Improving Bit Error Rate Under Burst Noise in OFDM Power Line Communications

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Abstract

Noise in Power Line Communications is typically impulsive, with impulses being a fraction of the OFDM symbol length. Because of their large duration the impulse can also be called bursts. The short duration of the burst compared with the OFDM symbol length implies that there is a strong correlation between the noise at different carriers, given a determined burst position. The position can be determined using an estimate of the noise after a first demodulation. The high correlation is used to develop demodulators with a reduced bit error rate in comparison with conventional demodulators, so increasing the capacity. The demodulators use a smoothed estimate of the noise signal or a new metric for the distance based on the new correlation matrix. About half of the bit errors can be corrected in this way, corresponding to a 1 dB improvement in Signal Noise Ratio (SNR). How to split the OFDM symbol without increasing the overhead due to the circular prefix is also shown. Noise measurements in power lines are presented. These measurements are used in the simulations.

Keywords: PLC, Impulsive Noise, Communications, Noise Reduction,
1. Introduction

Power Line Communications (PLC) [1][2] offers a network with no new wires and so it is a competitor with wireless, ADSL and other technologies to provide a network either to our and in our homes. However, the PLC channel is a difficult one. It has varying characteristic impedance, exhibits frequency selective fading, nulls in its frequency response, and high noise at low frequencies. The frequency selective fading can be dealt with using Orthogonal Frequency Division Multiplexing (OFDM) with adaptive modulation or other [3, 4, 5]. In the paper OFDM modulation is assumed. The power line channel is very noisy at low frequencies. The noise is mostly impulsive [6, 7], with impulses with a relatively long duration, but much shorter than the OFDM symbol length. Because of the long duration this could also be called bursts. Through the paper the words impulse or burst will be used interchangeably. In the paper we present techniques to improve the Bit Error Rate (BER) resulting in an increase in capacity under this kind of noise. Note that the algorithms presented are only capable of reducing the noise at the OFDM symbols where there is one single dominant burst, and are disabled for other symbols, but this is a common case. Through the text, $u_k$ is the element $k$ of vector $u$ for any vector $u$, and $C_{i,j}$ is the element at line $i$ and column $j$ of matrix $C$, for any matrix $C$. Also $|x|$ is the absolute value of $x$. 

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2. State of the Art

There are many papers dealing with impulsive noise reduction. However, they are not usually targeted to OFDM PLC, and normally deal with short impulses, one sample width, while in PLC these are much wider, more like bursts, sometimes reaching a large fraction of the OFDM symbol as shown in [6, 7]. Namely, the impulse noise model used is usually memoryless; for instance the Middleton Class A impulsive noise model, [8, 9]. However, note that bursts can be converted to a set of distant impulses by an interleaver.

The simplest methods for impulse noise reduction consist of clipping or nulling the impulses. In [10, 11] clipping and nulling are analysed, and in [12] a practical method for computing the clipping threshold is presented. In [13] Empirical Mode Decomposition (EMD) is used to reduce impulsive noise. They achieved a noise reduction of about 3 dBs, but tested the algorithm only in very simple signals. They classified the components after EMD with higher amplitude as noise.

Other methods are iterative. In [14] the noise is estimated after demapping and pilot insertion. Then a peak detector is used to find the impulses and the estimated impulsive noise is subtracted from the signal. De-mapping and pilot insertion are done again. This is repeated a predetermined number of times. Improvements of about 1 dB in the signal to noise ratio for real noise recorded from a hairdryer are reported. In [15] the signal is coded by applying a simple liner transformation ($G$) to an input vector. $G$ can be the Discrete Fourier Transform (DFT) as in OFDM modulation. A new iterative decoding algorithm is presented. The time domain received signal and the signal estimate is used to estimate the impulsive noise using
a MMSE estimator; then the impulsive noise estimate and the transform domain received signal is used to estimate the signal using a MMSE or MAP estimator, and this is done recursively. A Middletons Class A noise model is used for the impulsive noise in the time domains, and a Gaussian model for the impulsive noise in the transform domain. Large improvements in the BER are reported. In [16] convex programming techniques were used to determine impulse positions. Then their amplitudes are determined and the impulses are removed from the signal. The pilot and unused carriers are used to determine the impulse positions. Gains of up to 3 bits in the capacity are reported. The results are compared with the Gaussian-erasure channel.

