

Fuzzy Mechanisms For Causal Relations

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Abstract

Classical fuzzy logic does not allow the implementation of causal relations as defined in causal maps. Until now, Fuzzy Causal Maps (FCM) have been implemented by the use of mechanisms closer to neural networks that can not be mixed with classic fuzzy rule based systems. Rule based fuzzy cognitive maps importance resides in the fact that they could allow a more complete representation of cognition since relations other than causality would be possible.

This paper presents a method to implement Fuzzy Causal Relations that can be used in Rule Based Fuzzy Cognitive Maps (RBFCM). This method introduces a new fuzzy operation that simulates the “accumulative” property associated with causal relations – the Fuzzy Carry Accumulation (FCA). The FCA allows a great flexibility in the addition and removal of concepts and links among concepts while keeping compatibility with classic fuzzy operations.

Keywords: Causality; Fuzzy Causal Relations (FCR); Fuzzy Carry Accumulation (FCA); Fuzzy Cognitive Maps (FCM); Fuzzy sets.

1. Introduction

Decision makers usually face serious difficulties when approaching significant, real-world dynamic systems. Such systems are composed of a number of dynamic concepts or actors which are interrelated in complex ways, usually including feedback links which propagate influences in complicated chains. Axelrod work on Cognitive Maps (CMs)[1] introduced a way to represent these systems, and several methods and tools like [2] or [3] allow the analysis of causal maps structure. However, complete, efficient and practical mechanisms to analyse and predict the evolution of data in CMs are necessary [4] but not yet available for several reasons. System Dynamics tools like [5] could be a solution, but since numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [6], are a qualitative alternative approach to dynamic systems. They are a combination of Neural Networks and

Fuzzy Logic that allow us to predict the change of the concepts represented in CMs. They are fuzzy directed graphs with feedback, consisting of various nodes representing concepts and directed arcs that connect those nodes. However, in most applications, a FCM is indeed a man-trained Neural Network (Multilayer perceptron) which is not Fuzzy in a traditional sense, and doesn't explore usual Fuzzy capabilities. They don't share the properties of other fuzzy systems and can not be mixed with traditional fuzzy rules and operations. Although this is not a problem by itself since FCM perform rather well when trying to represent the evolution of management, organisational or socio-economic problems, FCM are limited to the representation of simple monotonic causal relations between concepts.

Causal relations advantages [7] reside essentially in the fact that causal associations are the major way in which understanding about the world is organised. However, in the area of social sciences and/or psychology we can find out that Cognitive Maps should use other kind of relations between concepts (like the Cognitive Base Schemes defined in [8] and [9]) in order to allow a better representation of real world systems that involve cognition.

The use of Fuzzy sets, logic and inference in its traditional rule based form, as introduced by Zadeh [11] and developed throughout these 30 years is particularly more adequate to represent qualitative knowledge involved in cognitive maps due to its linguistic nature [12]. So, it seems a straightforward solution to try to implement FCM starting from a traditional rule based fuzzy architecture with feedback in order to overcome FCM weaknesses. However, since traditional fuzzy operations can not emulate the effects of causality as it is usually defined in causal maps, there is one important problem to solve while trying to create a Rule Based Fuzzy Cognitive Map: the implementation of fuzzy mechanisms that allow causal relations.

2. Causal Relations in Cognitive Maps and Fuzzy Logic

There is a causal relation between two given concepts whenever a change in one of those concepts affects the other one. For example, there is a causal relation between

Police vigilance and Robbery: a major increase in Police Vigilance will probably cause a decrease in Robbery.

Causal associations appear to be the most widely used in Cognitive Maps. According to Huff [7], the following statements can easily explain this fact:

- Causal relations are the major way in which understanding about the world is organised;
- Causality is the primary form of *post hoc* explanation of events;
- Choice among alternative actions (decision processes) involve causal evaluation.

