Metadata of the chapter that will be visualized in SpringerLink

Book Title	15th European Workshop on Advanced Control and Diagnosis (ACD 2019)
Series Title	
Chapter Title	A Control Approach to Energy Supply–Demand Markets
Copyright Year	2022
Copyright HolderName	Springer Nature Switzerland AG

Corresponding Author	Family Name	**Costa**
	Particle	
	Given Name	**Bertinho A.**
	Prefix	
	Suffix	
	Role	
	Division	INESC-ID/IST/ULisbon
	Organization	Rua Alves Redol
	Address	9 1000-029, Lisboa, Portugal
	Email	bac@inesc-id.pt
Author	Family Name	**Lemos**
	Particle	
	Given Name	**João M.**
	Prefix	
	Suffix	
	Role	
	Division	INESC-ID/IST/ULisbon
	Organization	Rua Alves Redol
	Address	9 1000-029, Lisboa, Portugal
	Email	jlml@inesc-id.pt

Abstract

This paper addresses the automatic control of energy supply–demand markets and defines the conditions that drive a market to a stable price equilibrium. Aggregate supply, involving fixed and variable costs, and aggregate demand based on a budget constraint are considered to model an energy market. A dynamic model for a single supplier/single consumer market is initially considered and developed. Conditions for price equilibrium are obtained. This is followed by considering the negotiation dynamics based on the time price profiles imposed by the supplier. In this case, linear programming optimization is used to address the problem. The model of a single supplier/single consumer market is extended to include several consumers. Constraints from the supplier, which impose constraints on the consumer behaviours, are considered. This leads to the need for an information network to exchange a minimum of information among market agents to coordinate their actions. The stability of the energy supply–demand market is demonstrated.

Keywords (separated by '-'): Supply–demand - Energy markets - Price stability - Control

Chapter 57
A Control Approach to Energy Supply–Demand Markets

Bertinho A. Costa and João M. Lemos

Abstract This paper addresses the automatic control of energy supply–demand markets and defines the conditions that drive a market to a stable price equilibrium. Aggregate supply, involving fixed and variable costs, and aggregate demand based on a budget constraint are considered to model an energy market. A dynamic model for a single supplier/single consumer market is initially considered and developed. Conditions for price equilibrium are obtained. This is followed by considering the negotiation dynamics based on the time price profiles imposed by the supplier. In this case, linear programming optimization is used to address the problem. The model of a single supplier/single consumer market is extended to include several consumers. Constraints from the supplier, which impose constraints on the consumer behaviours, are considered. This leads to the need for an information network to exchange a minimum of information among market agents to coordinate their actions. The stability of the energy supply–demand market is demonstrated.

Keywords Supply–demand · Energy markets · Price stability · Control

*This work was supported by Fundação para a Ciência e a Tecnologia under the research project HARMONY AAC n.2/SAICT/2017 - 031411 and the pluriannual funding program UID/CEC/50021/2013.

B. A. Costa (✉) · J. M. Lemos
INESC-ID/IST/ULisbon, Rua Alves Redol, 9 1000-029, Lisboa, Portugal
e-mail: bac@inesc-id.pt

J. M. Lemos
e-mail: jlml@inesc-id.pt

E. Zattoni et al. (eds.), *15th European Workshop on Advanced Control and Diagnosis (ACD 2019)*, Lecture Notes in Control and Information Sciences - Proceedings,
https://doi.org/10.1007/978-3-030-85318-1_57

57.1 Introduction

The decreasing costs of renewable energy technology as well as the need to decrease the consumption of energy from fossil energy sources motivate the integration of renewable electric generation into local grids (microgrids) such as in residential areas. This integration requires a management strategy to coordinate local generation and the access to the main electric grid. Consumers may switch from the main grid to local consumption and, later, they can connect again to the main grid. One important point that characterizes the AC electric networks is that the amount of energy produced must be in balance with the amount of energy consumed, since otherwise the operation/service of the electric grid is degraded.

Several problems of this area have been previously addressed in the literature. In [1], the scheduling of energy storage to maximize the islanding time for microgrids using distributed predictive control with the alternating direction method of multipliers (ADMM) algorithm is described. In [2], the problem of microgrid islanded operation is addressed by considering operational details, such as frequency versus power drop. In [3], the aim is to obtain a balance of power in a group of prosumers based on a price mechanism, and optimal control is applied. Balance of the demand and supply in a heat network is addressed in [4].

In this paper, the aim is to describe the balance of energy in a network using the economic concepts, the supply-demand curves, which incorporate costs and budget constraints that are translated in nonlinear functions. Technical details of the operation of the network are assumed to be translated into higher level constraints, such as the minimum time interval to disconnect/connect a consumer from/to a energy network, or the maximum energy rate change allowed to consumers.