Other methods are modifications to forward error correction (FEC) coding. This can be using convolutional codes with erasure [17, 18], turbo codes [19, 20] or low density parity check codes [21, 22].

3. OFDM MODEM

In Fig. 1 the OFDM MODEM considered in the current work is presented. It is composed of a number of blocks. First the serial bit stream is converted to a parallel stream by the S/P block. Then the bits are combined into groups of $\log_2(K)$ for K-QAM, and modulated by a set of $(N/2 - N_G)$ QAM modulators, where $N$ is the OFDM symbol length minus the circular prefix (CP), and $N_G$ is the number of guard carriers. The guard carriers will be set to zero, and will appear in the lower and upper part of the spectrum. This will result in a vector, $\mathbf{T}(n)$, with $N/2$ complex values. Then the Mi (mirror) block will fill the remaining $N/2$ entries of this vector, with $\mathbf{T}_{N-k}(n) = \mathbf{T}_k^*(n)$ so that Inverse Fast Fourier Transform (IFFT) of $\mathbf{T}(n)$ will result in a real
signal. The IFFT block will do a \( N \) point IFFT as the name implies, resulting in the vector, \( t(n) \). After the IFFT a circular prefix of length \( N_P \) is added, by copying to the beginning of \( t(n) \) the last \( N_P \) samples of \( t(n) \). Finally the signal is transmitted through the channel. At the receiver, the circular prefix is removed by the CPR block, resulting in signal \( r(n) \). This includes a noise term \( u(n) \). Then, a \( N \) point Fast Fourier Transform (FFT) is applied to the signal, resulting in \( R(n) \). The SL (select) block discards the second half of \( R(n) \) since it is just the conjugate of the first half. The EQ (equalizer) compensates for the frequency response of the channel by multiplying the signal at each carrier by the inverse of the frequency response of the channel. The resulting signal, discarding the guard carriers, will pass through a set of QAM demodulators, resulting in a parallel bit stream. Finally, the bit stream is converted to a serial bit stream by the P/S block.

![Diagram](image)

Figure 1: OFDM MODEM.

4. Power Line Noise Measurements

In a previous paper [7] the authors presented a set of measurements of power line noise. These measurements, along with the work of [6] motivated
the work presented in this paper, and are used in the simulations. Fig. 2 presents a typical waveforms for the signal and noise, for a Signal no Noise Ratio (SNR) of about 7 dB, using the noise measured in [7].

![Figure 2: Plot of typical signal (in blue) and noise (in red) for a SNR of 7 dB and for the duration of one typical OFDM symbol. Samplig rate was 200 MHz.](image)

In [7] it is shown that shifting the impulses position to the origin in each received OFDM symbol results in high correlation between the noise at different carriers.

Namely, consider the noise of the received OFDM symbol, after circular prefix removal, \( \mathbf{u}(n) = [u_0(n) \ldots u_{N-1}(n)]^T \), and its DFT, \( \mathbf{U}(n) = [U_0(n) \ldots U_{N-1}(n)]^T \). Let \( d \) be calculated so that the correlation between different elements of \( \mathbf{U}_2(n) \) is maximized, with \( U_{2k}(n) = U_k \exp(2\pi kd/N i) \).

If the noise is a single impulse, then \( d \) is the impulse position, so this correspond to shifting the impulse to the origin in the time domain. Define the cross-correlation matrix of \( \mathbf{U}_2(n) \) by,

\[
\Sigma = \mathbb{E}[\mathbf{U}_2(n) \mathbf{U}_2^H(n)],
\]

(1)
and the matrix of correlation coefficients, $C$ by,

$$ C_{i,j} = \frac{\Sigma_{i,j}}{\sqrt{\Sigma_{i,i}\Sigma_{j,j}}}. \quad (2) $$

The values for correlation coefficients, the diagonals of $C$ are plotted in Fig. 3. It can be seen that the correlation coefficients of adjacent carriers take large values. The impulse position, $d$, that maximizes the correlation of the

