Synchronization of Mutually Coupled LC-Oscillators

A.Allam, I. M. Filanovsky
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, Canada
allam.igor@ece.ualberta.ca

Luís Bica Oliveira, Jorge R. Fernandes
I.S.Técnico/INESC-ID Lisboa:
R. Alves Redol 9 - 1000
Lisboa, Portugal
luis.b.oliveira,jorge.fernandes@inesc-id.pt

Abstract — It is shown that synchronization is similar to amplitude stabilization: both mechanisms involve creation of harmonics and frequency reduction. Synchronization of LC-oscillators can be achieved using similar synchronizing circuits in each oscillator connected in parallel with the tank. The output of the first synchronization circuit is routed to the second oscillator and vice versa. This scheme provides coupling using first, third and other odd harmonics. In addition, introducing coupling between common mode outputs of the synchronizing circuits provides second and other even harmonic coupling. This type of coupling may be also used between common mode outputs of amplitude stabilization circuits. The benefits of using simultaneous first and second harmonic coupling were verified by designing a low-power, low phase noise 5 GHz LC oscillator with quadrature outputs. The oscillator phase noise is lower than -121.2 dBc/Hz at 1-MHz offset over the tuning range with a power consumption of 1.8 mW, achieving a FOM lower than -192.2 dBc/Hz.

Index terms—Coupled LC oscillators, quadrature oscillators, phase noise, phase accuracy.

I. INTRODUCTION

The development of quadrature mutually coupled LC-oscillators is characterized by efforts to stay within the framework of linear theory. Yet, some methods of synchronization [1,2,3] may only be explained using the non-linear theory of mutual synchronization. Synchronization of LC-oscillators is a deeply non-linear phenomenon [4]. But the results of [4] are overlooked in the modern approach to the problem of synchronization, and the correct explanation of synchronization based on the non-linear behaviour of constituent oscillators is absent. Here, we try to fill this gap. We consider that synchronization is similar to amplitude stabilization: both processes reduce the oscillation frequency due to generation of harmonics.

To separate amplitude control and synchronization one introduces synchronizing circuits in the constituent oscillators. These synchronizing circuits have output currents with rich harmonic content. For symmetric synchronizing circuits one can make further separation and obtain differential and common mode current components. Then, the current differential component is close to sinusoidal, with the oscillator natural frequency; the current common mode component is also close to sinusoidal, but its frequency is double the natural frequency of the oscillator.

For synchronization, the current differential component provided by the synchronizing circuit should be in phase with the current supplied to the parallel LC-circuit. The synchronizing circuit will partially substitute the amplitude stabilization circuit, and this mechanism is needed for synchronization to occur. The common mode component may also be used for synchronization, but in this case one should find a method to use the second harmonic in the common mode output.

In this paper, the method of coupling using both first and second harmonics was verified by designing a low-power, low phase noise 5 GHz LC oscillator with quadrature outputs.

The system of non-linear differential equations is written under the condition that the synchronizing circuit is supplying a small synchronizing current component, the influence of which may only be seen after the second differentiation. This assumption allows one to find the approximate solution of the system, and to evaluate the amplitudes of the oscillations in the constituent oscillators and to find the frequency of synchronized oscillations.

The structure of this paper is the following. Section II shows that amplitude control and synchronizing mechanisms are deeply connected by reduction of frequency and generation of harmonics. This section also describes the general synchronization system using both synchronization and coupling circuits. The next section describes a circuit implementation using first and second harmonic coupling. Section IV presents simulations results, and gives the comparison with the previously published circuits. Finally, in section 5 we draw the conclusions.

II. STABILIZATION AND SYNCHRONIZATION

We consider the synchronization of two oscillators of the type shown in Fig. 1 a). Each oscillator consists of a parallel LC-circuit, and the circuit providing amplitude stabilization. The process of amplitude stabilization is connected with creation of harmonic distortion in the currents \( i_1 \) and \( i_2 \). The differential current \( i_d = i_1 - i_2 \) supplied to the LC circuit, in addition to first harmonic, includes the third and other odd harmonics. The current \( i_3 = i_1 + i_2 \) includes a DC component, second and other even harmonics. If the harmonic content of the oscillator current is known (it is represented by both differential and common mode components) then the oscillation frequency is given by [4]:

\[
\text{This work was partially supported by POSI and the Portuguese Foundation for Science and Technology (FCT) through project POCTI /58533/ESE/2001 and scholarship BD 10539/2002.)}
\]
\[ \omega = \omega_0 \sqrt{\left(\sum_{n=1}^{\infty} a_n^2 / \sum_{n=1}^{\infty} n^2 a_n^2\right)} \quad (1) \]

where \( \omega_0 = \frac{1}{\sqrt{LC}} \) and \( a_n \) is the amplitude of the \( n \)-th harmonic. Hence, if the coupling using higher harmonic is used, it requires that this harmonic is created. Contrary to the accepted opinion, this coupling will result in the reduction of the oscillation frequency as well.