This paper is organized as follows: In Sect. 57.2, the supply and demand functions are described based on simplified economic principles for a market with a single supplier and a single consumer. The dynamics of the negotiation process, the control, and convergence is addressed in Sect. 57.3. Section 57.4 addresses the negotiation dynamics based on time cost profiles. The extension of the single supplier/single consumer to a single supplier/multiple consumer and its control is described in Sect. 57.5. In this case, it is assumed that the supplier has a constraint on the energy rate change. This constraint is explored to formulate the control strategy of the market. Its implementation is based on the existence of an information network to broadcast information (current price and aggregate quantity) from the supplier to the consumers, and the computation, among the consumers, of the number of consumers that want to increase or decrease their consumption level. The behaviour of the control algorithm is illustrated with computer simulation results. Changes in aggregated demand over time and the supply curve of the producer define the price/unit in the market. From the point of view of consumers, it is important to find a strategy that minimizes their costs, but subject to the constraint the given amount of energy is needed for their needs, a strategy addressed in Sect. 57.6. Conclusions are drawn in the last section.

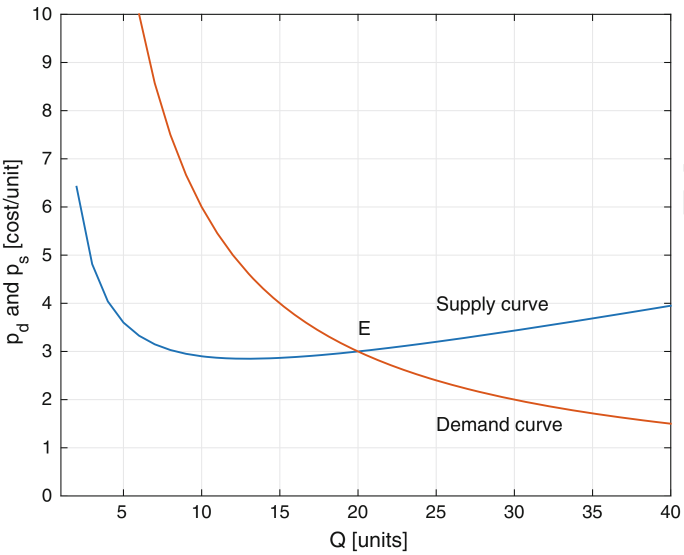

Fig. 57.1 Example of a supply curve $f_s(.)$ and a demand curve $g_d(.)$ that intercept at the point E, which defines the market price equilibrium

57.2 Modelling an Energy Market

The simplest energy market that can be considered has a single supplier and a single consumer. These market agents have economic objectives that are described by a supply curve (function) and by a demand curve (function).

A supply function $f_s(.)$ relates the quantity supplied $Q_s > 0$ to the price/unit defined by the supplier $p_s(Q_s) > 0$, that is $p_s(Q_s) = f_s(Q_s)$. In a similar way, a demand function $g_d(.)$ relates the quantity demanded, $Q_d > 0$, to the price $p_d(Q_d) > 0$ (cost/unit), that is $p_d(Q_d) = g_d(Q_d)$.

Goods are traded if there is an agreement between the supplier and the consumer, and this is achieved if, for a given quantity Q, the condition $p_s(Q) = p_d(Q)$ is fulfilled. This condition is represented in Fig. 57.1 where the point E represents the market equilibrium.

57.2.1 *Demand Function*

Several constraints may shape a demand function. A consumer may have a rigid consumption profile over a "large" period of time. An other consumer may assign a constant amount of money $\beta > 0$ to buy a product. This consumer may increase or decrease the consumption $Q > 0$ depending on the supplier's price, such that $p_d(Q)Q \leq \beta$,

$$p_d(Q) \leq \frac{\beta}{Q} \;\; with \;\; Q \in [Q_{min};\; Q_{max}] \,, \tag{57.1}$$

with $0 < Q_{min} \leq Q_{max}$. This function is used in this work, and it is represented in Fig. 57.1 where $\beta = 60$.

57.2.2 Supply Function

A supply function defines the price of a product as a function of quantity, and it depends on the production cost plus some profit. The total production cost depends on fixed costs $c_o > 0$ and running costs $c_v Q > 0$. The total cost ($C_T(Q)$) that the market "must" cover is formed by the total production cost plus a gain $gQ > 0$, where $g > 0$ is a profit parameter,

$$C_T(Q) = c_o + c_v(1 + \alpha Q)Q + gQ \,, \tag{57.2}$$

where the term $\alpha Q > 0$ represents an increasing cost due to a loss of efficient when the production is large. In the supply function, the price per unit is obtained by computing $C_T(Q)/Q$ that yields

$$p_s(Q) = \frac{c_o}{Q} + c_v(1 + \alpha Q) + g \,. \tag{57.3}$$

This function is represented in Fig. 57.1, where $c_0 = 10$, $c_v = 1$, $\alpha = 0.06$, and $g = 0.3$.

