A Multiple Criteria Nominal Classification Method Based on the Concepts of Similarity and Dissimilarity

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Abstract

In this paper, we propose a new multiple criteria decision-aiding method for nominal classification problems, where the categories are pre-defined and no order exists among them. A multiple criteria nominal classification problem consists of assigning each action, assessed according to multiple criteria, to at least one category. The new method, designated CATegorization by Similarity-Dissimilarity (Cat-SD), is based on the concepts of similarity and dissimilarity. We propose a way of modeling similarity and dissimilarity between two actions, which includes the possibility of taking into account interaction between criteria. Each category is characterized by the set of reference actions most representative of that category. The proposed method follows a decision-aiding constructive approach. First, the reference actions should be defined through a co-constructive interactive process between the analyst and the decision-maker. Then, the assignment of an action depends on the comparison of such an action to the sets of reference actions. For that, a degree of similarity-dissimilarity is computed and membership degrees allow an action to be assigned to the most adequate categories. The fundamental properties of the method and their proofs are provided. A numerical example is presented to illustrate the manner in which the proposed method can be applied. Robustness concerns are also considered in our work.

Keywords: Multiple criteria, Decision support systems, Nominal classification, Similarity, Dissimilarity, Interaction between criteria.

1. Introduction

Sorting, categorizing, classifying, or clustering actions into homogeneous categories have a long tradition in humankind's activities and crucial importance for the development and evolution of our societies; they are present in several aspects, not only in our private daily life, but also in the management of organizations and institutions with a strong impact on the life of populations. Modern societies are very competitive; they are currently faced with excellence in science, fierce industrial leadership competition, and challenging societal problems (social networks, terrorism threats, local issues related to intelligent cities, and so on). Our present societies are constantly looking for patterns, homogeneity, resemblance, for better adaptation or adjustment of their policies, strategies, and objectives to the needs of our times, allowing them to be governed in effectively and efficiently.

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Multiple criteria classification or clustering problems are two of the main decision "problem statements" or "problematics" (Roy, 1985), considered as pertaining to the domain of decision-aiding. They can be simply defined as an activity or process involving the assessment of objects or events (hereafter called actions) according to several criteria and consequent assignment to homogeneous groups, categories, or classes (hereafter called categories). When categories are pre-defined, we are in the presence of general classification problems. Those may be ordinal or sorting problems (when the categories are rank ordered) or nominal problems (when no order exists among the categories); some mixed cases may also occur. In clustering problems, the categories are defined *a posteriori*, as a result of the assignment process (for more details, see Doumpos and Zopounidis, 2002; Zopounidis and Doumpos, 2002).

1.1. State of the art in nominal classification problems

The literature on multiple criteria nominal classification is not vast. Among the existing methods, we can find procedures based on different approaches, such as verbal decision analysis (Furems, 2013), and outranking relations (see, for example, Rigopoulos et al., 2010). Most of the current interesting proposals are outranking-based procedures based on an indifference relation: Belacel and Boulassel (2004), Belacel (2000), Henriet (2000), and Perny (1998). This indifference relation leads in general to forming classes of equivalence, exceptionally the relations proposed in the method by Henriet (2000). However, the fact they are indifference outranking-based relations implies that construction of the threshold functions is rather technical, since it serves to model the imperfect knowledge of data (see Roy et al., 2014) and does not depend on an interaction between the analyst and the decision-maker. Threshold functions can be seen as very particular cases of general similarity-dissimilarity functions, which require subjective information from decision-makers in order to be adequately built. These are then different from the threshold functions of outranking-based methods, which are very common in published work, such as in a very interesting paper by Słowiński and Vanderpooten (2000), where the similarity concept was discussed and modeled in the context of the rough sets theory. Other researchers have used the concept of similarity in nominal classification problems (Léger and Martel, 2002; Goletsis et al., 2004). However, in those works, the similarity relation is considered as symmetric.

1.2. Similarity-dissimilarity in nominal classification problems

In this paper, we shall deal with nominal classification problems. These problems are frequently encountered in a broad range of fields: ecology, genetics, medicine, psychology, safety, economics, business and finance management, education and training, physics, geology, land management, geographical information systems, energy management, and so on. It is widely accepted that the assignment of actions to nominal categories is mainly based on the similarity and dissimilarity aspects of the actions (Ashby and Lee, 1991; Chater and Hahn, 1997; Markman and Gentner, 1993; Tversky, 1977). This principle can be stated as follows.

Principle 1. (Similarity-dissimilarity.)
When comparing two actions, both the similarity *and* dissimilarity *aspects between them should be taken into account.*

The similarity aspects or criteria are, in general, what count most towards the assignment of actions to homogeneous categories. However, in some situations, dissimilar actions may be the most desirable. When dissimilarity matters, the homogeneous groups are formed by dissimilar actions, which will be subject to the same treatment (for example, in the case of genetics, in general, it is common to select dissimilar individuals apt to serve as parents to generate offspring, thus avoiding consanguinity).

Similarity and dissimilarity judgments are rather subjective concepts (Tversky, 1977). In this context, judgments about similarity or dissimilarity are not necessarily symmetric (*e.g.*, it is not because the son is like the father that the father should be like the son), neither is transitivity required (*e.g.*, this is not because the son is like the father and the father like the grandfather that the son should be like the grandfather). As a direct consequence, a similarity-dissimilarity category is not a class of equivalence in mathematical terms.

An interesting model based on the above principle was proposed by Tversky (1977). It is based on the intrinsic qualities of the actions rather than on some of their more or less artificial continuous properties. This model can be stated as follows.

Model 1. (Contrast model.)
The feature ConTrast *(CT)* model *can be stated as a set-theoretical function of three arguments,*

$$f\big(A \cap B, A \setminus B, B \setminus A\big)$$

The function f is used to measure the similarity between two actions, say a and b. It takes into account the common features or criteria of both actions that contribute to the similarity $(A \cap B)$, the criteria present in the first and not in the second $(A \setminus B)$, and the reverse situation, where both contribute to an opposition to the similarity $(B \setminus A)$. What generally matters in this model for creating an operational method to measure similarity is the number of criteria that both actions have in common/not in common, the intensity or importance of the criteria, and the weights of the opposition to the similarity. Note that this function is not a metric, there is no symmetry and nor is there triangle inequality.

1.3. Proposal

This paper intends to generalize the *CT* model by Tversky (1977) in order to encompass the possibility of including interaction effects among criteria. In what follows, we propose a more general method for nominal classification, by making it possible to use a more general *per*-criterion similarity-dissimilarity function.

Model 2. (A generalized feature contrast model.)
A generalized feature contrast *(GTC)* model *is an extended version of the CT, which can be presented as follows:*

$$f\Big(\big([A \cap B], o([A \setminus B]), o([B \setminus A])\big),\ \big([A \setminus B], o([A \cap B]), o([B \setminus A])\big),\ \big([B \setminus A], o([A \cap B]), o([A \setminus B])\big)\Big)$$

Several types of interaction can be modeled within the sets, $A \cap B$, $A \setminus B$, and $B \setminus A$, and the power of opposition from the sets in $o(\cdot)$. In general, only the interaction between some pairs of criteria is considered. In our proposal, actions are compared against reference actions (or prototypes) representing each category. A degree of similarity-dissimilarity is computed and membership degrees allow actions to be assigned to the most adequate categories.