![Figure 3: Values for Correlation coefficient of adjacent OFDM carriers, with a distance of one (diagonal 1), carriers with a distance of two (diagonal 2), three and four. The values are taken from the corresponding diagonal of the correlation coefficient matrix. The correlation decreases with increasing distance. The length of the OFDM symbols was 2467 samples at 200 MHz.](image)

noise between carriers can be calculated from [7],

$$ 2\pi d/N = \angle \left( \sum_{k=0}^{N-2} (U[k]U[k+1]^H) \right). \quad (3) $$

where $\angle(x)$ stands for the angle of the complex number $x$, and gives results in $0$ to $2\pi$. Note that if $u(n)$ is a single impulse at the origin, then $U(n)$ is constant, so the correlation coefficients between its elements are equal to
5. BER Improvement Techniques

In this section we present three techniques to reduce the BER in PLC with impulsive noise. The first two are based on the frequency domain correlations of the time shifted noise signals, and the last one is based on changing the OFDM symbol length while keeping a low overhead due to the circular prefix. We call the algorithms in section 5.1 and section 5.2 algorithms I and II and the technique in section 5.3 technique III.

5.1. Frequency Domains Noise Smoothing and Subtracting

The frequency domain shifted noise signal at the receiver, $U_2[k]$ varies slowly with $k$, because it is low frequency, since it is the DFT of an impulse or burst at the origin. When a signal is received in a OFDM MODEM it will be the sum of the transmitted signal at the receiver, $s(n)$, plus the noise signal, $u(n)$, $r(n) = s(n) + u(n)$. The transmitted signal will normally follow a QPSK or QAM constellation. At a first iteration the transmitted signal can be estimated by simply demodulation of the QPSK or QAM signal. This will give us an estimate of the received and noise signals. The estimation will not be perfect because of demodulation errors in the case of high noise signals. In cases where the noise signal is dominated by a simple impulse we can proceed as follows. If we take the OFDM symbol vector of the noise estimate time shifted so that the impulse is at the origin, $\hat{u}_2[n]$, then its DFT, $U_2[k]$, should be smooth. This is not always the case because of decision errors on the demodulation process. But it means that these errors can be easily spotted, since there will be abrupt changes in an otherwise slow varying
signal, see Fig. 4. This is not that simple for several reasons, for instance, the errors can appear together in regions where the noise signal takes large values. However, this still suggests the following procedure to reduce the impulsive noise:

1. Demodulate the signal and estimate the noise signal.
2. Estimate the impulse position using the estimated noise signal and (3).
3. Low-pass filter the time shifted noise signal to remove sudden changes and then reverse shift it.
4. Subtract the new noise estimate from the received signal.
5. Repeat until there are no more changes.

In the paper, we use a size 3 moving average filter as the low pass filter. Other filters and other sizes could also be used.

It remains to explain how the OFDM noise symbols with a single impulse are detected. This is done as follows. It is taken that, when there is a large uncertainty in calculating the impulse position using (3) it means that there are zero or more than one impulses and no impulsive noise reduction is done on this symbol. The uncertainty is calculated using the standard deviation of the unit norm normalization of the term inside the summation of (3). That is, one calculates \( \rho \), given by,

\[
\gamma_k = \frac{|\mathbf{U}_i[k] \mathbf{U}_i[k+1]^H|}{|\mathbf{U}_i[k] \mathbf{U}_i[k+1]^H|}
\]

\[
\rho = \sqrt{\sum_{k=0}^{N-2} \frac{|\gamma_k - \bar{\gamma}_k|^2}{N-1}}/(2\pi |\bar{\gamma}_k|)
\]
and then do the noise reduction procedure when \( \rho < \rho_{\text{min}} \). Experimentally it was determined that a good value for \( \rho_{\text{min}} \) is 0.2. Note that \( \bar{\gamma}_k \) is the mean of \( \gamma_k \), namely \( \bar{\gamma}_k = \frac{\sum_{k=0}^{N-2} \gamma_k}{(N - 1)} \).