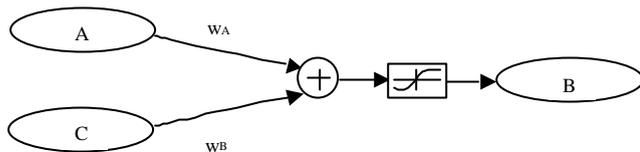


Figure 1 Causal reasoning in a FCM

Usually, causal relations in causal maps always involve change. The result of a causal effect is always a variation in one or more concepts.

Another property of causal relations is that, by nature, causality is “accumulative”. For example, using “natural language” we can say that if concept A and concept B each cause concept C to increase “little”, then C will increase “more than a little”. If two concepts A and B have the exact opposite effect on C, then C will not change. If A affects “little” and B affects “much”, then C will increase “more than much”. The effect when both decrease is similar.

In contrast, a traditional fuzzy relation tends to reinforce itself. If A and B cause C to increase “little” with a belief of 0.3 and 0.6, then concept C will increase “little” with a stronger belief (0.9). If A causes C to increase “little” and B causes C to increase “much”, then C would increase somewhere between “little” and “much”. Opposed effects tend to nullify each other (as in a causal relation).

This essential intrinsic difference causes a total incompatibility in the use of classical fuzzy to represent causal relations. Therefore, in order to introduce causal relations in fuzzy systems, it is necessary to find new ways to make both worlds compatible.

It is important to note that several fuzzy additive systems exist (like Kosko’s SAMs [6] for instance), which are not accumulative in the above sense. Those systems add the beliefs of the variables (Y-Axis), not the values in their universe of discourse (UoD – X-Axis), which means that they do not present the intended causal behaviour.

To represent causality in FCM, Kosko [6] used a different approach that is not compatible with classic fuzzy systems: To obtain the value of a concept, the value of each of its inputs (concepts) [1..1] is multiplied by a weight [1..1];

then the results are added and passed by a non-linearity, just like a common neuron in a Neural Network (Figure1).

As we mentioned earlier the purpose of this paper is to propose fuzzy mechanisms that allow the implementation of causality by the use of fuzzy *If...Then* rules that can be mixed with traditional fuzzy operations.

3. Fuzzy Causal Relations (FCR)

3.1 Fuzzy Carry Accumulation

The idea behind the property of fuzzy causal relations can be easily explained using singletons. Let us imagine that the fuzzy set *Increase* of concept *Robbery* is a singleton at $x = 0.5$ (Figure 2) and we have the following rules:

- **If** Police_Vigilance *Decrease* **Then** Robbery *Increase*;
- **If** Wealth_of_Residents *Increase_Much* **Then** Robbery *Increase*

If the consequent of the application of the two rules is *Increase* ($\mu=0.7$) and *Increase* ($\mu=0.5$), how should these consequents be combined in order to produce the accumulative effect (as we saw above, the result should represent a variation larger than *Increase*)?

In a traditional fuzzy system, the result would always be (after defuzification) $x = 0.5$, even when the sum of the beliefs is greater than 1. Here, we introduce the concept of the **Fuzzy Carry Accumulation (FCA)**:

- If the sum of the beliefs is lower or equal to 1, then we have a standard fuzzy operation - If we have *Increase* ($\mu=0.3$) and *Increase* ($\mu=0.5$), then the result is *Increase* with $\mu=0.8$, since I do not fully believe that the result should be more than *Increase*.

However,

- If the sum of the beliefs is greater than 1, then there is an overflow of the reminder (just like a carry in a sum operation) towards a value representing a larger variation.

Figure 2 represents this operation. With this overflow of the excess, after defuzification, the result of *Robbery* will be larger than *Increase*.

If the rules involved represent a decrease, then the carry is performed in the opposite direction.

If there are several rules that cause large reminders, whenever the reminder exceeds 1 there is another overflow towards a larger variation.