1.4. Structure of the paper

This paper is organized as follows. Section 2 is devoted to the motivation, namely the presentation of some potential applications, and the main notation used in the rest of the paper. Section 3 is related to our proposal to model a broader concept of similarity and dissimilarity including the interaction between criteria. Section 4 presents a new nominal classification method and the proof of its main properties. Section 5 is related to robustness concerns. Finally, the conclusions and lines of future research are provided.

2. Motivation: Potential applications

This section presents some more or less realistic applications. They are important to see the main features of multiple criteria nominal classification problems. After presenting these applications, we shall discuss such features. Finally, a numerical example is introduced, which will be used in the rest of this paper.

2.1. Examples of applications and their main features

The following applications are related to recruiting soldiers for special forces, health care alerts in social networks, medical diagnosis, policy instruments for environmental issues, and risk classification in enterprise risk management.

Application 1. (Recruiting soldiers for special forces.) The growing threat of terrorist attacks targeting private citizens and public assemblies, drug trafficking, insurgency or rebellion groups, hostages taken by force, among others, raise several security issues for our societies and governments. The need for more secure societies is a major challenge governments are faced with at the beginning of this century. Over the last few years, several countries have made a great effort to reinforce their military special forces. This requires particular attention to the recruitment process (assessment and selection) before enlisting candidates as soldiers. Each candidate or applicant is assessed according to multiple individual features, for instance, physical fitness, intelligence, motivation, teamwork skills, and mental skills (sharpness and ability to learn, and maturity and resilience). The candidates may be assigned to one of several special core skills task units (snipers, breachers, communications operators, heavy weapons operators,...), *i.e.*, they will be subject to selective training. All the soldiers within each task unit will be subject to special training courses. Before selection, it is necessary to know the suitability of candidates for the task units. How to identify the most adequate unit(s) for each candidate?

Application 2. (Health care alerts in social networks.) The risk of unexpected occurrences of diseases in restricted geographical areas or even an outbreak over several countries in the form of epidemics is a societal problem. There is a constant need to keep people well informed. Making announcements in social networks may have a strong impact on people's awareness of the risk of an epidemic. More and more health care organizations provide information on public health events and risks, and develop strategies and initiatives to assess emerging and re-emerging epidemic diseases, in order to limit their international spread. In this sense, for more effective communication and social mobilization, alerting people through social networks should be carried out differently according to the kind of user. Whenever possible, users are characterized by taking into account various aspects, for example, age, health condition, frequency of travel, degree of dependence on social networks. Users can be assigned to one of various social groups (roughly speaking, "younger", "middle-aged", "elderly",...). All the users of a particular social group will receive the same type of announcement. With the purpose of publishing health care information in social networks differently targeting each group, first it is necessary to know the adequacy of users to the type(s) of announcement. How to identify the most suitable type of announcement for each user?

Application 3. (Medical diagnosis.) The complexity of medical decision-making has increased over the last years, due to the vast amount of knowledge generated by medical advances. Moreover, inaccurate or incomplete diagnoses can be made by physicians as a result of the high complexity of medical diagnosis and their cognitive limitations. There is a need for medical diagnostic decision support with the ultimate goal of assisting physicians to improve diagnostic accuracy. Indeed, physicians have recognized that diagnostic decision support is a valuable medical decision-making aid for diverse medical specialties. In a scenario of this kind, each patient is assessed based on his/her symptoms (*e.g.*, fever, pain, muscle weakness, cough), in order to be assigned to one of several classes of disease diagnosis. According to multiple symptoms, groups of patients can be diagnosed with the same disease and, therefore, might be subject to an identical medical procedure. In order to define the medical procedure and prescribe the appropriate treatment, firstly physicians need to perform the medical diagnosis. How to identify the most accurate disease class(es) for each patient?

Application 4. (Policy instruments for environmental issues.) Environment-related decisions have become one of the most complex challenges our societies and governments must face in pursuit of a more sustainable future. Policies have a key role in addressing complex environmental and health problems, and consequently improving the state of the environment. Determining the type of instrument(s) for environmental policy best suited to manage each environmental issue is crucial to achieve desired outcomes strategically, effectively and efficiently. In recent years, several environmental issues have become increasingly evident, such as overpopulation, loss of biodiversity, ocean acidification, climate change, air, water and soil pollution, and deforestation. Environmental issues can be assessed according to multiple relevant characteristics, for example, type of situation, risks, social and environmental impacts, and urgency. Each particular environmental issue may be assigned to one (or more) of various policy instrument domains (*e.g.*, regulatory approaches, market-based instruments, education and information, and voluntary approaches). The same type of policies will be implemented for the environmental issues in each category. Before policy-makers in government and industry create environmental policies, they need to know the most effective type of policy instruments for each specific issue at hand. How to identify the most appropriate policy instrument(s) for each environmental issue?

Application 5. (Risk classification in enterprise risk management.) A crucial aspect in risk management is the assignment of responsibilities for risk treatment. When implementing an Enterprise Risk Management (ERM) framework, organizations need to ensure that stakeholders are responsible and have the necessary authority and competences for risk treatment. The assignment of responsibilities is typically established through the identification of risk owners - a person or entity with the responsibility to manage the assigned risk. In ERM, especially in organizations with complex hierarchical structures, identifying that can be complex due to the multitude of contexts where the risk can occur and impact. Risks are characterized by a set of risk attributes, for instance, affected asset or goal, consequence, source, etc. According to the contextual and technical nature of these attributes and expertise required to deal with them, risks may be assigned to different risk owners (human resources staff, finance staff, information technology staff, etc.). This is relevant because the set of risks assigned to each risk owner might be subject to the same type of analysis. To ensure separation of concerns and responsibilities, a risk should be assigned to a single risk owner. However, in ERM, risks identified at a high level might also have to be assigned to multiple risk owners depending on their intrinsic nature. How to identify the most advisable risk owner(s) for each risk in these cases?

The previous applications contain three essential aspects, which constitute the basic data of any nominal classification problem:

1. The *actions* (candidates, users, patients, environmental issues, and ERM risks), which are in fact the objects of the decision; let $A = \{a_1, \ldots, a_i, \ldots\}$ denote the set of actions (not necessarily known *a priori*);
2. The elements (features, aspects, symptoms, characteristics, or attributes) that allow construction of the *criteria* used to assess the performance of the actions; let $G = \{g_1, \ldots, g_j, \ldots, g_n\}$ denote the set of criteria; and, $g_j(a_i)$ is the performance of action a_i on criterion g_j;
3. The *categories* (task units, social groups, disease classes, policy domains, or risk owners) are conceived to receive the actions; let $C = \{C_1, \ldots, C_h, \ldots, C_q, C_{q+1}\}$ denote the set of categories, where category C_{q+1} is a category that contains actions that can not be assigned to the other categories.