57.2.3 Market Equilibrium Point

The market equilibrium point(s) is obtained by solving the equation

$$p_d(Q) = p_s(Q) \,, \tag{57.4}$$

subjected to the constraint $Q \in [Q_{min};\, Q_{max}]$. For the functions described ((57.1) and (57.3)), the problem can be solved by finding the positive solution ($Q^\star$) of the second-order equation

$$c_v \alpha Q^2 + (c_v + g)Q + (c_o - \beta) = 0, \tag{57.5}$$

and by checking if it belongs to the interval $[Q_{min}; Q_{max}]$. This second-order equation has a real positive solution if $\beta - c_o > 0$. This can be concluded by inspecting the algebraic expression of the solutions. In the special case that $1 + \alpha Q^\star$ is approximately 1, the solution is given by

$$Q^\star = \frac{\beta - c_o}{c_v + g}\ , \tag{57.6}$$

where $\beta - c_o > 0$. The profit is defined by $L = gQ^\star$ that yields

$$L = (\beta - c_o)\frac{g}{c_v + g}, \tag{57.7}$$

where the parameter g can be used by the supplier to increase the profit. In this particular case, the maximum profit is obtained by increasing g towards $+\infty$, $Q^\star \to 0$ causing $p^\star \to +\infty$. To avoid this situation, a lower bound on Q must be selected. AQ2

57.3 Negotiation Dynamics

The negotiation is an iterative process that has the objective of driving the price to an equilibrium value defined by the aggregate supply function and the aggregate demand function of the market.

To describe the negotiation between the supplier and the consumer, the following assumptions are used:

1. The negotiation happens over discrete-time intervals of equal length (h).
2. The consumer asks the supplier the price (per unit) for a quantity Q.
3. The supplier sends the price to the consumer.
4. The consumer decides to increase the quantity Q if $p_d(Q) >= p_s(Q)$, or to decrease quantity if $p_d(Q) < p_s(Q)$.

This iterative procedure has the purpose to find the solution of an algebraic equation. To describe this negotiation process at the discrete time k, a tracking error variable $e[k]$ is defined by the difference between the consumer's price and the supplier's price,

$$e[k] = p_d(Q[k]) - p_s(Q[k]). \tag{57.8}$$

Considering now the tracking error at discrete time $k + 1$, it relates to $e[k]$ by

$$e[k+1] = e[k] + [g_d(Q(k+1)) - g_d(Q(k))] - [f_s(Q(k+1)) - f_s(Q(k))]. \tag{57.9}$$

Using Eqs. (57.1) and (57.3) in Eq. (57.9) yields

$$e[k+1] = e[k] - \left(\frac{\beta - c_o}{Q(k+1)Q(k)} + c_v\alpha\right)(Q(k+1) - Q(k)). \tag{57.10}$$

Considering (57.10), the purpose is to adjust the quantity $(Q(k+1) - Q(k))$ over time such that $e[k] \to 0$. One possible solution is to select a linear control law such that

$$Q(k+1) - Q(k) = h\theta e[k]\,, \tag{57.11}$$

where h is the discrete-time interval and $\theta > 0$ is an adjustable parameter that can be used to accelerate the convergence speed.

Lemma 1 *The closed-loop system defined by (57.10) and the control law (57.11) are stable if θ is chosen according to*

$$0 < \left(\frac{\beta - c_o}{Q(k+1)Q(k)} + c_v\alpha\right) < \frac{1}{h\theta}. \tag{57.12}$$

Proof Using Eq. (57.11) in Eq. (57.10) yields

$$e[k+1] = \left[1 - \left(\frac{\beta - c_o}{Q(k+1)Q(k)} + c_v\alpha\right)\right] h\theta e[k]. \tag{57.13}$$

By applying stability criteria to this discrete-time closed-loop nonlinear model and $Q[.] > 0$, the output will converge to zero if the condition (57.12) is fulfilled. Note that $Q[k] \to Q^*$.

From (57.12), it can be concluded that θ can be chosen small enough to fulfil the stability condition. But for very small values, the convergence may be too slow. In this case, the "optimal" value for θ depends on the knowledge of the parameters of the supply and the demand functions.

Lemma 2 *The continuous-time version of the closed-loop system defined by (57.10) and (57.11) is asymptotically stable.*

Proof Equations (57.10) and (57.11) can be written as

$$\frac{e[k+1] - e[k]}{h} = -\left(\frac{\beta - c_o}{(Q(k) + h\theta e[k])Q(k)} + c_v\alpha\right)\theta e[k]. \tag{57.14}$$

Note: Considering that k corresponds to t, then the discrete time $k+1$ corresponds to $t+h$.

Letting $h \to 0$ yields

$$\frac{de(t)}{dt} = -\left(\frac{\beta - c_o}{Q^2(t)} + c_v\alpha\right)\theta e(t)\,, \tag{57.15}$$

and

$$\frac{de_q(t)}{dt} = \theta e(t)\,, \tag{57.16}$$

where $e_q(t) = Q(t) - Q^*$ and Q^* represents the quantity of energy at the equilibrium. As the term $\left(\frac{\beta - c_o}{Q^2(t)} + c_v \alpha\right)$ is positive and $Q(.) > 0$, the solution of (57.15) tends to zero and $Q(t) \to Q^*$.

