, a first order approximation:

$$\begin{aligned}\mathbf{C}(\lambda) &= \mathbf{C}_0 + \textstyle\sum_{i=1}^{Q} \lambda_i \mathbf{C}_i \\ \mathbf{G}(\lambda) &= \mathbf{G}_0 + \textstyle\sum_{i=1}^{Q} \lambda_i \mathbf{G}_i\end{aligned} \tag{1}$$

where $\mathbf{C}_0$ and $\mathbf{G}_0$ are the nominal values for the matrices, $\mathbf{C}_i$ and $\mathbf{G}_i$ are the sensitivities w.r.t. to the i-th parameter variation λ_i, and Q is the number of parameters.

A. BEM-based capacitance extraction

When the Boundary Element Method (BEM) is applied to capacitance extraction, the surfaces of conductors are discretized into *panels*. Capacitances between these discretized panels are called *partial short-circuit capacitances*, denoted $\bar{\mathbf{C}}$ in this context. Each entry $\bar{C}_{ij}$ is equal to the charge on panel i when panel j is held at a unit potential and all other panels are short-circuited to the ground. The nominal capacitances $\mathbf{C}_0$ used in (1) are calculated by accumulating the associated partial short-circuit capacitances.

These partial short-circuit capacitances can be obtained from the inversion of the *Green's function matrix* $\mathcal{G}$, whose entry $\mathcal{G}(i,j)$ amounts to the potential induced at panel p_i by a unit charge at panel p_j. Equation (2) shows the Green's function for a uniform dielectric of infinite dimensions:

$$\mathcal{G}(i,j) = \frac{1}{4\pi\varepsilon|p_i - p_j|} \tag{2}$$

with ε the permittivity, and $|p_i - p_j|$ the Euclidian distance between panels p_i and p_j.

Thus the capacitance is a non-straightforward function of the panel position or the dimensions of wires. Further, since capacitance is a mutual property between panels (or wires), adding or removing a panel or changing its position can theoretically induce variations in all capacitances in the system.

These two facts make the computation of capacitance sensitivities very complicated. Any technique whose computational complexity depends on the number of parameters or the number of capacitances would not be feasible, for instance, the perturbation method. In the following, a summary is given, regarding the sensitivity computation technique which will be used in this paper. The computational complexity of this technique does not depend on the number of parameters nor the number of capacitances.

B. Capacitance sensitivities

As shown in [4], the sensitivity of the capacitance between nodes i and j w.r.t. a parameter variation λ_p is given by

$$\frac{\partial C_{ij}}{\partial \lambda_p} = \sum_{k\in s_{\lambda_p}} \left(\frac{1}{\varepsilon A_k} \sum_{a\in N_i} \sum_{b\in N_j} \bar{C}_{k,a}\bar{C}_{k,b} \right) \tag{3}$$

where s_{λ_p} is the set of *victim* panels incident to parameter p. In other words, these are the panels whose positions are changed due to the variation λ_p. A_k is the area of panel k and ε is the material permittivity around panel k. $\bar{C}_{k,a}$ is an entry in the partial short-circuit capacitance matrix $\bar{\mathbf{C}}$, representing the capacitance between two panels k and a. And $\sum_{a\in N_i} \bar{C}_{k,a}$ expresses the capacitance between a panel k and a node i.

The above discussion shows:

- Sensitivities between different nodes can be obtained simply by assembling different sets of associated partial short-circuit capacitances, i.e., $\sum_{a\in N_i} \bar{C}_{k,a}$.
- Sensitivities w.r.t. different parameter variations are local for different sets of victim panels, i.e., $k \in s_{\lambda_p}$.
- The nominal capacitances are computed using the partial short-circuit capacitances $\bar{\mathbf{C}}$.

It follows that the data needed for the sensitivity computation and for the nominal capacitance calculation is the same, i.e., the partial short-circuit capacitances $\bar{\mathbf{C}}$. Therefore, all the sensitivities between various nodes w.r.t. multiple variations can be computed simultaneously with the nominal capacitance extraction. The representation $\mathbf{C}(\lambda)$ in (1) can be obtained with a single extraction procedure. It has been shown in [2], [4] that the extra computational time for the sensitivity is negligible compared to that of the standard capacitance extraction. The high efficiency of capacitance sensitivity computation is essential because during the parasitic extraction, the overall performance is ruled by the capacitance extraction instead of the conductance extraction. As can be seen in the next section, both the nominal conductance extraction and its sensitivity computation are simpler and more straightforward compared to that of the capacitances.