The multiple criteria *nominal classification problem* consists of assigning each action a, evaluated according to the criteria in G, to at least one category in C under the assumptions below. This assignment should be performed in the most adequate (suitable, accurate, appropriate, advisable) way, meaning that in most cases the preferences of decision-makers should be taken into account.

Assumption 1. *The set of categories to which the actions must be assigned is not ordered.*

Assumption 2. *Each category is defined by a set of reference actions, B_h, which contains the most representative actions of the category, $B_h = \{b_{h1}, \ldots, b_{h\ell}, ..., b_{h|B_h|}\}$, for $\ell = 1, ..., |B_h|; h = 1, \ldots, q$.*

Assumption 3. *Each category is defined* a priori *to receive actions, which will be or might be processed in the same way (at least in a first step,* e.g.*, the same training courses, the same type of announcement, the same medical procedure, the same type of policies, or the same type of analysis).*

The next sub-section introduces a numerical example, where it will be possible to see the features of a nominal classification problem.

2.2. An illustrative numerical example

Based on Application 1 presented in the previous section, a numerical example is designed in this section. This example is thus related to the recruitment process of soldiers for special forces: Candidates are assessed on the basis of their individual features and assigned to special core skills task units. With the purpose of identifying the most adequate task unit(s) for each candidate, a set of six criteria is built ([min] and [max] are assigned to the criteria to be minimized and maximized, respectively):

g_1: Physical fitness [min];
g_2: Mental sharpness [max];
g_3: Mental resilience [max];
g_4: Intelligence [max];
g_5: Teamwork skills [max];
g_6: Motivation [max].

The performances of criterion g_1 (physical fitness) are physical screening test scores, and the performances of criterion g_2 (mental sharpness) are percentile scores related to word knowledge, paragraph comprehension, arithmetical reasoning, and mathematical knowledge (quantitative scales). The performances of criterion g_3 (mental resilience), which takes into account performance strategies, psychological resilience, and personality traits, are expressed on a four-level qualitative scale. The performances of criteria g_4 (intelligence), g_5 (teamwork skills), and g_6 (motivation) are expressed on a seven-level qualitative scale. Moreover, criterion g_4 (intelligence) is related to the ability to perceive information and apply the retained knowledge to adaptive behaviors; criterion g_5 (teamwork skills) includes communication skills, temperament, and camaraderie; and criterion g_6 (motivation) is related to determination and dedication. In this example, the criteria scales are the following:

$E_1 = \{370, 371, ..., 1281, 1282\}$;
$E_2 = \{35, 36, .., 98, 99\}$;
$E_3 = \{$low (1), medium (2), high (3), very high(4)$\}$;
$E_4, E_5, E_6 =$
$= \{$very low (1), low (2), rather low (3), medium (4), rather high (5), high (6), very high (7)$\}$.

It should be noted that in the qualitative scales, E_3, E_4, E_5, and E_6, the values in parentheses are used to code the different verbal statements, which plays a purely ordinal role in the computations.

Each candidate is assessed according to the six criteria and must be assigned to at least one of five categories: snipers (C_1), breachers (C_2), communications operators (C_3), heavy weapons operators (C_4), and not-assigned candidates (C_5). Notice that these categories are not ordered. Intrinsic relative (non-normalized) weights $k_j, j = 1, ..., 6$, are associated with the corresponding criteria. Such criteria weights may be different for each category, thus a distinct set of weights, $k^i, i = 1, ..., 4$ ($k^i = (k^i_1, k^i_2, ..., k^i_j)$, for $i = 1, ..., 4$ and $j = 1, ..., 6$), is considered, as presented in Table 1.

Table 1: Weights of criteria for each category

Categories	Sets of weights	k^i_1	k^i_2	k^i_3	k^i_4	k^i_5	k^i_6
Snipers	k^1	10	15	20	20	5	15
Breachers	k^2	25	15	20	5	10	5
Communications operators	k^3	10	20	15	20	15	10
Heavy weapons operators	k^4	25	5	15	5	20	10

Each category is defined by a set of reference actions, which contain the most representative action(s). Table 2 displays the set of reference actions for each category.

In the rest of the paper, this example will continue to be used to clarify the reader's understanding of the application of the method proposed herein to a nominal classification problem.

3. Modeling similarity and dissimilarity

This section starts by presenting a definition of the *per*-criterion similarity-dissimilarity function as a way of making operational the principle of similarity-dissimilarity for each criterion. Then, it introduces a way of modeling interaction effects between some pairs of criteria according to the philosophy of the *GCT* model proposed in the introduction. It is then necessary to aggregate

Table 2: Sets of reference actions for each category

Categories	Sets of reference actions	Reference actions	g_1	g_2	g_3	g_4	g_5	g_6
Snipers	B_1	b_{11}	700	80	4	6	4	6
		b_{12}	750	75	4	7	4	7
Breachers	B_2	b_{21}	800	70	3	6	6	6
Communications operators	B_3	b_{31}	1000	85	2	5	4	4
		b_{32}	950	80	2	5	4	5
Heavy weapons operators	B_4	b_{41}	700	60	3	5	6	5

similarity and dissimilarity; two sub-sections are devoted to both of these aspects. A comprehensive model of similarity-dissimilarity will be presented in the next sub-section. Finally, this section ends with a definition of a similarity binary relation.

3.1. Modeling the per-*criterion similarity-dissimilarity*

The following is a general definition of a way of model the similarity-dissimilarity of each criterion. Let E_j denote the scale of criterion g_j, which in general is bounded from below by $g_j^{\min}$ and from above by $g_j^{\max}$.

Definition 1. (per-*criterion similarity-dissimilarity modeling function.)*
*A per-*criterion similarity-dissimilarity modeling function *is a real-valued function $f : E_j \to [-1, 1]$ defined as follows:*

$$f_j\big(g_j(a)\big) = \begin{cases} \text{is a non-decreasing function of } g_j(a), & \text{if} \quad g_j(a) \in [g_j^{\min}, g_j(b)]; \\ \\ \text{is a non-increasing function of } g_j(a), & \text{if} \quad g_j(a) \in [g_j(b), g_j^{\max}]. \end{cases}$$

When $f_j\big(g_j(a)\big)$ is non-negative, a per-criterion similarity coefficient *can be defined as $s_j(a,b) = f_j\big(g_j(a)\big)$, but when its value is non-positive, the* per-criterion dissimilarity coefficient *can be stated as $d_j(a,b) = f_j\big(g_j(a)\big)$. Furthermore, if $s_j(a,b) \geqslant 0$, then $d_j(a,b) = 0$; analogously, if $d_j(a,b) \geqslant 0$, then $s_j(a,b) = 0$. When the performance of action a outperforms the performance of action b, the notation $d_j^{+}(a,b)$ is used to define a dissimilarity coefficient in favor of a; in the reverse situation, $d_j^{-}(a,b)$, denotes the dissimilarity coefficient in favor of b.*

Remark 1. *The value of the coefficients $s_j(a,b)$, $d_j^{+}(a,b)$, and $d_j^{-}(a,b)$ should remain the same whenever the scale E_j changes. It is easy to see that these coefficients are meaningful in the sense presented by Martel and Roy (2006).*

The construction of the function $f_j(\cdot)$ is subjective and should be done with a constructive interaction process between the analyst and the decision-maker.