![Figure 4: The noise estimate and the demodulation errors. It can be seen that the noise signal is smooth with two abrupt changes corresponding to the demodulation errors.](image)

### 5.2. Using a New Distance Metric

In this technique for impulsive noise reduction an approximation for the value of frequency domain OFDM symbol vector cross-correlation matrix \( \hat{\Sigma} \), is used for the demodulation of the received signal. The value of \( \hat{\Sigma} \) is used in Maximum Likelihood (ML) [23] demodulation. Let’s say that the frequency domain transmitted symbol after a circular time shift of \( d \) is \( S_2[k] \) and that the channel is characterized by additive Gaussian noise with covariance matrix \( \Sigma \). Then the distribution of time shifted receiving signal, \( R_2[k] \), is multi-variable Gaussian. Define the vectors formed by the real and complex components of the time shifted receiver and transmitted signals,

\[
S_2 = [\text{re}(S_2[0]), \text{im}(S_2[0]), \ldots, \text{re}(S_2[N - 1]), \text{im}(S_2[N - 1])]^T
\]  

(6)
\[ \mathbf{R}_2 = [\text{re}(r_2[0]), \text{im}(R_2[0]), \ldots, \text{re}(R_2[N - 1]), \text{im}(R_2[N - 1])]^T \]  

(7)

the noise signal is, \( \mathbf{U}_2 \),

\[ \mathbf{U}_2 = \mathbf{R}_2 - \mathbf{S}_2 \]

(8)

the covariance matrix of the noise is,

\[ \mathbf{\Sigma} = \mathbf{E}[\mathbf{U}_2 \mathbf{U}_2^T] \]

(9)

the PDF of \( \mathbf{R}_2 \) is given by,

\[ P_r = \frac{1}{\sqrt{(2\pi)^N|\mathbf{\Sigma}|}} \exp \left( -\frac{1}{2} (\mathbf{R}_2 - \mathbf{S}_2)^T \mathbf{\Sigma}^{-1} (\mathbf{R}_2 - \mathbf{S}_2) \right) , \]

(10)

where \( |\mathbf{\Sigma}| \) is the determinant of \( \mathbf{\Sigma} \). Note that \( \mathbf{\Sigma} \) is the covariance matrix of the noise after the impulse has been moved to the origin. If the impulse were symmetric around its center then its DFT after moving to the origin would have only a real component. This is not generally the case, since the impulses can have arbitrary shapes, so the DFT of the noise will have arbitrary real and complex components with different powers. This means that the diagonal entries of \( \mathbf{\Sigma} \) that give the power of real and imaginary components will be different. Also the entries that correspond to the correlation between real and imaginary components will be zero. This is because the real component of the noise will be related to the even part of the impulse and the imaginary component will be related to the odd part of the impulse, and the two are unrelated. Maximizing the value of \( P_r \) in (10) in respect to \( \mathbf{S}_2 \) corresponds
to minimizing the value of

\[(R_2 - S_2)^T \Sigma^{-1} (R_2 - S_2)\]  \hspace{1cm} (11)

that corresponds to minimizing the distance between \(R_2\) and \(S_2\) but in a new metric given by \(\Sigma\). This is a Mahalanobis distance. An approximation for \(\Sigma\), \(\hat{\Sigma}\), instead of the real \(\Sigma\) is actually used.

The technique starts by detecting if there is one and only one impulse on the OFDM symbol, as in section 5.1, using (5). Note that impulses are bursts. Then impulse position must be determined, using the techniques presented in previous sections namely, using (3).

The high correlation and low frequency nature of \(U_2[k]\) means it varies slowly, or is close to constant if we take only a small number of adjacent carriers. Note that this is not the case of \(U[k]\) since it will correspond to an impulse at an arbitrary position, \(d\), and so will be closer to \(e^{-2\pi kd/N} i\).

Then we take three carriers at a time and do the demodulation assuming that \(U_2[k]\) is constant across the three carriers. This corresponds to using as the estimate for the covariance matrix of the noise in the three carriers, the matrix,

\[
\hat{\Sigma} = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1
\end{bmatrix}.
\]  \hspace{1cm} (12)
The choice of $\hat{\Sigma}$ will influence which is the symbol that is selected as the most likely transmission (the one that is closer to the received in the new metric), but it can be multiplied by a scalar constant without influencing the result. This will only affect the value of the distance but not its minimum. So the matrix could have been filled with any value instead of the ones shown, that the demodulator would perform in the same way. Making the matrix diagonal equal to one simply means that the power of the noise at the three carriers is taken to be the same. The values of the correlations between real and imaginary components are all zero for the reasons already stated. In this case the correlation between real or imaginary parts of different carriers will be equal to the power of the noise at the different carriers, since the noise is taken to be constant. That is $E[U_2[k]U_2[k + l]] = E[U_2[k]^2] = 1$.