When the consequents involve different fuzzy sets – for example *Increase* and *Increase_Much* – then since we know that the result must be larger than the largest consequent, the solution is to shift the smaller consequent towards the former. This shift operation arises several problems and constraints that will affect the implementation of fuzzy causality, since there must be a way of retaining and

distinguish the original value of the shifted set. Due to this

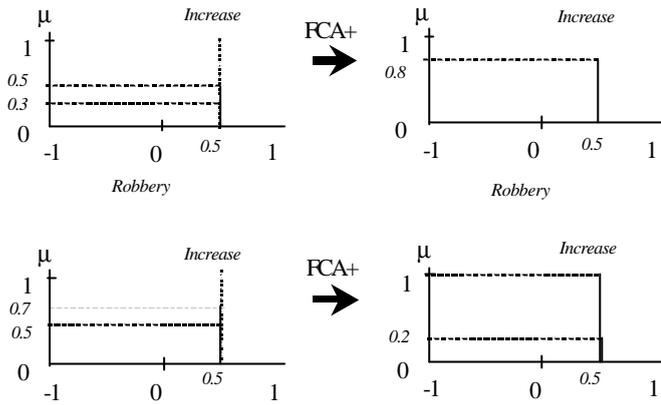


Figure 2 Fuzzy Carry Accumulation with singletons

constraints, fuzzy causal operations can not be implemented with singletons. Singletons were used as a simpler way to show the FCA principles.

When we use typical fuzzy sets, the FCA operation is applied at every point of the UoD. The overflow of the sum at each point is added to the sum of the μ of both sets at the next point of the UoD. While overflow exists, it is carried over to the next point, provoking an increase of the value of the resulting set centroid.

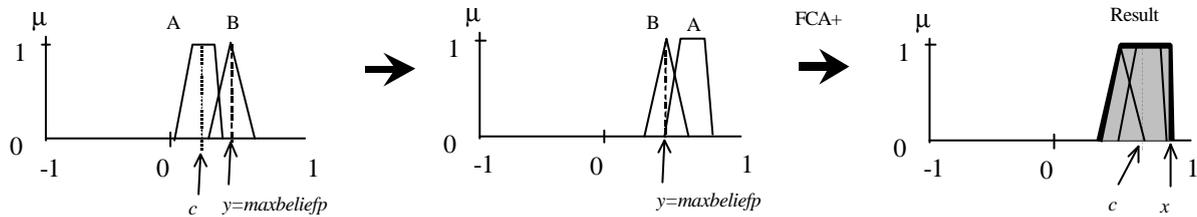


Figure 3 Fuzzy Carry Accumulation

As with singletons, it is necessary to shift the set that represents a lower variation towards the one with the largest variation. Advantages and disadvantages of several ways to define which set is the largest and where to shift the lowest set are discussed in [14]. Presently we use the following methods:

a) Use the maximum belief point of each set to find the set that represents the largest change:

given x such as $\mathbf{mb}(x) = \text{MAX}(\mathbf{mb})$;

given y such as $\mathbf{mb}(y) = \text{MAX}(\mathbf{mb})$;

if $\text{abs}(x) > \text{abs}(y)$ **then** A is higher than B , and $\text{maxbeliefp} = x$

Else B is higher than A , and $\text{maxbeliefp} = y$.

Where $\mathbf{mb}(z)$ is the degree of belief of membership function A at point z .

If there are several points of maximum belief (which is usual), maxbeliefp will be the one with the highest absolute value.

b) Shift the set that represents the lower change until its lowest point coincides with maxbeliefp

Figure 3 shows how to apply the shift and FCA operation between fuzzy sets A and B . A is the result of the application of a previous rule. c represents the value of the centroid of the concept (output). The application of the input that produces set B shifts c to the right, which indicates an increase in the change value of the concept.

The Positive or Negative Fuzzy Carry Accumulation – **FCA±**, maintains the total area of the involved fuzzy sets while there is no saturation (as we approach maximum variation allowed for the concept some area is lost), and will move the centre of gravity of the variation of the concept towards “1” or “-1”.

3.2 Evaluation of the rules (FOS - Fuzzy Output set of each input)

In order to guarantee that the FCA is commutative and associative, which are obvious requirements, and due to the shift operation, it is necessary to transform the fuzzy sets that result from a set of rules between 2 concepts in a fuzzy set with certain characteristics. In 3.3 we explain why is this transformation necessary.