III. Parameterized conductance extraction

This section presents the generation of the parameterized conductances. Firstly, a brief summary of the nominal conductance extraction using the Finite Element Method (FEM) is given (see [15] for more details). Then, the computation of sensitivities w.r.t. geometric variations is presented.

A. FEM-based conductance extraction

To perform FEM, the interconnect polygons are firstly discretized into retangles or triangles, which are often called *tiles*. Computation is then conducted for each tile to place a conductance element to each branch, as illustrated in Fig. 1(a).

For for rectangle tile, the conductance element on a branch between two vertices $(x_i,\ y_i)$ and $(x_j,\ y_j)$ is computed as

$$G_{ij} = \begin{cases} G_{sh}\frac{(x_i-x_j)^2}{2A} & \textit{if } y_i = y_j \\ G_{sh}\frac{(y_i-y_j)^2}{2A} & \textit{if } x_i = x_j \\ 0 & \textit{otherwise} \end{cases} \tag{4}$$

where A is the area of the tile, and G_{sh} is the *sheet conductance*, i.e. the inverse of the *sheet resistance*, of the material. Sheet conductances or sheet resistances of different layers are usually defined in the technology file provided by the foundry. As to the triangle tiles defined by three vertices $(x_i,\ y_i)$, $(x_j,\ y_j)$ and $(x_k,\ y_k)$, the conductance element G_{ij} can be calculated as

$$G_{ij} = G_{sh}\frac{(x_k - x_i)(x_k - x_j) + (y_k - y_i)(y_k - y_j)}{4A}. \tag{5}$$

It is known that the sheet conductance G_{sh} is defined as a product of the conductivity σ and the layer thickness t

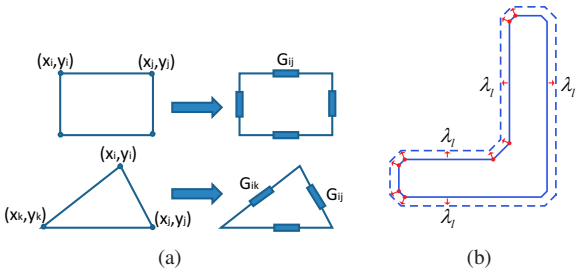


Fig. 1. a) Illustration of the FEM for conductance extraction; b) Illustration of the layout perturbation for the sensitivity computation.

$$G_{sh} = \sigma t. \tag{6}$$

Thus Equation (4) and (5) can be summarized in the following expression:

$$G_{ij}(t_\alpha, l_\alpha) = G_{sh}(t_\alpha) F(l_\alpha). \tag{7}$$

where i and j are indices of the vertices of a tile α, t_α is its corresponding layer thickness, and l_α represents the incident vertices. It follows that the dependencies of G_{ij} on the layer thickness (z-dimension) and the layout (x, y-dimensions) are separate, which is very convenient for the sensitivity computation.

B. Conductance sensitivities

Unlike capacitances, there are practically only two geometric parameters relevant to each conductance, namely the thickness of its corresponding layer t_α and its corresponding tile dimension l_α. Furthermore, as concluded from Equation (7), the sensitivities of a conductance w.r.t. these two parameter variations can be computed separately, i.e.,

$$\frac{\partial G_{ij}}{\partial \lambda_t} = F \frac{\partial G_{sh}(t)}{\partial \lambda_t}; \qquad \frac{\partial G_{ij}}{\partial \lambda_l} = G_{sh} \frac{\partial F(l)}{\partial \lambda_l}. \tag{8}$$

1) Variation in the layer thickness: The calculation is simple and straightforward. Considering the sensitivity is for the nominal or designed system, the substitution of (6) in the left equation of (8) leads to:

$$\frac{\partial G_{ij}}{\partial \lambda_t}|_{t_0} = F\sigma = \frac{F\sigma t_0}{t_0} = \frac{G_{ij}(t_0)}{t_0} \tag{9}$$

where t_0 is the nominal layer thickness and $G_{ij}(t_0)$ is the nominal conductance.