In practice we have to consider coupling involving the first and the second harmonics only. If oscillators are synchronized by the first harmonic, they should be described by the system of non-linear differential equations:

\[ \begin{cases} 
-2(\delta_0 - \delta_2 x^2)x + \omega_0^2 x + \alpha_1 y = 0 \\
-2(\gamma_0 - \gamma_2 y^2)y + \omega_0^2 y + \alpha_2 x = 0 
\end{cases} \quad (2) \]

where \( x = \omega_1 v_d = \omega_0 (v_1 - v_2) \) for the first oscillator and \( y \) is the corresponding variable for the second oscillator. \( \delta_0 \) and \( \delta_2 \) are defined approximating \( i_d (v_d) \) by a third order polynomial. \( \omega_0 \) is the frequency of the first oscillator and \( \alpha_1 \) is the coupling coefficient and shows the influence of the second oscillator on the first one. Equations (2) are valid for the case of minimal coupling influence of one oscillator on another, when this coupling becomes visible after second differentiation only. In a similar way, the coefficients and variables are defined in the second equation for the second oscillator (for equal oscillators \( \gamma_0 = \delta_0 \), and \( \gamma_2 = \delta_2 \)).

If coupling involves the second harmonic only, then \( \alpha_1 = \alpha_2 = 0 \), and other coefficients are periodic functions of second harmonic [5]. If both coupling mechanisms are present, then (2) preserves constant \( \alpha_1 \) and \( \alpha_2 \) and has periodicity of other coefficients.

For constant coefficients in (2) the approximate synchronous quadrature solution includes \( x \) and \( y \) that are nearly in quadrature, with possible small error, so that [4]:

\[ \begin{cases} 
x = A \cos(\omega t) \\
y = B \sin(\omega t) - e \cos(\omega t) = B \sin(\omega t - \varphi) 
\end{cases} \quad (3) \]

where \( \varphi \) is the quadrature phase error.

Using the energy balance method (the details of this method can be found in [4]; the basic idea is that all internal forces in the nonlinear system are reduced to the external forces applied to a linear oscillating system of close frequency), one can find that:

\[ \frac{2 \Lambda \alpha (\delta_0 - \frac{1}{4} \delta_2 A^2)}{2 \alpha_1 A} + \alpha_2 \omega^2 b = 0 \]
\[ \frac{\omega_0^2 - \omega^2}{A - \alpha_1 \omega^2 c} = 0 \]
\[ \frac{\omega_0^2 - \omega^2}{b + 2 \omega_0 (\gamma_0 - \frac{1}{4} \gamma_2 B^2)} = 0 \]
\[ \frac{\omega_0^2 - \omega^2}{e + 2 \omega_0 (\gamma_0 - \frac{1}{4} \gamma_2 B^2)} = \alpha_2 \omega^2 A = 0 \quad (4) \]

This solution can be rearranged in terms of the amplitudes and the quadrature phase error as:

\[ \begin{cases} 
\delta_0 - \frac{\alpha_1 \omega B}{2 A} \cos \varphi \\
\omega^2 - \omega_1 = \alpha_1 \omega_0 \frac{2 B}{A} \sin \varphi \\
\omega^2 - \omega_2 = \alpha_2 \omega_0 \frac{2 B}{A} \sin \varphi \\
\gamma_0 - \frac{\alpha_2 \omega A}{2 B} \cos \varphi 
\end{cases} \quad (5) \]

The frequency of oscillation can be found from the equation:

\[ \left( \omega^2 - \omega_1^2 \right) \left( \omega^2 - \omega_2^2 \right) - \alpha_1 \omega_0 \alpha_2 \omega^4 \sin^2 \varphi = 0 \quad (6) \]

From this equation, one can see that the synchronized frequency is lower than \( \omega_0 \) and \( \omega_2 \), even in the case of minimal influence of one oscillator on the other. It is possible to show that the full synchronized solution should include the harmonics as well (they simply omitted in assumption (3)). This means that the synchronization process is similar to amplitude stabilization in terms of harmonics content (i.e. generation of harmonics and reduction of the oscillation frequency).