Figure 1 presents an example of a possible function according to the condition of Definition 1. In this figure, the function assumes non-negative values within the range $[t_j^{'}\big(g_j(b)\big), t_j\big(g_j(b)\big)]$. It means there is a positive contribution to the similarity when the difference $|g_j(b) - g_j(a)|$ is within this range. The function assumes negative values within the ranges $]g_j^{\min}, u_j^{'}\big(g_j(b)\big)[$ and $]u_j\big(g_j(b)\big), g_j^{\max}[$, which means there is a negative contribution to the similarity. We have a negative dissimilarity when b outperforms a, and a positive dissimilarity when a outperforms b.

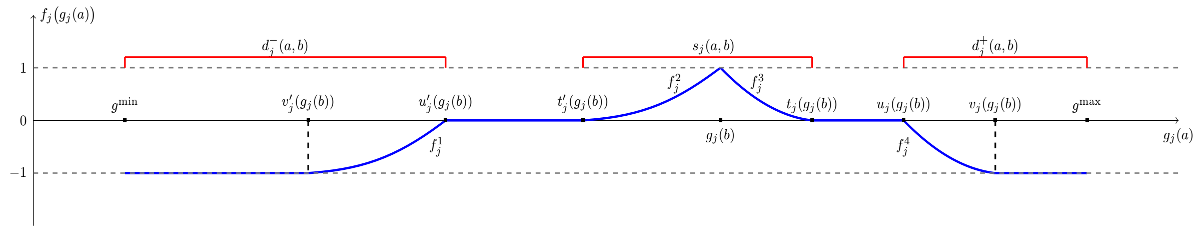

Figure 1: *A per*-criterion similarity-dissimilarity function

The process of eliciting the function, in particular the different points in the criterion axis, can be done like the elicitation of veto thresholds in outranking methods (see Roy et al., 2014). Then, particular attention should be devoted to the four components of the functions, f_j^1, f_j^2, f_j^3 and f_j^4. This elicitation process is not an easy task and will be presented in a separate paper.

3.2. On the interaction between criteria

In practice, the interaction effects among several criteria are rather difficult to understand for decision-makers. In general, it makes more sense to consider only the interaction effects between a small number of criteria (see Figueira et al., 2009).

In the *GCT* model proposed, there are several possible ways of considering interactions; the following seems very intuitive:

1. Mutual-strengthening and mutual-weakening effects within the set $(A \cap B)$: The two criteria are in favor of similarity;
2. Mutual-strengthening and mutual-weakening effects within the set $\big((A \setminus B) \cup (B \setminus A)\big)$: The two criteria are in favor of dissimilarity;
3. Antagonistic effects of $(A \cap B)$ against $\big((A \setminus B) \cup (B \setminus A)\big)$: A criterion favoring similarity is against the second criterion, which favors dissimilarity;
4. Antagonistic effects of $\big((A \setminus B) \cup (B \setminus A)\big)$ against $(A \cap B)$: A criterion favoring dissimilarity is against the second criterion, which favors similarity.

Indeed, we could also consider, for example, antagonistic effects of $(A \setminus B)$ against $(B \setminus A)$ and *vice-versa*, but this seems more difficult to justify in our framework.

Definition 2 presents the interaction effects we shall consider (see Figueira et al., 2009).

Definition 2. (Interaction effects.)

i) Mutual-strengthening effect *between the pair of criteria* $\{g_j, g_\ell\}$*:* $k_j + k_\ell < k_j + k_\ell + k_{j\ell}$*, with* $k_{j\ell} > 0$ *(*$k_{j\ell} = k_{\ell j}$*);*

ii) Mutual-weakening effect *between the pair of criteria* $\{g_j, g_\ell\}$*:* $k_j + k_\ell < k_j + k_\ell + k_{j\ell}$*, with* $k_{j\ell} < 0$ *(*$k_{j\ell} = k_{\ell j}$*);*

iii) Antagonistic effect *between the ordered pair of criteria* (g_j, g_p)*:* $k_j + k_{jp}$*, with* $k_{jp} < 0$.

In what follows, consider the following additional notation:

– M denotes the set of all pairs $\{j, \ell\}$ (for mutual interaction effects between the pair of criteria $\{g_j, g_\ell\}$);

– O denotes the set of all ordered pairs (j,p) (for antagonistic effects between the ordered pair of criteria (g_j, g_p)).

The following condition is necessary to guarantee that the weights of criteria k_j never become negative after considering the interaction effects (Figueira et al., 2009).

Condition 1. (Non-negativity.)

$$k_j \quad - \sum_{\big\{\{j,\ell\}\in M\,:\,k_{j\ell}<0\big\}} |k_{j\ell}| \;-\; \sum_{(j,p)\in O} |k_{jp}| \;\geqslant\; 0, \; \textit{for all}\; j \in G.$$

3.3. Modeling comprehensive similarity
A similarity function can be defined as follows.

Definition 3. (Similarity function.)
A similarity function *is a real-valued function* $f^s \;:\; [0,1]^n \times [-1,0] \times [-1,0] \to [0,1]$, *which can be stated as follows:*

$$s(a,b) = f^s\big(s_1(a,b),\ldots,s_j(a,b),\ldots,s_n(a,b),\diamondsuit,\blacklozenge\big),$$

where $\diamondsuit$ *and* $\blacklozenge$ *are related to the interaction power of opposition to the similarity.*

An example of the function in Definition 3 is presented next.

Example 1. (A non-additive similarity function.)

$$s(a,b) = \frac{1}{K(a,b)}\left(\sum_{j\in G} k_j s_j(a,b) + \sum_{\{j,\ell\}\in M} z\big(s_j(a,b),s_\ell(a,b)\big)k_{j\ell} + \sum_{(j,p)\in O} z\big(s_j(a,b),s_p(b,a)\big)k_{jp}\right), \tag{1}$$

where

$$K(a,b) = \sum_{j\in G} k_j + \sum_{\{j,\ell\}\in M} z\big(s_j(a,b),s_\ell(a,b)\big)k_{j\ell} + \sum_{(j,p)\in O} z\big(s_j(a,b),s_p(b,a)\big)k_{jp},$$

and $z \;:\; [0,1]\times[0,1]\to[0,1]$ *is a real-valued function, which can take the form* $z(x,y)=xy$.

3.4. Modeling comprehensive dissimilarity
As in the previous case, we can define the comprehensive positive and negative dissimilarity functions as follows.