A detailed analysis can be done by considering the Lyapunov candidate function

$$V(t) = \frac{1}{2}e^2(t) + \frac{1}{2}e_q^2(t) \,. \tag{57.17}$$

The time derivative of $V(t)$ can be written as

$$\frac{dV(t)}{dt} = -\left(\frac{\beta - c_o}{(e_q(t) + Q^*)^2} + c_v\alpha\right)\theta e^2(t) + e(t)e_q(t) \,. \tag{57.18}$$

By inspecting Fig. 57.1, it can be concluded that $e_q(t) < 0$ for $e(t) > 0$, $e_q(t) > 0$ for $e(t) < 0$, and $e(t)e_q(t) \leq 0$. This implies that $\frac{dV(t)}{dt} < 0$ for $e(t) \neq 0$. In the case that $\frac{dV(t)}{dt} = 0$ implies that $e(t) = 0$ and $e_q(t) = 0$. Thus, the closed-loop nonlinear dynamic system is asymptotically stable. AQ3

57.3.1 Example—Illustration of the Convergence of the Negotiation

The results obtained with computer simulation are shown in Fig. 57.2, where the initial value for $Q(1) = 3$, $h = 1$, and $\theta = 0.1$. Figure 57.3 shows the time evolution of the prices in the supply–demand plane, and this to be compared with Fig. 57.1 that shows the supply and demand functions.

57.3.2 The Cobweb Model

The cobweb model is a dynamic model that is used to describe the behaviour of markets. As such, it is interesting to compare the results obtained by the approach followed in this work and the cobweb model.

Figure 57.4 shows, on the left, the behaviour obtained by applying the control law $Q[k+1] = Q[k] + h\theta e[k]$. The system evolved along Q (represented by vertical lines) in the price/quantity space, having a “smooth” convergence. In the case of the cobweb model, which is represented on Fig. 57.4 on the right, it is assumed an initial high demand/high price (point 1); this causes a large production (point 2); in consequence, the demand side decreases the price (point 3); the producer acts by decreasing the production (point 4), which leads to an increase in the price (point 5), ... , until the demand and supply are equal or until a limit cycle is obtained. This describes oscillatory behaviour.

468241_1_En_57_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:5/10/2021 Pages: xxx Layout: T1-Standard

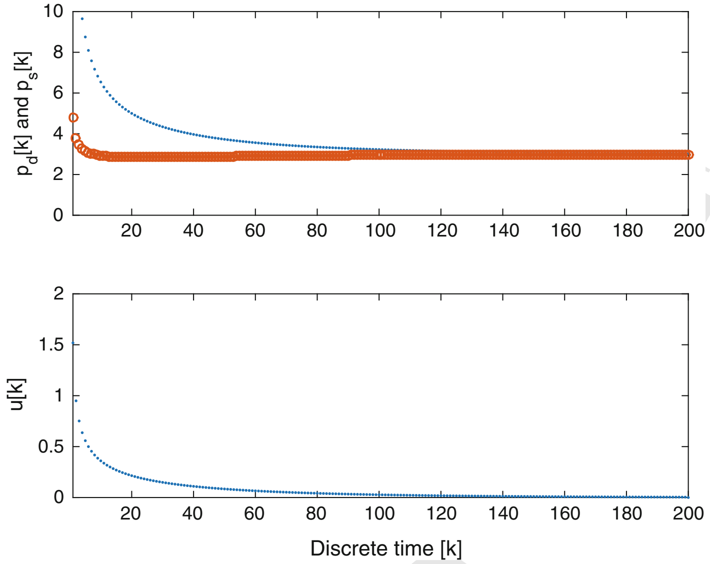

Fig. 57.2 Illustration of the negotiation process where $p_d(k)$ and $p_s(k)$ converge to the same value

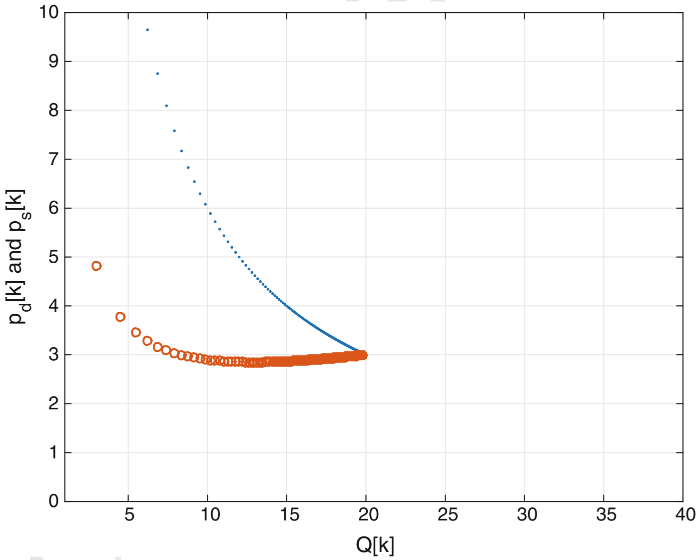

Fig. 57.3 Evolution of the negotiation process along the supply and demand functions towards the price equilibrium. This figure should be compared to Fig. 57.1

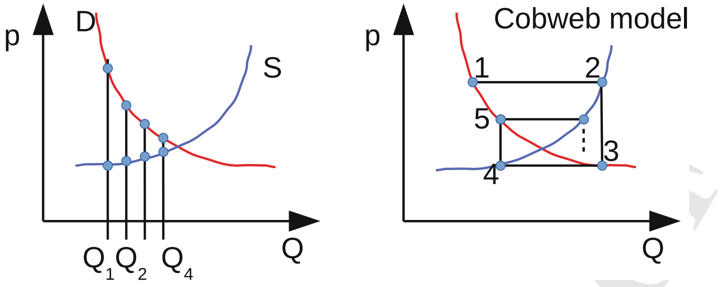

Fig. 57.4 Comparing the results obtained with the proposed negotiation model and the behaviour of the cobweb model

The main conclusion is that by defining a negotiation process and by using a control law with adequate properties, smooth behaviour is obtained that avoids the oscillatory behaviour present in the cobweb model.