To separate these two processes, it is reasonable to augment each oscillator by a special synchronizing circuit. This synchronizing circuit should provide unilateral transmission of energy from the first oscillator to the second and vice-versa. The synchronizing should be based on non-linear active elements and reactance elements only. It is rational to introduce separate coupling between the even harmonics of the common mode outputs of the synchronizing circuits, and, if possible, to introduce coupling between the common mode outputs of the amplitude stabilization circuits (if these even harmonics exist anyway, let us use them!). In this way one can reduce the load of synchronizing circuits on oscillator. These two synchronization mechanisms are represented in Fig. 2.
Amplitude stabilization circuit

Synchronization circuit

a)

b)

Figure 1. LC Oscillator without (a) and with (b) synchronizing circuit

Synchronization using odd and even harmonics

III. CIRCUIT IMPLEMENTATION

From the previous theoretical analysis different circuits can be designed, and different harmonics can be used for synchronization of LC oscillators. In order to validate the theory we consider a coupling scheme using both the first and second harmonics. The circuit implementation is shown in Fig. 3. The first harmonic coupling is similar to using conventional cross-coupling circuits [1,6,7,8] but here we do not use extra bias current sources. The energy required for the synchronization operation is taken from the first oscillator and injected into the second oscillator and vice-versa. The transistors in the coupling circuits work as nonlinear unilateral capacitors required to differentiate the variable supplied from one oscillator to another. Synchronization of LC oscillators usually requires strong coupling, entailing the use of high bias currents in the coupling circuits. By omitting the synchronizing circuit bias current, the shift in the oscillation frequency is minimal compared to the oscillation frequency of the stand-alone oscillator; hence, the degradation in oscillator phase noise can be minimized.

The second harmonic coupling is implemented with a narrow band passive circuit resonating at double the oscillation frequency (10 GHz) connected between the common mode nodes of the amplitude stabilization circuits. The main advantage of this second harmonic coupling, is that, as the coupling circuit is not connected to the tank circuit nodes, it will not force any additional change in the synchronized oscillation frequency. Therefore, there will be no additional phase noise degradation due to further shifts from the resonant frequency. Using both weak first harmonic coupling (since no bias current is consumed by the coupling circuits) and second harmonic coupling (using a passive narrow band circuit) one may benefit from the good phase noise performance of a single LC oscillator, and, at the same time, achieve accurate quadrature outputs [4,7,8]. The circuit was designed for a 0.18 µm CMOS technology, for oscillation frequency 5 GHz. The circuit parameters are the following. Inductors $L/2 = 1\, \text{nH}$ are designed using a thick Al metal, $M$ transistors have $(W/L) = 80 \, \mu\text{m} / 0.18\, \mu\text{m}$, $M_{SC}$ (synchronization transistors) have $(W/L) = 40 \, \mu\text{m} / 0.18\, \mu\text{m}$, $I_T = 500 \, \mu\text{A}$. For oscillator tuning we use varactors with PMOS transistors of $10 \times 40 \, \mu\text{m} / 0.18\, \mu\text{m}$. The resonant LC circuit is realized using an inductor of 1 nH and a capacitor of 25 fF. The supply voltage is 1.8 V.

IV. SIMULATION RESULTS

The designed oscillator had a tuning range from 4.85 to 5.24 GHz, and the quadrature relationship error was less than 2 degrees over the tuning range (considering 0.5% error in tank frequencies). The oscillator phase noise at 5.24 GHz oscillation frequency is shown in Fig. 4. We simulated the phase noise over the tuning range and we used, for comparison, the following FOM [9]:
The oscillator phase noise at 1 MHz offset is lower than -121.2 dBc/Hz (the best is lower than –192.2 dBc/Hz, and is better at higher frequencies (the best value is –193.8 dBc/Hz). Table I summarizes the oscillator performance. Table II compares this quadrature oscillator performance with that of previously published results. One can see that this circuit has the lowest power consumption and the highest FOM. All FOMs are calculated using the worst-case phase noise over the tuning range.

$$ FOM = L_{\text{simulated}} + 10 \log \left( \frac{\Delta f}{f_{\text{osc}}} \right)^2 \text{Power} \left[ \frac{\text{dBc/Hz}}{\text{mW}} \right] $$

V. CONCLUSION

In LC oscillators both amplitude stabilization and synchronization result in reduction of oscillation frequency and a high harmonic content. Stand-alone oscillators can be synchronized using both the first and second harmonic. These conclusions are confirmed by the design of a 5 GHz low power quadrature oscillator using both first and second harmonic coupling. The circuit inherits the best features from the first and second harmonics coupling schemes, leading to a high performance oscillator in terms of power consumption and FOM. The circuit has a simulated phase noise lower than –121.2 dBc/Hz over the tuning range, and a FOM lower than -192.2 dBc/Hz. The quadrature relationship error is lower than 2 degrees over the tuning range considering 0.5% error in tank frequencies. The power consumption is 1.8 mW.

REFERENCES