In order to determine the transmitted symbol one wishes to minimize (11). Assuming that the transmitted symbol is QPSK then one wishes to determine which point of the possible constellation points minimize the distance in the new metric. This is done by calculating the distance for each possible constellation point and determining the minimum. This is more computationally costly than the classic Euclidean distance case, where the determination of the closest constellation point can be done by simple rounding. In the case of QPSK there are 4 constellation points in three carriers resulting in an evaluation of $4^3 = 64$ points, corresponding to 6 bits. Note that the search has to be done in the three carriers. Although more costly, this is not prohibitive. In the case of QAM the same procedure could be used, but for instance for 16-QAM the number of evaluations would grow to $16^3 = 4096$. So instead of searching in all the possible constellation points only the four
points surrounding the received signal at each carrier are searched. This will keep the number of evaluations at 64, while still giving good performance since it is unlikely that the best constellation point would be very far from the received point.

The matrix $\hat{\Sigma}$ is the covariance matrix of the time shifted impulsive noise signal. In order to use it for the demodulation, and since we time shift the noise signal, we must also time shift the transmitted signal. The transmitted signal is unknown. The goal is to determine it. So instead of time shifting the transmitted signal, one has to time shift each of the possible transmissions, which corresponds to time shift the signals in the constellations. The constellation will become distorted. The time shift will correspond to multiplying $S(k)$ by $e^{2\pi k d/N i}$ so this will correspond to a rotation dependent on $k$.

The matrix $\hat{\Sigma}$ as presented is singular. In order to make it invertible it is required to consider that the noise signal is composed by an impulse embedded in a small amount of white noise. The new $\hat{\Sigma}$ will be equal to $\hat{\Sigma}$ plus a diagonal matrix with the white noise power, $\sigma_n^2 I$ as in (13). Experimentally it was determined that a good values for the white noise power is $\sigma_n^2$ is 0.1.

$$\hat{\Sigma} = \hat{\Sigma} + \sigma_n^2 I$$  \hspace{1cm} (13)

The method for QAM demodulation using the new distance metric consists in the algorithm presented in table 1.

In order to understand better what is happening one can determine what are the regions of constant distance using the new metric dictated by the matrix $\hat{\Sigma}$. In the classical Euclidean distance with $\hat{\Sigma} = I$ this would be the sur-
for each carrier $k$

// received signal without time shift
$S_r = R(k - 1 : k + 1)$

// received signal with time shift
$S_{r1} = R_1(k - 1 : k + 1)$

$S_o, D = \text{QAMDemodulator}(S_r)$
// This is the center for the 4
// constellation points to be tested for each $k$
// and the decimal representation of its binary value

$S_{r2} = [\text{re}(S_{r1}(1)), \text{im}(S_{r1}(1)), \text{re}(S_{r1}(2)), \text{im}(S_{r1}(2)), \text{re}(S_{r1}(3)), \text{im}(S_{r1}(3))]'$

// test all 64 possible transmission combinations
// and find the minimum distance

$d_{\text{min}} = 1e99$
for $i=0$ to 63
    $b_t = \text{binary}(i)$ // It’s a vector
    $S_t = b_t \times 2 - 1 + S_o$

    // calculate the rotated constellation
    $S_{t1}(k - 1) = (S_{t1}(1) + S_{t1}(2)i)e^{\frac{2\pi(k-1)d}{N}i}$
    $S_{t1}(k) = (S_{t1}(3) + S_{t1}(4)i)e^{\frac{2\pi(k-1)d}{N}i}$
    $S_{t1}(k + 1) = (S_{t1}(5) + S_{t1}(6)i)e^{\frac{2\pi(k-1)d}{N}i}$