57.4 Negotiation Dynamics Based on a Time Price Profile

Energy consumption shows periodic behaviours in short (daily) and long (season) periods. A producer can explore these behaviours to impose a time price profile, which is associated with time instants, independently of the consumption of each consumer.

In this scenario, a consumer will try to consume or store energy during time periods where the price is low. Assuming that a consumer needs a given time-consumption profile $v[k] \geq 0$ with $k = 1, ..., n$, and receives (or learns) the time price profile $p[k] > 0$, he/she can decide to store energy $u[k] > 0$ in a device that has an energy level $x[k]$ such that $0 \leq x[k] \leq C$ by requesting energy $y[k]$ to the producer. If the price is high, the consumer will use the energy that is in the device by selecting $u[k] < 0$. The constant $C > 0$ represents the maximum storing capacity. The decision to store energy and the level of energy in the device can be modelled by the following discrete-time dynamic system:

$$\begin{aligned} x[k+1] &= x[k] + Tu[k] \\ y[k] &= w[k] + u[k] \,, \end{aligned} \tag{57.19}$$

where $T > 0$ is a normalization constant. In this work T is set to 1.

The problem consists in finding $u[.]$ such that the cost function

$$J(N) = \sum_{k=1}^{n} y[k]p[k] \tag{57.20}$$

is minimized subject to the constraints imposed by (57.19) and

$$\sum_{k=1}^{n} y[k] = \sum_{k=1}^{n} v[k] \,, \tag{57.21}$$

$$0 \leq y_{min}[k] \leq y[k] \leq y_{max}[k] \,, \tag{57.22}$$

$$0 \leq x[k] \leq C \,. \tag{57.23}$$

This problem can be translated in a Linear Programming (LP) minimization problem with a set of inequalities (due to the dynamics of the storing device)

$$AY \leq I_{n,1}(C - x(1)) + AW \,, \tag{57.24}$$

where $Y = [y[1], y[2], ..., y[n]]'$, $I_{n,1} = [1, 1, ..., 1]'$, $W = [w[1], w[2], ..., w[n]]'$ and,

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \ddots & 0 \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} , \tag{57.25}$$

and the equality due to equation $\sum_{k=1}^{n} y[k] = \sum_{k=1}^{n} v[k]$.

57.4.1 Example—Consumer Decision Based on Linear Programming Optimization

Figure 57.5 shows the results obtained by applying the LP minimization method. The upper plot shows the price profile ("+") and the requested energy $y[k]$ ("*") that occurs during time instants with low prices. The energy requested is stored in the device and it will be used during the time period where the prices are at a higher level, as shown in the lower plot. The energy level stored in the device, $x[k]$, is represented by the line with "*", the consumer profile $v[k]$ is represented by the line with "o", and the command $u[k]$ is represented by the line with "x".

Note, however, that the solution depends on the selection of the constraints, and if there is a large difference between the lower and the higher prices, the consumers rationally will shift the energy purchases to the time periods that have the lower prices. They will avoid the time periods that have higher prices. This creates synchronous behaviour in the consumers that may cause rush periods. Additionally, the producer may change the price profile to explore the new behaviour of the consumers, and this can destabilize the energy network.

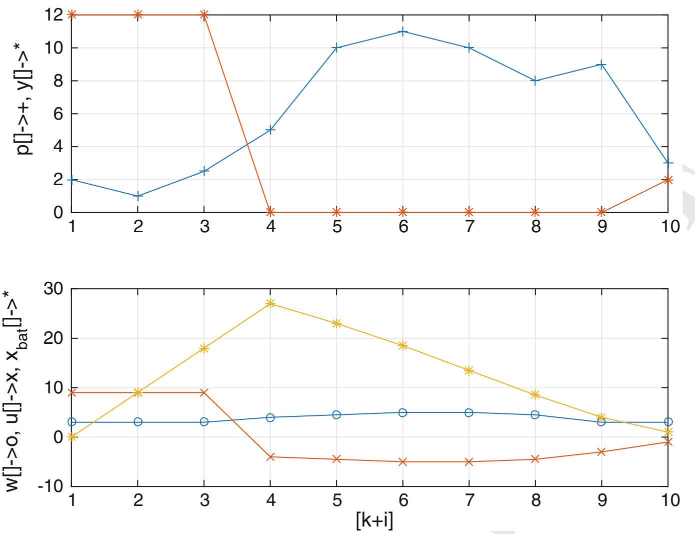

Fig. 57.5 Requesting energy based on Linear Programming Minimization. The upper plot shows the price profile ("+"), and the requested energy $y[k]$ ("*") which occurs during the time instants with low prices. The lower plot shows the command to store energy $u[k]$ ("x"), the energy level in the device $x[k]$ ("*") and the consumer profile $v[k]$ ("o")