2) Layout variation: The perturbation method is used to calculate the sensitivities w.r.t. the layout variation.

Unlike capacitance, conductance is a *self*-property of an interconnect, depending solely on its own dimension and material property. Thus the sensitivities of all the conductances in the system w.r.t their related layout variations can be obtained using only one finite difference (FD) computation with simply one extra system solve. In this case, the perturbation method is a very appropriate choice for its high accuracy and the modest additional computational complexity, i.e., $1\times$ the nominal extraction.

Using the perturbation method, the sensitivities of conductances w.r.t. the layout variation can be calculated as

$$\frac{\partial G_{ij}}{\partial \lambda_l} = \lim_{\lambda_l \to 0} \frac{G_{ij}(l_0 + \lambda_l) - G_{ij}(l_0)}{\lambda_l} \tag{10}$$

where λ_l is the layout perturbation, $G_{ij}(l_0)$ is the nominal conductance and $G_{ij}(l_0 + \lambda_l)$ represents the perturbed conductance. An issue related to the computation of (10) in a matrix form is that the size of the conductance matrix $\mathbf{G}$ has to remain the same. To do so, the dimension of the perturbed system is generated by adjusting the vertex coordinates of the original system, illustrated in Fig. 1(b). This implies that only the conductances related to the boundary nodes are affected and generate non-zero sensitivities. It can also be seen from the figure that irregular shapes can be handled as well. This has a practical meaning because the corners of wires or wire-like structures are usually no longer right angles after the lithography and the etching processes.

IV. Order Reduction of Parameterized Systems

This section summarizes the pMOR methods for the parameterized system generated in the previous sections, focusing on the structure-preserving technique to be applied in the proposed statistical analysis methodology.

A. Parameterized Model Order Reduction

In Section II and III, the sensitivity-based parameterized LPE technique generates the capacitance and the conductance matrix descriptors (1) for a parameterized system. Such a system has an associated parameter dependent frequency response that can be modeled via the transfer function

$$\mathbf{H}(s, \lambda) = \mathbf{E} \left[s\mathbf{C}(\lambda) + \mathbf{G}(\lambda) \right]^{-1} \mathbf{B} \tag{11}$$

where $\mathbf{C}(\lambda), \mathbf{G}(\lambda) \in \mathbb{R}^{n \times n}$ are the generated parameter dependent capacitance and conductance matrices, and $\mathbf{B} \in \mathbb{R}^{n \times m}$ and $\mathbf{E} \in \mathbb{R}^{p \times n}$ are the input and the output incidence matrices respectively.

For a complete analysis, a linear system of dimension n has to be solved for every point of the parameter plus frequency space. Thus when the size of the system n is large, the analysis of the function in (11) becomes prohibitive.

To overcome this issue, Parameterized Model Order Reduction (pMOR) approaches seek to efficiently generate a reduced order approximation, usually by projecting the system into a suitable reduced q-dimensional subspace, $q \ll n$, spanned by the columns of a projector $V \in \mathbb{R}^{n \times q}$ (see [10], [11], [12] for details). The projection generates a Reduced Order Model (ROM) with an associated reduced transfer function

$$\widehat{\mathbf{H}}(s, \lambda) = \widehat{\mathbf{E}} \left[s\widehat{\mathbf{C}}(\lambda) + \widehat{\mathbf{G}}(\lambda) \right]^{-1} \widehat{\mathbf{B}} \tag{12}$$

where $\widehat{\mathbf{C}}(\lambda), \widehat{\mathbf{G}}(\lambda) \in \mathbb{R}^{q \times q}$, $\widehat{\mathbf{B}} \in \mathbb{R}^{q \times m}$, and $\widehat{\mathbf{E}} \in \mathbb{R}^{q \times p}$ define the ROM of dimension $q \ll n$, which can be handled much more efficiently.