Definition 4. (Positive and negative dissimilarity functions.)
The positive and negative dissimilarity functions *are real-valued functions* $f^{d+}, f^{d-} \;:\; [-1,0]^n \times [0,1] \to [-1,0]$ *defined as follows:*

$$d^+(a,b) = f^{d+}\big(d_1^+(a,b),\ldots,d_j^+(a,b),\ldots,d_n^+(a,b),\circledcirc\big),$$

and

$$d^-(a,b) = f^{d-}\big(d_1^-(a,b),\ldots,d_j^-(a,b),\ldots,d_n^-(a,b),\circledcirc\big),$$

where $\circledcirc$ *is related to the interaction power of opposition to the dissimilarity. It should be noticed that, when there is no distinction between the positive and negative dissimilarity, the notation* $d(a,b)$ *is used instead.*

An example of a dissimilarity function can be stated as follows. In this example, and for the sake of simplicity, we do not make a distinction between positive and negative dissimilarity.

Example 2. (A non-linear dissimilarity function.)

$$d(a,b) = \prod_{j=1}^{n} \big(1 + d_j(a,b)\big) - 1. \tag{2}$$

3.5. A measure of comprehensive similarity-dissimilarity

The function below is used to assess the degree to which an action a is similar to an action b. We could also build an analogous function to assess the degree to which the two actions are similar, but this is a rather different question which requires symmetry. In our function, a is the subject of the comparison and b the referent (*i.e.*, the reference action to which a is compared to).

Definition 5. (Comprehensive similarity-dissimilarity function.)
A comprehensive similarity function *is a real-valued function* $f\ :\ [0,1] \times [-1,0] \times [-1,0] \to [-1,1]$ *as follows:*

$$\delta(a,b) = f\big(s(a,b), d^{+}(a,b), d^{-}(a,b)\big).$$

This function is a non-decreasing function of each one of its arguments with the following properties:

1. *Reflexivity:* $\delta(b,b) = 1$.
2. *Asymmetry:* $\delta(a,b) \not\Rightarrow \delta(b,a)$.
3. *Non-transitivity:* $\delta(a,b) \geqslant (\leqslant)\ \delta(b,c)$ *and* $\delta(b,c) \geqslant (\leqslant)\ \delta(a,c) \not\Rightarrow \delta(a,b) \geqslant (\leqslant)\ \delta(a,c)$.

It should be noticed that $s(a,b)$ and $d(a,b)$, and consequently $\delta(a,b)$, should not necessarily be symmetric. We shall call $\delta(a,b)$ a *degree of similarity-dissimilarity* of a with respect to b.

A simple example of this comprehensive function is presented next. It takes into account the function of Examples 1 and 2.

Example 3. (A multiplicative comprehensive similarity function.)

$$\delta(a,b) = s(a,b)\big(1 - d(a,b)\big). \tag{3}$$

3.6. A similarity-dissimilarity binary relation

The similarity-dissimilarity binary relation, S, is not necessarily symmetric. As stated in Słowiński and Vanderpooten (2000), symmetry and transitivity should not be imposed on similarity relations. The reflexivity is the minimal property of the similarity-dissimilarity relation. The relation S only depends on the performances $g_j(a)$ and $g_j(b)$, for all $j \in G$.

A *similarity-dissimilarity degree* between a and a reference set B_h can be defined as follows:

$$\delta(a, B_h) = \max_{\ell=1,\ldots,|B_h|} \left\{\delta(a, b_{h\ell})\right\}. \tag{4}$$

We say that a is similar to B_h, denoted $aS(\lambda_h)B_h$, if $\delta(a,B_h) \geqslant \lambda_h$, where λ_h is a membership degree. The parameter λ_h is a preferential parameter that can be viewed in the same sense as a majority measure allowing for classifying an action a into a given category C_h. Therefore, we have:

$$a \in C_h \Leftrightarrow \delta(a, B_h) \geqslant \lambda_h.$$

3.7. An illustrative example

Let us continue to use the numerical example introduced in sub-sub-section 2.2 to illustrate how to model similarity and dissimilarity. The following interaction coefficients between some pairs of criteria are considered:

- Mutual-strengthening effect between g_2 (metal sharpness) and g_3 (mental resilience): $k_{23} = k_{32} = 10$;
- Mutual-weakening effect between g_1 (physical fitness) and g_5 (teamwork skills): $k_{15} = k_{51} = -4$;
- Antagonistic effect between g_1 (physical fitness) and g_4 (intelligence): $k_{14} = -3$.

In this example, seven candidates (actions), $a_1, ..., a_7$, are analyzed, in order to find out their suitability for task units. The candidates' performances on the six criteria considered are provided in Appendix A. It should be noticed that a candidate may be assigned to one or more task units, or even not be assigned to any (as happens when the candidate is not suitable for any task unit). The similarities and dissimilarities of each candidate with respect to the sets of reference actions are assessed by comparing the performances on all the criteria of each pair of actions, a candidate (the subject) and a reference action (the referent). For each criterion, we use a *per*-criterion similarity-dissimilarity modeling function that takes into account the interaction effects between criteria. The functions utilized for each criterion are provided in Appendix B. Hence, a measure of comprehensive similarity-dissimilarity is obtained to compute the degree to which the two actions are similar. Table 3 presents the degree of similarity-dissimilarity for each candidate and sets of reference actions (for each ordered pair $(a_i, b_{h\ell})$, for $i = 1, ...,7$; $\ell = 1, ...|B_h|$; $h = 1, ..., 4$).

Table 3: Similiarity-dissimilarity degrees between candidates and sets of reference actions

Candidates	B_1		B_2	B_3		B_4
	b_{11}	b_{12}	b_{21}	b_{31}	b_{32}	b_{41}
a_1	0.748	0.835	0	0	0	0
a_2	0	0	0	0.778	0.688	0
a_3	0	0	0.375	0	0	0.778
a_4	0	0	0	0.594	0.850	0
a_5	0	0	0.884	0	0	0.705
a_6	0	0	0	0	0	0
a_7	0	0	0.667	0	0	0.500

According to the results displayed in Table 3, and using Equation 4, we have: $\delta(a_1, B_1) = 0.835$, $\delta(a_2, B_3) = 0.778$, $\delta(a_3, B_2) = 0.375$, $\delta(a_3, B_4) = 0.778$, $\delta(a_4, B_3) = 0.850$, $\delta(a_5, B_2) = 0.884$, $\delta(a_5, B_4) = 0.705$, $\delta(a_7, B_2) = 0.667$, and $\delta(a_7, B_4) = 0.500$; the remaining similarity-dissimilarity degrees between the candidates and the sets of reference actions are equal to zero.

Let us consider the following values of λ_h for each category: $\lambda_1 = 0.75$, $\lambda_2 = 0.60$, $\lambda_3 = 0.65$, and $\lambda_4 = 0.60$.

The next section introduces the assignment procedure and illustrates its application by using this example.