57.5 Management of a Market With a Single Producer and Several Consumers

In this section, a market composed of one energy producer and by several consumer agents is considered. This model can represent a local grid with local production (wind or solar photovoltaic and local storage) and a connection to the main electric power network. Consumer agents may switch from the main grid to local consumption, and after some time, will connect again to the main grid. One important point that characterizes AC networks is that the amount of energy produced must be in balance with the amount of energy consumed, since otherwise the operation of the electric grid is compromised. Thus, a producer cannot instantaneously change its energy output from one level to another level. This fact imposes constraints on how consumer agents interact with the producer. An information, a telecommunication/computer network must be used to broadcast information to the consumer agents to allow them to shape their interactions with the producer. AQ4

In this analysis, the information that is broadcasted to the consumer agents by the producer is the following:

1. The production output $Q[k]$ at discrete time k and the price $p_s(Q[k])$.
2. The production upper bound for the next time $k+1$, $Q_{max}[k+1] = Q[k] + \Delta_1[k]$, and the price $p_s(Q_{max}[k+1])$.

468241_1_En_57_Chapter ✓ TYPESET ☐ DISK ☐ LE ✓ CP Disp.:5/10/2021 Pages: xxx Layout: T1-Standard

3. The production lower bound for the next time $k+1$, $Q_{min}[k+1] = Q[k] - \Delta_2[k]$, and the price $p_s(Q_{min}[k+1])$.

With this set of information, the current segment of the supply function is disclosed to let users shape their requests for the next time interval $k+1$, and this includes the (production rate) constraint from the supplier,

$$Q[k+1] \in [Q[k] - \Delta_2[k]; Q[k] + \Delta_1[k]] \ . \tag{57.26}$$

This simplifies the search of the market equilibrium, and promotes the implementation of decentralized management of the market.

The next problem that must be addressed is how to split the increments Δ_1 and Δ_2 among the consumer agents. The aggregate demand (consumption) $Q[k]$ is given by

$$Q[k] = \sum_{i=1}^{m} Q_i[k] \,, \tag{57.27}$$

where m represents the number of consumer agents, and $Q_i[k]$ is the consumption of an agent i.

A solution to be considered involves the computation of the number of agents that want to increase the consumptions, n_1, the number of agents that want to keep their consumptions, n_o, and the number of agents that want to decrease the consumptions, n_2. The counting process can be implemented using the information network to exchange information among the consumer agents.

Knowing the values of $n_1[k]$, $n_o[k]$, and $n_2[k]$, each consumer agent can compute the increment

$$\Delta_{1i}[k] = \Delta_1[k]/n_1[k] \,, \tag{57.28}$$

$$\Delta_{2j}[k] = \Delta_2[k]/n_2[k] \,. \tag{57.29}$$

Other strategies can be used that explore the ratio $Q_i[.]/Q[.]$ to define the increments. A consumer agent that wants to increase the consumption must compare the price given by the supplier $p_s[Q[k] + \Delta_1[k])$ to $p_{di}(Q_i[k] + \Delta_{1i}[k])$:

- If $p_s(Q[k] + \Delta_1[k]) \leq p_{di}(Q_i[k] + \Delta_{1i}[k])$, the increment $\Delta_{1i}[k]$ is requested.
- If $p_s(Q[k]) < p_{di}(Q_{1i}[k] + \Delta_i[k]) < p_s[Q[k] + \Delta_1[k])$, the increment $\theta \times \Delta_{1i}[k]$ is requested, with $theta \in [0; 1]$.
- If $p_{di}(Q_i[k] + \Delta_{1i}[k]) < min(p_s[Q[k], p_s[Q[k] + \Delta_1[k]))$, then $Q_i[k+1] = Q_i[k]$ and a decision can be taken to decrease the consumption in the discrete time $k+2$.

A similar approach can be applied by a consumer agent that wants to decrease the consumption.

This control strategy is a one-step-ahead control strategy since it involves a decision at time k based on the future behaviour of the market for time $k+1$. This strategy

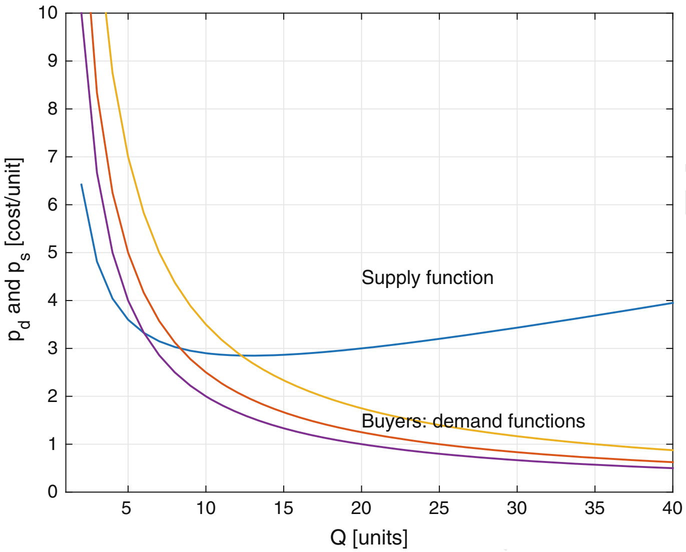

Fig. 57.6 Demand functions and supply function used to illustrate the behaviour of the market where the supplier has a rate production constraint. The supplier defines the price and buyers adjust the amount Q_i according to their demand functions and the constraints imposed by the supplier