B. Explicit Parameter Matching

Standard projection based pMOR approaches generate a ROM with an equivalent Taylor series formulation for the $\widehat{\mathbf{C}}(\lambda)$ and $\widehat{\mathbf{G}}(\lambda)$ matrices, which is useful in terms of compatibility. This allows for a fast evaluation of the system matrix for any parameter and frequency point, but still requires solving the system in order to obtain the system response.

Any modification of the frequency or parameter values implies another solve. For statistical analysis with potentially a large number of Monte Carlo (MC) samples this could be expensive.

A different pMOR approach which is interesting in terms of statistical analysis is the one presented in [14], valid for the cases in which the output behavior w.r.t. the parameters is smooth, which as can be seen is indeed the case. It proposes a reformulation of the system as a Taylor series approximation of the transfer function w.r.t. the parameters.

This is achieved by a Taylor series representation of the matrices $\mathbf{C}(\lambda)$ and $\mathbf{G}(\lambda)$ as in (1), plus an expansion of the state vector in Taylor series w.r.t. the parameters, but not w.r.t. the frequency:

$$\mathbf{x}(s,\lambda) = \mathbf{x}_0(s) + \sum_{i=1}^{Q}\sum_{j=1}^{T} \lambda_i^j \mathbf{x}_{ij}(s) \tag{13}$$

with $\mathbf{x}_0$ the nominal state vector, and $\mathbf{x}_{ij}$ the sensitivity of order j w.r.t. parameter λ_i. This formulation, presented here without cross terms, can be extended to any desired order, including cross terms. As an example, for a single parameter λ_1, using the first order sensitivities in (1) (i.e. $\mathbf{G}_0, \mathbf{C}_0, \mathbf{G}_1$ and $\mathbf{C}_1$), the states can be approximated as

$$\begin{aligned}
&\mathbf{x}(s,\lambda_1) = \mathbf{x}_0(s) + \lambda_1\mathbf{x}_1 + \lambda_1^2\mathbf{x}_2 + \dots \\
&\mathbf{x}_0(s) = (\mathbf{G}_0 + s\mathbf{C}_0)^{-1}\mathbf{B}u \\
&\mathbf{x}_1(s) = -(\mathbf{G}_0 + s\mathbf{C}_0)^{-1}(\mathbf{G}_1 + s\mathbf{C}_1)\mathbf{x}_0(s) \\
&\mathbf{x}_2(s) = -(\mathbf{G}_0 + s\mathbf{C}_0)^{-1}(\mathbf{G}_1 + s\mathbf{C}_1)\mathbf{x}_1(s) \\
&\dots
\end{aligned} \tag{14}$$

This explicit parameter dependence can be shifted to the output, which generates a parameterized transfer function as

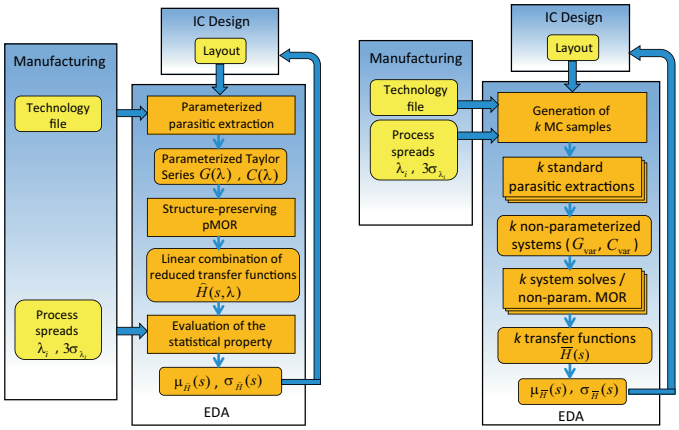
$$\mathbf{H}(s,\lambda) = \mathbf{H}_0(s) + \sum_{i=1}^{Q}\sum_{j=1}^{T} \lambda_i^j \mathbf{H}_{ij}(s) \tag{15}$$

where $\mathbf{H}_{ij} = \mathbf{E}\mathbf{x}_{ij}$ are the frequency dependent transfer function sensitivities, each one related to one sensitivity of the states $\mathbf{x}$. In other words, the parameterized transfer function can be written as the contribution of the nominal transfer function plus the contribution of each one of the non-parameterized transfer function sensitivities w.r.t. the parameters, i.e. a linear combination of the multiple non-parameterized transfer functions weighted by the parameter variation.