4. Cat-SD: A new nominal classification method

In this section, we propose a new nominal classification method, designated Cat-SD (from CATegorization by Similarity-Dissimilarity). This section introduces the Cat-SD method, including the characterization of categories, as well as the assignment procedure and its mathematical properties.

4.1. Characterization of categories and the assignment procedure

Definition of the categories clearly depends on the nominal classification problem we have at hand. We propose to use several reference actions to characterize a category. Such reference actions must be representative of the actions that should be assigned to a given category. Definition of the categories requires intervention by the decision-maker. The categories are also defined within a co-constructive interactive process between the analyst and the decision-maker.

Definition 6. (Characterization of the categories.)
Each category C_h is characterized by a set of reference actions, $B_h = \left\{b_{h1}, \ldots, b_{h\ell}, ..., b_{h|B_h|}\right\}$, for $\ell = 1, ..., |B_h|; h = 1, \ldots, q+1$.

It should be noticed that C_{q+1} is conceived to receive actions that cannot be assigned to the remaining categories. Thus, $B_{q+1} = \emptyset$. At least, one reference action must be used for each category C_h, for $h = 1, ..., q$. Therefore, the set B_h contains the most representative action(s) of the category C_h, $h = 1, ..., q$. Let $B = \{B_1, ..., B_h, ..., B_{q+1}\}$ denote the set of all sets of reference actions. As in the case of ordinal classification problems (Almeida-Dias et al., 2010, 2012), in our settings, defining the following operations is important for introducing the structural requirements of the procedure.

Definition 7. (Merging and splitting operations.)
The merging and splitting operations are defined as follows:

i) Merging: Two categories, C_r and C_s, are merged to become a new one, C_t, which is characterized by the set $B_t = B_r \cup B_s$ (assumption), and $\lambda_t \leqslant \min\{\lambda_r, \lambda_s\}$, with $r \neq s$, such that:

1. *If $C_r = C_{q+1}$, then $B_t = B_s$ and $\lambda_t = \lambda_s$;*
2. *If $C_s = C_{q+1}$, then $B_t = B_r$ and $\lambda_t = \lambda_r$.*

ii) Splitting: The category C_t is split into two new categories C_r and C_s, which are characterized by two new sets of reference actions, B_r and B_s, respectively, such that:

1. *If $|B_t| = 1$, then $B_r = B_t$ and $\lambda_r = \lambda_t$ or $B_s = B_t$ and $\lambda_s = \lambda_t$;*
2. *If $|B_t| > 1$, then there is at least one $b_{h\ell} \in B_t$, such that $b_{h\ell} \in B_r$ and $\lambda_r = \lambda_t$ or $b_{h\ell} \in B_s$ and $\lambda_s = \lambda_t$, for $h = 1, ..., q$;*
3. *When $C_t = C_{q+1}$, new sets B_r and B_s, and new values of λ_r and λ_s, must be defined.*

Both operations involve, in general, changing the set of reference actions B through the interaction between the analyst and the decision-maker. After a merging operation and a splitting operation, the new sets become $C^{'} = \{C_1, \ldots, C_t, \ldots, C_q, C_{q+1}\}$, characterized by the new set $B^{'} = \{B_1, \ldots, B_t, \ldots, B_q, B_{q+1}\}$, and $C^{'} = \{C_1, \ldots, C_r, \ldots, C_s, ..., C_q, C_{q+1}\}$, characterized by the new set $B^{'} = \{B_1, \ldots, B_r, \ldots, B_s, ..., B_q, B_{q+1}\}$, respectively. It should be noticed that adding or removing a category are particular cases of those two operations.

The Cat-SD method has been conceived to verify a set of structural requirements.

Definition 8. (Structural requirements.)
The following are natural requirements for the CAT-SD *method:*

1. *Possibility of multiple assignments: Each action a is assigned to at least one category;*
2. *Independence: The assignment of an action does not depend on the assignment of the other actions;*
3. *Conformity: Each reference action $b_{h\ell}$ must be assigned to category C_h, for $\ell = 1, ...|B_h|; h = 1, ..., q$;*
4. *Homogeneity: If two actions compare the same way with respect to each set of reference actions, then they must be assigned to the same category or categories (i.e., two actions must be assigned to the same category or categories, when they have the same similarity-dissimilarity degrees with respect to all reference actions);*
5. *Stability: When changing the set of reference actions B by applying either a merging or a splitting operation, assignment of the actions to the non-modified categories may not change (i.e., after a merging or splitting operation, the actions previously assigned to the non-modified categories will be assigned to the same categories or to the new modified category).*

The assignment procedure provides the set of possible categories (at least one category) to which an action a can be assigned. The assignment of an action a to a category C_h is based on how such an action compares with the reference actions b_{hl}, for $h = 1, ..., q$. Note that the categories $C_1, ..., C_q$ can have distinct sets of weights and membership degrees among them. A different set of criteria for each category can also be considered in the method.

Definition 9. (Similarity assignment procedure.)
Given $\lambda_h \in [0.5, 1]$, for $h = 1, \dots, q$, the similarity assignment procedure *can be stated as follows:*

i) *Compare action a with set B_h, for $h = 1, \dots, q$;*

ii) *Identify $K = \{k \ : \ aS(\lambda_k)B_k\}$;*

iii) *Assign action a to category C_k, for all $k \in K$;*

iv) *If $K = \emptyset$, assign action a to category C_{q+1}.*

It should be noticed that action a is assigned to category C_{q+1} if and only if is not assigned to any C_h, for $h = 1, ..., q$ ($C_h \cap C_{q+1} = \emptyset$, for $h = 1, ..., q$).

Remark 2. *It is important to state that this problem can be viewed as a successive resolution of dichotomic sorting problems with two categories: accepted and rejected.*

Theorem 1. (Properties of the assignment procedure.)
The CAT-SD *assignment procedure fulfills the requirements of the possibility of multiple assignments, independence, conformity, homogeneity, and stability.*

Proof.

1. Possibility of multiple assignments: The proof is trivial. The similarity assignment procedure only depends on $\delta(a, B_h)$. Therefore, if $\delta(a, B_h) \geqslant \lambda_h$, then action a is assigned to C_h, for any $\lambda_h \in [0.5, 1]$ and $h = 1, ..., q$. Otherwise, action a is assigned to category C_{q+1}. Thus, each action a may be assigned to one or more categories, but at least a is assigned to one category.

2. Independence: The proof is also trivial. The assignment of each action only depends on $\delta(a, B_h)$, for $h = 1, ..., q$, thus it does not depend on the assignment of the other actions.

3. Conformity: According to the property of reflexivity of the comprehensive similarity-dissimilarity function, we have $\delta(b_{h\ell}, b_{h\ell}) = 1$, for $h = 1, ..., q$. This implies that $\delta(b_{h\ell}, B_h) = 1$, for $h = 1, ..., q$. When we apply the similarity assignment procedure, the reference action $b_{k\ell}$ is assigned to category C_k, since we have $\delta(b_{k\ell}, B_k) \geqslant \lambda_k$, for any $\lambda_k \in [0.5, 1]$.