allows a consumer to request energy, enables the competition based on the demand profile of each consumer, and addresses the rate producer constraint.

57.5.1 Example—Illustration of the Proposed Strategy

To illustrate the results with this control approach, the following scenario is considered. Three buyers are present in the market; their demand functions have $\beta_1 = 25$, $\beta_2 = 35$, and $\beta_3 = 20$. The supplier function and the demand functions are represented in Fig. 57.6.

The decision prices of a consumers $p_{di}[k]$, demand quantities $Q_i[k]$, the $Q_s[k]$,[1] and the price $p_s[k]$ of the supplier are shown in Fig. 57.7. At the beginning of the computer simulation, the initial values assigned to $Q_i[k]$ are $Q_1[1] = 2$, $Q_2[1] = 3$, and $Q_3[1] = 1$. The increments $\Delta_1 = \Delta_2 = 0.5$. Buyer 1 and buyer 2 start from the beginning to increase Q_i according to their demand functions. Buyer 3 maintains a constant value $Q_3[1] = 1$ until time $k = 50$; after this moment, buyer 3 starts to adjust $Q_3[k]$. At time $k > 100$, buyer 1 decides to leave the market and starts decreasing $Q_1[k]$ according to the constraints imposed by the supplier.

[1] In this example, the aggregate quantity $Q[k]$ is represented as $Q_s[k]$.

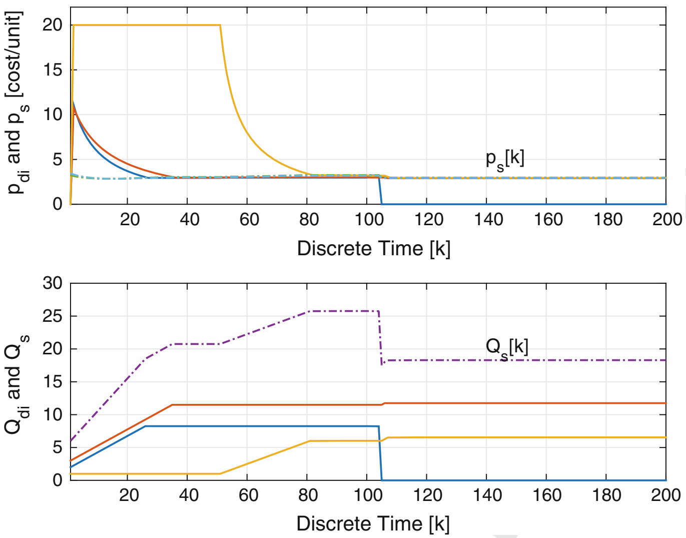

Fig. 57.7 Time evolution of the market with three buyers and one supplier. The price $p_s[k]$ is defined by the supplier based on the aggregate quantity $\sum_{i=1}^{n} Q_i[k]$. The aggregate quantity depends on the number of buyers present in the market

The speed of convergence to equilibrium depends on the value of Δ_1 and Δ_2, but because they represent a working constraint imposed by the supplier it is not possible to speed up the convergence process.

57.6 Strategy to Coordinate the Demand

The price per unit depends on the aggregated demand and on the supply curve of the producer. As illustrated in the previous section, consumer behaviours cause demand variations over time that affect the price/unit and the total cost to pay. From the point of view of the consumers, it is important to find a strategy that minimizes their costs, but subjected to the constraint that a given amount of energy is needed for their needs. For this purpose, the following cost function is considered:

$$J_1(N) = \sum_{k=1}^{n} Q[k]p[k] \,, \qquad (57.30)$$

subject to the constraint $\sum_{k=1}^{n} Q[k] = Q_n$, where Q_n represents the total consumption for the time period length n (n can represent a time interval of 24h), $Q[i]$ and $p[i]$ are the consumption, and the price/unit at the discrete time k.