    $S_{t2} = [\text{re}(S_{t1}(k - 1)), \text{im}(S_{t1}(k - 1)), \text{re}(S_{t1}(k)), \text{im}(S_{t1}(k)), \text{re}(S_{t1}(k + 1)), \text{im}(S_{t1}(k + 1))]$

    // calculate the distance
    $\text{distance} = (S_{r2} - S_{t2})' \hat{\Sigma}^{-1} (S_{r2} - S_{t2})$

    if (distance < $d_{\text{min}}$)
        $d_{\text{min}} = \text{distance}$
        $b_b = b_t$
    end
end

bits = convert to gray ($D, b_b$)

place bits in bit vector

end

Table 1: The algorithm for QAM demodulation using a new distance metric.
face of a sphere. With the new metric dictated by (11) it will be an ellipsoid. This can be seen by diagonalizing \( \Sigma \) resulting that in the new coordinate system the equations for fixed distance will simply become \( \sum_i (x_i - y_i)^2 / \lambda_i = d \), which is the equation for an ellipsoid. Fig. 5 represents a four point constellation with the four candidates to the transmitted signal, the received signal, and an ellipse corresponding to a constant distance in the new metric. It can be seen that, using Euclidean distance the most likely candidate for the transmitted signal would be the point in the lower right corner, while using the new metric the most likely candidate is the point in the lower left corner.

Figure 5: Four constellation points (o) the received signal (x) and the ellipse marking the constant distance in the new metric.

We can also discuss the binary case where it is simple to see that assuming the noise is approximately constant across the three carriers will allow correcting decision errors. If the noise is constant it means that the transmitted signal will simply be shifted up or down in its entirely, by the noise. This means that any transition from high to low or low to high will identify without doubt ones and zeros in a binary transmission. For a binary signal all signals would be identified without errors but the 000 and 111, that are constant.

5.3. Changing the OFDM Symbol Length

The length of the OFDM symbol influences the equivalent noise level at the frequency domain caused by impulsive noise. Namely consider, without
loss of generality, that in the OFDM MODEM, the normalized DFT is used so that the energy of the signals is equal before and after the DFT. In time domain an impulse is usually spread through a small number of samples, and in the frequency domain through most of the carriers. Since its energy is equal in both cases this means that for a larger number of carriers the impulsive noise will spread through a larger number resulting in lower noise levels. The signal on the other hand is equally spread in time and frequency domain. So the signal to noise in each carrier will increase with the number of carriers. This up to the point where one starts to get more than one impulse (or burst) per OFDM symbol. This will actually result in a lower error probability if the spreading is enough to make the noise lower than the values that produce demodulation errors, although the number of carriers affected by a single impulse is larger. On the other hand, if the demodulation was in the time domain a short impulse would only affect a few samples while in the frequency domain it will affect most of the carriers. So, when moving from time to frequency domain there is a spreading of the noise, but this can lead to a higher or lower error probability depending on the case. If the noise level is high it is better to keep it concentrated in time domain so that it only affects a few bits, instead off all the carriers. If the noise level is low, then, spreading it, will still lower its level and possibly remove errors. This means that in high noise scenarios it will be desirable to reduce the OFDM symbol length. This can be confirmed by the simulations. See Fig. 11 and 12.

However, reducing the OFDM symbol length will reduce the efficiency of the MODEM since it will increase the overhead due to the circular prefix.
The OFDM symbol is often chosen to be much larger than the circular prefix, so that it does not represent a large penalty in performance. In this section we present a technique that is equivalent to reduce the OFDM symbol length but without increasing the overhead due to the circular prefix. It consist of adding a set of $N/M$ FFTs of dimension $M$ before the IFFT of the OFDM MODEM at the modulator and then doing the reverse at the demodulator by adding a set of IFFT blocks at the output of the OFDM FFT, as presented in Fig. 6. This has the effect of splitting the OFDM symbol in $M$ parts. One can see what happens to the impulsive noise signal at the receiver. It will pass through a FFT that will spread the noise through the carriers but then it will pass through a IFFT that will concentrate the noise again in time slots. The full, length $N$, OFDM symbol will be spread through frequency and time. This will reduce the error rate in the cases of high noise levels.