4. Homogeneity: Two different actions, a and b, compare in the same way with respect to the set of reference actions B_h if and only if $\delta(a, B_h) = \delta(b, B_h)$. Since the similarity assignment procedure only depends on $\delta(a, B_k)$, actions a and b are assigned to category C_k, for any $\lambda_k \in [0.5, 1]$.

5. Stability: According to the merging operation (see Definition 7), the new category C_t will be characterized by the new set of reference actions $B_t = B_r \cup B_s$. An action a verifying $\delta(a, B_r) \geqslant \lambda_r$ and $\delta(a, B_s) \geqslant \lambda_s$, will also verify $\delta(a, B_t) \geqslant \lambda_t$, with $r \neq s$. Therefore, after merging categories, when we apply the assignment procedure, any action a previously assigned to category C_r or C_s (or even to both), will be assigned to category C_t, with $r \neq s$. According to the splitting operation (see Definition 7), any action a will be assigned to one of the new categories, C_r and C_s, or possibly to both, since we have $\delta(a, B_r) \geqslant \lambda_r$ and/or $\delta(a, B_s) \geqslant \lambda_s$, for any $\lambda_r, \lambda_s \in [0.5, 1]$.

$\square$

4.2. An illustrative example

In this sub-section, we use the data of the numerical example presented in sub-sections 2.2 and 3.7 to illustrate how the proposed method can be used. By applying the similarity assignment procedure, we obtain the following assignment results (see also Table 4):

- Snipers: $C_1 = \{a_1\}$;
- Breachers: $C_2 = \{a_5, a_7\}$;
- Communications operators: $C_3 = \{a_2, a_4\}$;
- Heavy weapons operators: $C_4 = \{a_3, a_5\}$;
- Not-assigned candidates: $C_5 = \{a_6\}$.

Table 4: Candidates' assignment

Candidates	C_1	C_2	C_3	C_4	C_5
a_1	✓				
a_2			✓		
a_3				✓	
a_4			✓		
a_5		✓		✓	
a_6					✓
a_7		✓			

Let us observe that the intersection between category C_2 (breachers) and C_4 (heavy weapons operators) is $\{a_5\}$. This means that the candidate a_5 is suitable for both task units. However, the most appropriate task unit for this candidate is C_2 (breachers), since the value of the similarity-dissimilarity degree for C_2 is greater than for C_4 (*cf.* Table 3). Note that candidate a_6 is assigned to category C_6, which means that a_6 is not suitable for any task unit.

As illustrated in this numerical example, the CAT-SD method allows us to address nominal classification problems by assessing the performances of the actions according to multiple criteria, and assigning them to pre-defined and non-ordered categories, based on similarity-dissimilarity.

5. Robustness concerns: Scenario Analysis

This section is devoted to robustness concerns. In decision-aiding, all possible ways that allow us to formulate synthetic recommendations according to robust conclusions is a robustness concern (Figueira et al., 2016). As stated by Roy (2010), robustness is a crucial issue in the field of operational research and decision-aiding (for more details about robustness concerns in this context, see also Doumpos et al., 2016). Motivated by this fact, we addressed the robustness of the results obtained when the CAT-SD method is applied, using the numerical example presented in the previous sections.

In general, the values assigned to the parameters are not perfectly defined. We are interested in providing recommendations concerning the categorization of candidates that remain acceptable for a wide range of the values of the parameters used in the CAT-SD method. Thus, robustness with respect to different scenarios was assessed, by changing some preference parameters, for each category, $C_1, ..., C_4$. Indeed, we performed a scenario analysis, by analyzing a total of 180 scenarios.

Let us use the data of the example presented in the previous sections to illustrate how robust the classification proposed by the model is. Instead of using only seven candidates, which is a small number for this purpose, we used data of twenty dummy candidates, including those previously considered (see Appendix A).

Some parameters of categories $C_1, ..., C_4$ were analyzed, in order to assess the robustness of the assignment results obtained through the application of the proposed method. Several scenarios were considered by making changes in the following preference parameters:

1. The weights vectors (sets $k^i, i = 1, ..., 20$);
2. The set of interaction coefficients (sets of coefficients k_{jl} and k_{jp});
3. The membership degree ($\lambda_h, h = 1, ..., 4$).

Table 5: New sets of criteria weights for each category

Categories	Sets of weights	k_1^i	k_2^i	k_3^i	k_4^i	k_5^i	k_6^i
Snipers	k^5	8	11	16	16	4	11
	k^6	9	13	18	18	4	13
	k^7	11	17	22	22	6	17
	k^8	12	19	24	24	7	19
Breachers	k^9	21	11	16	4	8	4
	k^{10}	23	13	18	4	9	4
	k^{11}	27	17	22	6	11	6
	k^{12}	29	19	24	7	12	7
Communications operators	k^{13}	8	16	11	16	11	8
	k^{14}	9	18	13	18	13	9
	k^{15}	11	22	17	22	17	11
	k^{16}	12	24	19	24	19	12
Heavy weapons operators	k^{17}	21	4	11	4	16	8
	k^{18}	23	4	13	4	18	9
	k^{19}	27	6	17	6	22	11
	k^{20}	29	7	19	7	24	12

Table 5 displays the additional values of the criteria weights considered in this analysis. Besides the set of interaction coefficients for some pairs of criteria considered in the numerical example (see sub-section 3.7), we used the following two additional sets for the same pairs of criteria:

$$\begin{cases} k_{23} = k_{32} = 9, \\ k_{15} = k_{51} = -3, \\ k_{14} = -2. \end{cases} ; \begin{cases} k_{23} = k_{32} = 8, \\ k_{15} = k_{51} = -2, \\ k_{14} = -1. \end{cases}$$

In addition to the values of membership degree used in the numerical example (see sub-section 3.7), the following new values for each category were considered:

- C_1: $\lambda_1^1 = 0.70$ and $\lambda_1^2 = 0.80$;
- C_2: $\lambda_2^1 = 0.55$ and $\lambda_2^2 = 0.65$;
- C_3: $\lambda_3^1 = 0.60$ and $\lambda_3^2 = 0.70$;
- C_4: $\lambda_4^1 = 0.55$ and $\lambda_4^2 = 0.65$.

For each category, and according to Remark 2, we carried out separately the robustness analysis with five sets of weights, three distinct values of membership degree, and three sets of interaction coefficients between some pairs of criteria. Therefore, for each category, 45 scenarios (including the four scenarios considered in the numerical example) were tested to analyze stability to the change of those parameters; in total, 180 scenarios were analyzed. Table 6 contains the results of the analysis for all scenarios. It provides the percentage of scenarios in which each candidate is assigned to a given category. Note that the values displayed in each column of Table 6 are the percentage of scenarios for the respective category. Thus, for example, 100 percent means that a candidate is assigned to a given category in all 45 scenarios examined for such a category, and 67 percent means that a candidate is assigned to a given category in only 30 scenarios.