In order to perform the minimization with the constraint, the Lagrange multiplier technique is applied. For that sake, define the Lagrangian function

$$J_2(N) = \sum_{k=1}^{n} Q[k]p[k] + \lambda(\sum_{k=1}^{n} Q[k] - Q_n)\ , \tag{57.31}$$

where λ is the Lagrange multiplier. Using the equation of the total cost in (57.31) yields

$$J_2(N) = \sum_{k=1}^{n}[c_o + c_v(1 + \alpha Q[k])Q[k] + gQ[k]] + \lambda(\sum_{k=1}^{n} Q[k] - Q_n)\ . \tag{57.32}$$

Computing the partial derivatives in order to $Q[k]$ and λ, and equating to zero yields

$$\frac{\partial J_2(n)}{\partial Q[k]} = c_v + g + 2\alpha Q[k] + \lambda = 0 \quad for\ \ k = 1...n\ , \tag{57.33}$$

$$\frac{\partial J_2(n)}{\partial \lambda} = \sum_{k=1}^{n} Q[k] - Q_n = 0\ . \tag{57.34}$$

By algebraic manipulations, the Lagrange multiplier is given by

$$\lambda = -\frac{n(c_v + g) + 2c_v\alpha Q_n}{Q_n}\ , \tag{57.35}$$

and

$$Q^\star[k] = \frac{Q_n}{n} \quad for\ \ k = 1...n\ . \tag{57.36}$$

This last equation defines the best strategy for the consumers to keep the total cost at the minimum value. The aggregate demand must be uniform over the time interval. This result implies cooperation among the consumers, meaning that they must coordinate the scheduling of their consumptions, and they must use devices to store energy during the time periods when their real consumption is below $Q^\star$, and to use the stored energy when the real consumption is above $Q^\star$. Note however that, in this analysis, no costs were assigned to the operation of the energy storage devices. If the cost of operating the energy storage devices is too high, there is no economic incentive to use them to compensate for the energy cost changes.

57.7 Conclusion

In this paper, the dynamics of a supply–demand market is considered. The aim is to describe the time evolution of the market price towards an equilibrium, where the negotiation is executed automatically. As a starting point, a dynamic model for the negotiation process of a simplified market with a single supplier and single consumer is tackled. Stability conditions were found that render the negotiation process stable, in which case it converges to the intersection of the supply curve with the demand curve. Linear programming minimization is used to address the demand for energy if the supplier imposes a time price profile. The simplified market model was expanded to include several consumers that are simultaneously present in the market. Additionally, a constraint production rate is addressed which motivates a different control approach to handle the stability of the market. In this case, the speed of convergence depends on the constraint production rate from the supplier side. Computer simulation results illustrate the working principles of the proposed control algorithms.

References

1. Braun, B., Faulwasser, T., Grüne, L., et al.: Maximal islanding time for microgrids via distributed predictive control. In: 22nd International Symposium on Mathematical Theory of Networks and Systems, Minneapolis, MN, USA (2016)
2. Lopes, J.A.P., et al: Defining control stragegies for microgrids islanded operation. IEEE Trans. Power Syst. **21**(2), (2006)
3. Larsen, G.K.H., et al.: Distributed control of the power supply-demand balance. IEEE Trans Smart Grid **4**(2), 828-836 (2013)
4. Scholten, T.W., De Persis, C., Tesi, P.: Modeling and control of heat networks with storage: the single-producer multiple-consumer case. In: 2015 European Control Conference (ECC), Linz Austria (2015)

Author Queries

Chapter 57

Query Refs.	Details Required	Author's response
AQ1	Please approve the edits made to the chapter title.	
AQ2	Please approve the edits made to the sentence "To avoid this...".	
AQ3	Please check the sentence "In the case..." for clarity.	
AQ4	Please check the sentence "An information..." for clarity.	
AQ5	Please check the sentence "Computing the partial..." for clarity.	

MARKED PROOF

Please correct and return this set

Please use the proof correction marks shown below for all alterations and corrections. If you wish to return your proof by fax you should ensure that all amendments are written clearly in dark ink and are made well within the page margins.

Instruction to printer	*Textual mark*	*Marginal mark*
Leave unchanged	• • • under matter to remain	Ⓘ
Insert in text the matter indicated in the margin	⋌	New matter followed by ⋌ or ⋌Ⓧ2
Delete	/ through single character, rule or underline or ⊢⊣ through all characters to be deleted	∂/ or ∂/Ⓧ2
Substitute character or substitute part of one or more word(s)	/ through letter or ⊢⊣ through characters	new character / or new characters /
Change to italics	— under matter to be changed	⌐
Change to capitals	≡ under matter to be changed	≡
Change to small capitals	= under matter to be changed	=
Change to bold type	~ under matter to be changed	~
Change to bold italic	≂ under matter to be changed	≂
Change to lower case	Encircle matter to be changed	≢
Change italic to upright type	(As above)	⊹
Change bold to non-bold type	(As above)	⊹
Insert 'superior' character	/ through character or ⋌ where required	Y or ⋌ under character e.g. Y or ⋌
Insert 'inferior' character	(As above)	⋌ over character e.g. ⋌
Insert full stop	(As above)	⊙
Insert comma	(As above)	,
Insert single quotation marks	(As above)	Y or ⋌ and/or Y or ⋌
Insert double quotation marks	(As above)	Y or ⋌ and/or Y or ⋌
Insert hyphen	(As above)	⊢⊣
Start new paragraph	⌐	⌐
No new paragraph	⌒	⌒
Transpose	⊔⊓	⊔⊓
Close up	linking ⁀ characters	⁀
Insert or substitute space between characters or words	/ through character or ⋌ where required	Y
Reduce space between characters or words	| between characters or words affected	⊤