The work in [14] also presents a compact state-space formulation for the complete system generating the individual transfer functions in (15), plus a scheme for efficiently reduce each transfer function independently (via preservation of the system structure), in order to maintain the explicit parameter dependence (see [14] for details).

A very important property of this representation and reduction is that it maintains an explicit parameter dependence on the output. Every reduced transfer function sensitivity, which only depends on the frequency, can be solved independently. The parameterized response is obtained by a linear combination of the multiple transfer functions, as shown in (15).

This specific structure-preserving property enables a very fast way to evaluate the statistical properties, for instance the standard deviation of the system response, which is a main topic of this paper presented in Section V.



(a) The proposed method. (b) The Monte Carlo approach.

Fig. 2. Comparison of the design flows.

V. Statistical Analysis of RC Nets

In this section, a statistical analysis methodology for RC nets is proposed. By combining the parameterized parasitic extraction and the structure-preserving MOR techniques, the transfer function and its standard deviation due to process variations can be obtained with a significant speed up, compared to the traditional Monte Carlo approach.

A. Design flow

The design flow of the proposed method is shown in Fig. 2(a). With the input of the layout information from designers and the technology file from the foundry, parameterized RC extraction generates the first order parameterized Taylor series $\mathbf{G}(\lambda)$ and $\mathbf{C}(\lambda)$ as in (1).

As addressed earlier, the computational complexity of setting up the parameterized conductance representation is two times the complexity of the nominal conductance $\mathbf{G}_0$ extraction and the complexity of setting up the parameterized capacitance representation is the same as the nominal capacitance extraction. The computation of capacitances rather than conductances dominates the overall computational burden. Therefore, the asymptotic complexity of the parameterized RC extraction, including the nominal values $\mathbf{G}_0$, $\mathbf{C}_0$ and their sensitivities $\mathbf{G}_i$, $\mathbf{C}_i$ w.r.t. multiple parameters, is the same as the asymptotic complexity of the nominal capacitance extraction. The high efficiency of the parameterized extraction technique is one essential advantage of the proposed method.

Using the generated parameterized matrix descriptors (1), the structure-preserving pMOR technique is applied to calculate the reduced transfer function with an explicit parameter dependence, expressed as

$$\widehat{\mathbf{H}}(s,\lambda) = \widehat{\mathbf{H}_0}(s) + \sum_{i=1}^{Q}\sum_{j=1}^{T} \lambda_i^j \widehat{\mathbf{H}_{ij}}(s) \tag{16}$$

where i represents the parameter index and j represents the order in the Taylor series expansion.

Applying various parameter settings, the induced variability of the transfer function can be easily obtained from (16).

Furthermore, and unlike standard projection based pMOR approaches, any change of the parameter variations simply requires evaluating the linear combination, with no need for additional system solution. This implies a major boost in the efficiency of the variational analysis.

More importantly, given the process spreads of the parameters ($3\sigma_{\lambda_i}$) from the manufacturer, the statistical properties such as the standard deviation of the transfer function (16) can be evaluated with a negligible complexity. This is the other essential advantage of the proposed method and will be discussed in Section V-B.

At last, the obtained mean and the standard deviation of the transfer function $\mu_{\widehat{H}}(s)$ and $\sigma_{\widehat{H}}(s)$ are fed back to the designers so that adjustments or improvements can be carried out before the tape-out. Thus the proposed method is very convenient for design exploration and optimization.

Traditionally, to compute the statistical properties of the transfer function, one has to perform a Monte Carlo simulation which implies a huge computational burden, as indicated in Fig. 2(b). A parameter sampling has to be conducted using the layout and the technology based on the particular process spread, obtaining k problem instances for the layout parasitic extractor. Then k standard parasitic extractions are performed to generate k non-parameterized systems, i.e., k groups of matrix descriptors ($\mathbf{G}_{var}$, $\mathbf{C}_{var}$). This is followed by k SPICE simulations or non-parameterized MOR procedures. Finally, the mean and the standard deviation of the transfer function can be calculated from the generated k transfer functions, denoted $\bar{\mathbf{H}}(s)$. Since the sampling number k has to be large enough to ensure a reliable statistical distribution, the workload of this traditional design flow is enormous due to the k-fold parasitic extraction and the k-fold SPICE simulation or standard MOR procedure.