When a fuzzy causal rule fires, we use the Max-Dot inference method in order to maintain the shape of each membership function (mbf) - triangular, bell, etc. Then we sum the outputs of each rule (fuzzy union). Due to the characteristics of the used membership functions (see 3.3), a maximum of 2 rules is fired between each concept, and the result of the sum of the resulting fuzzy sets at each point is never greater than 1. Then the sum is replaced by a fuzzy set with the **same area, the same centroid, maximum belief at centroid = 1** and the **shape of the mbf** of the concept.

Let us call the obtained Fuzzy Output Set of each input, **FOS**. Figure 4 shows the method to obtain the FOS of an input whose rules fire with Increase(0.3) and Increase_Much(0.7). The resulting FOS area is larger than the fuzzy set Increase area, and smaller than the fuzzy set Increase_Much area.

As a special case, if only one rule fires, its degree of activation is necessarily “1” and we obtain the FOS immediately.

The FOS is the result of the application of the rules that define a causal relation between two concepts. Whenever there are 2 or more concepts affecting another one, the shift and fuzzy carry accumulation is applied to the FOS resulting from each involved causal relation. Example: In Figure 5, we

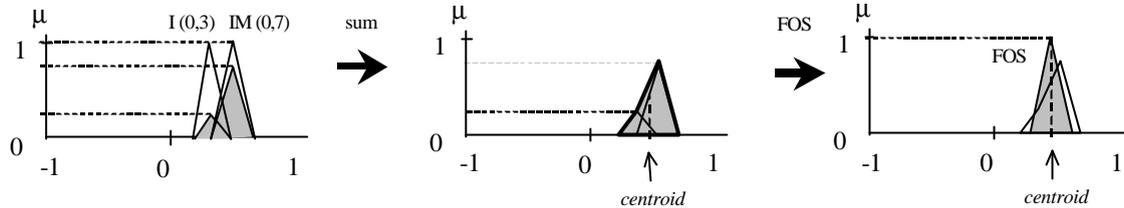


Figure 4 Combination of rule results

have 4 concepts. When the rules that define the causal relation between each concept and Robbery are activated, we obtain 3 different FOS. We must combine these FOS by the use of shift and FCA in order to obtain the desired causal “accumulative” effect.

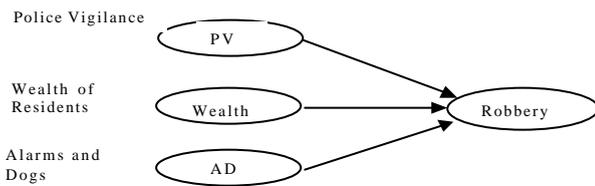


Figure 5 FCR example

3.3 Conditions for fuzzy causality

In order to implement fuzzy causality, the membership functions that define the change in each concept and the fuzzy operations that we introduced earlier must meet the following restrictions and conditions:

- Membership functions (mbf) which represent higher variations must have a larger area.
- Membership functions must cross their closest neighbours at $\mu=0.5$, and at any point of the UoD the sum of the membership degree of all mbf must be “1”, except for c).
- Increase and decrease mbf can not cross, not even being neighbours.
- The inference method must be the Max-Dot in order to maintain the shape of each membership function (triangular, bell, etc).
- The domain of the fuzzy set that results from the

application of all causal rules relating two concepts (FOS) must be related to the amount of intended change. The domain of the FOS is defined at the points where μ FOS is non-zero.

- Increase and decrease rules must be treated separately and the total resulting area of change involved in either direction (increase/decrease) must be stored.

Since the FCR algorithm shifts the output sets, there must

be some way to distinguish their meaning after the shift operation. Conditions a), b), c) and d) guarantee that the area of the FOS is related with the intended amount of change. The higher the amount changes, the larger is the area of the FOS [13]. The input *Increase(35%) / Increase_Much(65%)* produces an FOS with an area marginally larger than the input *Increase(36%/Increase_Much(64%)*. The relation change/area depends on the relation of the areas of the 2 involved membership functions.