Table 6: Results of the scenario analysis

Candidates	% of scenarios			
	C_1	C_2	C_3	C_4
a_1	100	0	0	0
a_2	0	0	100	0
a_3	0	0	0	100
a_4	0	0	100	0
a_5	0	100	0	100
a_6	0	0	0	0
a_7	0	100	0	0
a_8	0	0	100	0
a_9	0	33	0	100
a_{10}	100	0	0	0
a_{11}	0	0	0	0
a_{12}	67	0	0	0
a_{13}	0	100	0	100
a_{14}	0	0	0	0
a_{15}	0	0	0	0
a_{16}	0	0	100	0
a_{17}	0	67	0	100
a_{18}	0	0	0	0
a_{19}	67	0	0	0
a_{20}	0	100	0	0

Table 6 reveals that the candidates' classifications remain largely unchanged. It should be noticed that when we consider all 45 scenarios for each category $C_1, ..., C_4$, and a candidate is not assigned to any of those categories, we can conclude that such a candidate is assigned to category C_5 in all possible scenarios. This is the case of candidates $a_6, a_{11}, a_{14}, a_{15}$, and a_{18}. These results show that the proposed method leads to robust classification of the candidates according to the changes in the preference parameters.

6. Conclusions

In this paper, we proposed a new method for addressing multiple criteria nominal classification problems, in which actions are assessed according to multiple criteria and must be assigned to unordered categories. Each category is characterized by a set of reference actions, which are the most representative actions of the category. The proposed method, CAT-SD, is based on the concepts of similarity and dissimilarity. A way to model similarity and dissimilarity was presented. The possibility of interaction between some pairs of criteria is also taken into account in the proposed method.

Application of the CAT-SD method should follow a decision-aiding constructive approach, which means that an interactive process between the analyst and the decision-maker should be followed during application of the method. This interaction ensures that the preferences of the decision-maker are properly represented in the model.

The CAT-SD method fulfills a certain number of structural requirements (fundamental properties). We presented these fundamental properties of the method and provided their proofs.

Furthermore, a numerical example was used to illustrate the main theoretical results provided by the method.

In our work, robustness concerns were also considered, by performing a scenario analysis. The results show that this method can lead us to elaborate robust conclusions regarding the categorization of actions. Thus, we show that the proposed method is suitable to deal with classification problems in which the categories are not ordered and are characterized by reference actions.

The following extensions of our work can be lines for future research. We intend to study the elicitation of the *per*-criterion similarity-dissimilarity functions through a co-constructive process and the elicitation of these functions through aggregation-disaggregation processes. A more complete robustness analysis by using simulation may be a relevant focus of study (as in Corrente et al., 2014). It could also be interesting to study learning procedures in order to infer the reference actions and the parameters of preference, such as the weights of criteria, interaction coefficients, and membership degrees (in the same line of approach as Mousseau and Słowiński, 1998). Extending the method to group decision-making is also a promising line for future research. Considering the hierarchy of criteria as, for example, in Corrente et al. (2016), may be another avenue of potential research. The proposed method needs to be supported by appropriate software. Thus, future research may also depend on implementing this method in a computational framework.

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Appendix A. Performances of the candidates

Table A.7: Candidates' performances

Candidates	g_1	g_2	g_3	g_4	g_5	g_6
a_1	740	74	4	7	4	6
a_2	950	82	2	4	4	4
a_3	720	58	3	5	5	5
a_4	920	78	2	5	5	5
a_5	850	66	3	5	6	5
a_6	1100	70	4	5	5	6
a_7	710	73	3	6	5	6
a_8	1000	82	2	4	4	4
a_9	720	65	3	5	5	5
a_{10}	740	78	4	6	4	7
a_{11}	790	71	4	5	6	7
a_{12}	700	80	4	7	5	6
a_{13}	780	67	3	6	6	5
a_{14}	860	90	4	7	6	6
a_{15}	830	92	4	6	6	7
a_{16}	940	87	2	5	5	5
a_{17}	750	54	3	6	5	5
a_{18}	1200	86	3	5	5	4
a_{19}	670	84	4	7	4	6
a_{20}	840	77	3	6	7	6

Appendix B. *Per*-criterion similarity-dissimilarity functions

$$f_1\big(g_1(a)\big) = \begin{cases} 1, & \text{if } |g_1(a) - g_1(b)| \leqslant 50; \\ \frac{100-|g_1(a)-g_1(b)|}{50}, & \text{if } 50 < |g_1(a) - g_1(b)| \leqslant 100; \\ 0, & \text{if } |100 < |g_1(a) - g_1(b)| \leqslant 150; \\ \frac{150-|g_1(a)-g_1(b)|}{50}, & \text{if } 150 < |g_1(a) - g_1(b)| \leqslant 200; \\ -1, & \text{if } |g_1(a) - g_1(b)| > 200. \end{cases}$$

$$f_2\big(g_2(a)\big) = \begin{cases} 1, & \text{if } |g_2(a) - g_2(b)| \leqslant 5. \\ \frac{10-|g_2(a)-g_2(b)|}{5}, & \text{if } 5 < |g_2(a) - g_2(b)| \leqslant 10; \\ 0, & \text{if } g_2(b) - 20 < g_2(a) \leqslant g_2(b) - 10 \text{ or } g_2(b) + 10 < g_2(a) \leqslant g_2(b) + 15; \\ \frac{g_2(a)-g_2(b)+20}{5}, & \text{if } g_2(b) - 25 < g_2(a) \leqslant g_2(b) - 20; \\ \frac{-\big(g_2(a)-g_2(b)\big)+15}{5}, & \text{if } g_2(b) + 15 < g_2(a) \leqslant g_2(b) + 20; \\ -1, & \text{if } \ g_2(a) \leqslant g_2(b) - 25 \text{ or } g_2(a) \geqslant g_2(b) + 20; \end{cases}$$

$$f_3\big(g_3(a)\big) = \begin{cases} 1, & \text{if } g_3(a) = g_3(b); \\ 0, & \text{if } |g_3(a) - g_3(b)| = 1; \\ -1, & \text{if } |g_3(a) - g_3(b)| \geqslant 2. \end{cases}$$

$$f_4\big(g_4(a)\big) = \begin{cases} 1, & \text{if } g_4(a) = g_4(b); \\ 0, & \text{if } |g_4(a) - g_4(b)| = 1; \\ -1, & \text{if } |g_4(a) - g_4(b)| \geqslant 2. \end{cases}$$

$$f_5\big(g_5(a)\big) = \begin{cases} 1, & \text{if } g_5(a) = g_5(b); \\ 0, & \text{if } |g_5(a) - g_5(b)| = 1; \\ -1, & \text{if } |g_5(a) - g_5(b)| \geqslant 2. \end{cases}$$

$$f_6\big(g_6(a)\big) = \begin{cases} 1, & \text{if } g_6(a) = g_6(b); \\ 0, & \text{if } |g_6(a) - g_6(b)| = 1; \\ -1, & \text{if } |g_6(a) - g_6(b)| \geqslant 2. \end{cases}$$

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