The above two issues are conquered by the parameterized LPE and the structure-preserving pMOR techniques respectively. As addressed earlier and as shown in Fig. 2(a) both techniques avoid the parameter sampling. Therefore, the proposed method is highly efficient as a variation-aware modeling approach, especially for statistical analysis.

B. Statistical property computation

This section explains the computation of the mean and the standard deviation of the reduced transfer function (16). Both the nominal transfer function $\widehat{\mathbf{H}_0}(s)$ and the transfer function sensitivities $\widehat{\mathbf{H}_{ij}}(s)$ are vectors of the system response to frequency samples. The following computation is conducted for each frequency sample s_k. For the ease of discussion, some short-hand notations are used:

$$a_0 = \widehat{H_0}(s_k), \quad a_{ij} = \widehat{H_{ij}}(s_k), \quad F(\lambda_i) = \sum_{j=1}^{T} a_{ij}\lambda_i^j \tag{17}$$

The expectation of $\widehat{H}(s_k, \lambda)$ can then be expressed as

$$E[\widehat{H}(s_k, \lambda)] = E[a_0] + \sum_{i=1}^{Q} E[F(\lambda_i)] \tag{18}$$

where the expectation of $F(\lambda_i)$ can be computed as

$$E[F(\lambda_i)] = \int_{-\infty}^{\infty} \sum_{j=1}^{T} a_{ij}\lambda_i^j f(\lambda_i) d\lambda_i = \sum_{j=1}^{T} a_{ij} \int_{-\infty}^{\infty} \lambda_i^j f(\lambda_i) d\lambda_i \tag{19}$$

with $f(\lambda_i)$ the probability density function of λ_i. It is common to assume the distribution of the geometric parameter variations to be Gaussian with a mean of zero, i.e., $\lambda_i \sim \mathcal{N}(0, \sigma_{\lambda_i})$. Then $\int_{-\infty}^{\infty} \lambda_i^j f(\lambda_i) d\lambda_i = E[(\lambda_i - \mu_{\lambda_i})^j]$, $\mu_{\lambda_i} = 0$ is the central moment of the Gaussian distribution with a zero mean of order j. Therefore Equation (19) is a linear combination of the Gaussian central moments from order 1 to T, weighted by the corresponding transfer function sensitivities. The mean of the transfer function ($\mu_{\widehat{H_0}}(s_k)$) can then be computed by substituting (19) into (18).

As for the standard deviation, it is interesting to note that the variations of different parameters are usually independent since they are originated from different process steps. Assuming this is the case, the standard deviation of $\widehat{H}(s_k, \lambda)$ can be computed as

$$\sigma_{\widehat{H}}(s_k) = \sqrt{\sum_{i=1}^{Q} Var(F(\lambda_i))} \tag{20}$$

where the variance of $F(\lambda_i)$ is calculated as follows, using the computed expectation of $F(\lambda_i)$, i.e., $\mu_{F(\lambda_i)}$,

$$Var(F(\lambda_i)) = E[(F(\lambda_i)) - \mu_F)^2] = -\mu_{F(\lambda_i)}^2 + E[F^2(\lambda_i)], \tag{21}$$

with

$$E[F^2(\lambda_i)] = \sum_{j=1}^{T} a_{ij}^2 E[\lambda_i^{2j}] + 2\sum_{j=1}^{T}\sum_{l=j+1}^{T} a_{ij}a_{il}E[\lambda_i^{j+l}]. \tag{22}$$

Note that $E[\lambda_i^{2j}]$ and $E[\lambda_i^{j+l}]$ are also central moments of Gaussian distributions with zero means. Thus substituting (22) into (21) solves the variance of $F(\lambda_i)$, which is a combination of the Gaussian central moments with various orders, weighted by the incident transfer function sensitivities. At last, the standard deviation of the transfer function of each frequency sample $\sigma_{\widehat{H}}(s_k)$ can be obtained by substituting (21) into (20).