Condition e) is essential due to the nature of the shift and FCA± operation. Fuzzy sets that are accumulated at the same point of maximum belief must be distinguished by their domain size in order to represent the intended amount of change. This condition is guaranteed by conditions a), b), d) and the procedure to obtain the FOS (see 3.2). Since all FOS share the same shape, same maximum μ value at centroid and their area is related with the amount of change they represent, then the domain is also related with the amount of change [13].

These conditions also guarantee that the FCR is associative and commutative [13], which allows the introduction or removal of concepts and/or rules in a Rule Based Fuzzy Causal Map without special requisites.

Condition f) is necessary due to the “saturation” effect. Whenever the amount of change reaches its maximum, it is necessary to guarantee that any excess is not lost. This way, even if there are rules opposing the maximum change, their effect is not considered until they cancel the excess.

4. Results and conclusions

Fuzzy Causal Relations were developed with Rule Based Cognitive Maps in mind. Application to different systems, results of simulation of several causal maps and

Order of Application of Rules	Robbery (PV = 40, Wealth = 40; AD = 40)	Robbery (PV = 48, Wealth = 36; AD = 40)	Robbery (PV = 36, Wealth = 45; AD = 36)
1, 2, 3, 4, 5, 6	-36 (Decrease)	-43 (Decrease/DMuch)	-23 (Decrease / DFew)
2, 1, 5, 6, 3, 4	-36	-43	-23
6, 5, 4, 3, 1, 2	-36	-43	-23

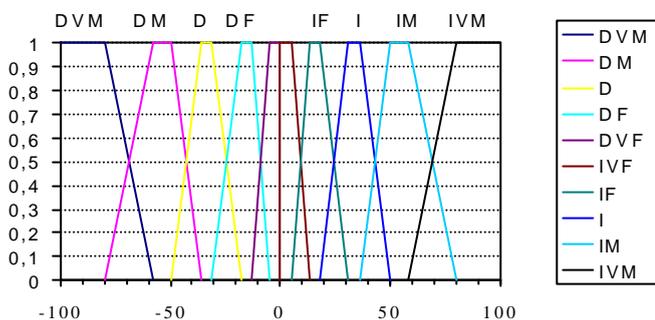
Table 1 Application of a FCR results

comparisons with FCM can be found in [14], [15] and [16]. The intended causal effect and the commutative and associative properties of FCR can be seen in the following example that uses the concepts and relations shown previously in Figure 5:

a) Relevant rules of rulebase:

- (1) If PV Increase_Much then Robbery Decrease
- (2) If PV Increase then Robbery Decrease_Few
- (3) If Wealth Increase then Robbery Increase
- (4) If Wealth Increase_Much then Robbery Increase_Much
- (5) If AD Increase then Robbery Decrease_Much
- (6) If AD Increase_Much then Robbery Decrease_Very_Much

b) Membership functions of concepts:



Considering variations in PV, Wealth and AD that only activate the above rules, we obtain the same variation to Robbery no matter the order of application of the rules (see Table 1).

If we eliminate the relation between AD and Robbery (maintaining PV=W= 40), then Robbery Increases_Few (12) as expected, since according to the rules an increase in Wealth has more influence than the same increase in PV.

Although the Fuzzy Carry Accumulation is a debatable option that can be improved, it is a solution to the problem of fuzzy causality, even if not as close to the classical fuzzy operations as originally intended.

One might argue that FCR computation is complicated and intensive, needing lots of rules, which is true when compared to the simplicity and elegance of FCM. However, traditional applications of causal maps are not “real time” applications that need to present immediate solutions, and the capabilities of actual computers are more than enough to solve most problems. That said, it is important to mention that we can isolate the user of FCR from this complexity, since it is possible to pre-define a certain number of fuzzy causal relations (rule bases), that can be expressed with simple sentences like “A affects much B” or “A affects B exponentially”.

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