![Figure 6: OFDM MODEM block structure with a equivalent shorter time duration of the OFDM symbol.](image)

### 6. Simulation Results

In this section the simulation results from the algorithms presented in the previous sections are shown. Simulations from the algorithm by Zhidkov [14]
are also shown for comparison. All the algorithms were tested with real noise measured in power lines and presented in section 4. The sampling frequency was 20 MHz, the OFMD symbol length was 256 samples corresponding to 128 carriers with 12 upper and 12 lower frequency guard carriers. In the Zhidkov algorithm the impulse detection threshold parameter, $C$ was set to 5 and the number of iterations of the algorithm was also set to 5. This were the parameters that resulted in best performance.

![Probability (Pe) of error versus SNR for algorithm I for QPSK modulation.](image)

Figure 7: Probability (Pe) of error versus SNR for algorithm I for QPSK modulation. $\bigcirc$ is with impulsive noise reduction; $\bigtriangledown$ is without impulsive noise reduction and $\times$ is the Zhidkov algorithm.

We plot the uncoded bit error probability, $P_e$, versus the SNR. Results are shown for QPSK and 16-QAM modulations. The lines for the $P_e$ of the MODEMs with impulse noise reduction is always below the line for the MODEMs without impulse noise reduction in all Figs. It can be seen that for a typical uncoded bit error probability of $10^{-2}$ there is a gain of about 1 dB in the SNR for the algorithms I and II (figs. 7, 8, 9 and 10). This corresponds to a reduction in the error probability to about half, meaning that half of the original decoding errors are corrected by the algorithms to deal with
Figure 8: Probability (Pe) of error versus SNR for algorithm I for 16-QAM modulation. ⭕ is with impulsive noise reduction; ▽ is without impulsive noise reduction and × is the Zhidkov algorithm.

Figure 9: Probability (Pe) of error versus SNR for algorithm II for QPSK modulation. ⭕ is with impulsive noise reduction; ▽ is without impulsive noise reduction and × is the Zhidkov algorithm.

impulsive noise. The algorithm by Zhidkov show even better performance achieving about 2 dB gain in the SNR. Note that the proposed algorithms and Zhidkov algorithm are very different and that is should be possible to use both simultaneously, resulting in even better performance.

The results corresponding to the technique III are presented in figs. 11
Figure 10: Probability (Pe) of error versus SNR for algorithm II for 16-QAM modulation. ○ is with impulsive noise reduction; ▽ is without impulsive noise reduction and × is the Zhidkov algorithm.

Figure 11: Probability (Pe) of error versus SNR for technique III for QPSK modulation. ○ is with impulsive noise reduction and ▽ is without impulsive noise reduction.

and 12. The number of division of the OFMD symbol, M, corresponding to the length of the added small FFT and IFFT in Fig. 6 was 8. In this case for high noise levels corresponding to an error probability of about $10^{-1}$ there is a gain in the SNR of about 5 dB.

Note that this algorithm has the effect of concentrating the bursts in
shorter OFDM symbols. Then there are two cases to be considered: the noise level is high or the noise level is low. If the noise level is high then a single impulse will be able to cause errors in all carriers of an OFDM symbol, having shorter symbols means less carriers and lower errors. If the noise level is low, then the noise may not be enough to cause errors in the carriers but, in a shorter symbol the burst will have more importance, degrading the SNR in each carrier and increasing the probability of error.

7. Discussion

Methods I and II show similar results although the computational complexity of methods II is much higher. Method III is very different and achieves great improvement in the BER for low SNR. The algorithm proposed by Zhidkoc [14] shows better performance than methods I and II, but the proposed methods uses different techniques; they are new methods and may be possible to use it together with the one by Zhidkov or other, resulting in even
better performance.

8. Conclusion

The proposed demodulators are able to correct about half of the bit errors in the presence of burst noise from real power line measurements. This corresponds to a 1 dB improvement in the SNR. In the high noise level case, splitting the OFDM symbol 8 times while keeping the same number of CP results in reducing the BER, corresponding to a 5 dB improvement in the SNR. The new demodulators use the high correlation between the noise at different carriers given the burst position. The burst position is determined using an estimate of the noise after a first demodulation. Algorithm I improves the estimate of the noise by smoothing and subtracting it from the original signal. Algorithm II uses a Mahalanobis distance metric based on the covariance matrix.

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