C. Experiment and result

To demonstrate the efficiency and the accuracy of the proposed method, a two-terminal RC structure is studied. As shown in Fig. 3, the example has two layers consisting of a meandering poly resistor connected to terminal **A** and a fork-structure metal capacitor connected to terminal **B**. This example is initially modeled with 410 states.

This structure depends on six geometric parameters, namely the poly and the metal thicknesses, the layout variations and the dielectric thicknesses of the two layers. The process spreads ($3\sigma_{\lambda_i}$) of these parameters are assumed to be 10% of their nominal values.

To verify the results of the proposed method, a Monte Carlo (MC) simulation with 700 samples is performed according to the flow in Fig. 2(b). The experiment is conducted on a 3.00GHz Intel 2 Core CPU. Results are summarized in Fig. 4. The mean of the transfer function computed by the proposed

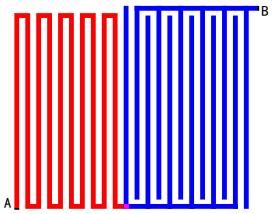
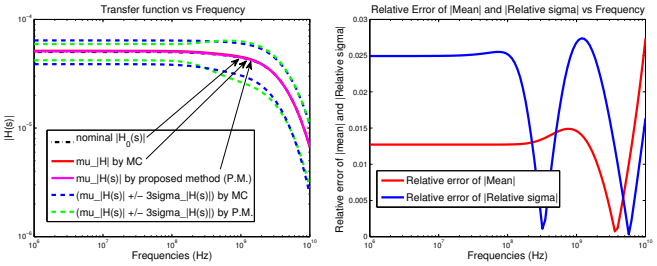


Fig. 3. Layout of the RC example with a poly resistor (red) and a metal capacitor (blue).



(a) (b)

Fig. 4. Results of the proposed method compared to the Monte Carlo approach.

method shows a perfect agreement with the result obtained from the MC simulation (see Fig. 4(a)). To indicate how much is the effect of the assumed process spreads, Fig. 4(a) also shows the induced variability of the transfer function, i.e., $\mu_H(s) \pm 3\sigma_H(s)$, given by the proposed method and the MC approach respectively. Fig. 4(b) shows the accuracy of the proposed method. The relative error of the computed mean $\mu_{\hat{H}}(s)$ is very small, with an average over the frequency being 0.0125 and the maximum being 0.0274. The computed relative standard deviation $\frac{\sigma_{\hat{H}}}{\mu_{\hat{H}}}(s)$, known as the *mismatch* by designers, has an average relative error (absolute value) over the frequency of 0.0206 and a maximum of 0.0274. Therefore, the proposed method nicely captures the effect of the physical process variations on the system response.

More importantly, the proposed method achieves a significant speed up over the traditional Monte Carlo approach. Table I shows the elapsed time and speed ups in the extraction procedure and the evaluation of the statistical properties, for the traditional MC analysis, the traditional MC analysis plus non-parameterized MOR (in this case PRIMA [16]) on each extracted system, and the proposed approach. Note that the speed up, which already achieves two orders of magnitude for a middle size example (410 states), will further increase as the increase of the size of the system and the number of samples.

VI. Conclusion

This paper presents a highly efficient statistical analysis methodology for RC nets subject to manufacturing variabilities. It achieves *zero* parameter sampling, based on the combination of a sensitivity-based parameterized parasitic extraction technique and a structure-preserving pMOR technique. Given the layout and the process spreads of the technology, the statistical properties such as the mean and the standard deviation of the system response can be obtained extremely fast. As such, the proposed method provides a very convenient tool for design exploration and optimization.

TABLE I
CPU time of the proposed method and the MC approach

	Extraction	Evaluation	Total
MC	$55h49'$ $(1\times)$	$46'12''$ $(1\times)$	$56h35'$ $(1\times)$
MC + MOR	$55h49'$ $(1\times)$	$3'30''$ $(13\times)$	$55h52'$ $(1.01\times)$
Proposed	$4'49''$ $(695\times)$	$0.43''$ $(6446\times)$	$4'50''$ $(701\times)$

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