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Preface

This book addresses the design of a model predictive control algorithm for performing spacecraft rendezvous manoeuvres. Although hundreds of rendezvous missions have been successfully carried out, the development of new guidance and control algorithms remains an active area of research, motivated by the demand for improved efficiency, safety, and autonomy of these manoeuvres, which this book attempts to consolidate. The book is accessible to those new to the topics covered, regarding both orbital rendezvous and model predictive control but also presents compelling subjects for researchers and professionals in the aerospace industry, including some contributions to this area of research. In addition, the book is a useful complement to courses on model-based predictive control.

The present work was initially developed and adapted from the first author’s (A. Botelho) MSc dissertation, performed under the supervision of the remaining authors. The authoring team counts with the experience of a full professor (J. M. Lemos) at the University of Lisbon with expert knowledge in optimal control, as well as two professionals (B. Parreira and P.N. Rosa) of the aerospace company Elecnor Deimos, a European Space Agency partner and a strong player in the European aerospace industry. The thesis was motivated as a feasibility study on the application of the aforementioned techniques to the ESA PROBA-3 mission Rendezvous Experiment (RVX), led by Deimos Engenharia.

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Lisbon, Portugal
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Chapter 1
Introduction

Orbital rendezvous is a procedure in which two separate spacecraft meet at the same orbit, as illustrated in Fig. 1.1, thereby approximately matching their orbital velocity and position [1]. Such manoeuvres allowed for the feasibility of the Apollo moon landing missions, with the rendezvous of the Lunar Excursion Module with the Command Module in lunar orbit, and for the construction and resupply of modular space stations, such as Mir and the International Space Station. Other applications include, for example, the exploration of smaller celestial bodies, such as asteroids, comets and small moons, the in-orbit servicing of satellites, for instance, the multiple repair missions to the Hubble Space Telescope, or the active removal of space debris. Often, the rendezvous process is followed by a docking or berthing procedure, that results in the physical connection of the two spacecraft.

The first attempt at rendezvous was performed in the Gemini 4 manned mission in 1965, which was unsuccessful due to the method of approach being simply ‘point-and-shoot’, resulting in a further separation of the spacecraft. This revealed the challenge in performing space rendezvous, and proved that the relative orbital dynamics involving the two spacecraft must be taken into consideration. Since then, rendezvous missions have been performed successfully hundreds of times, both by manned and unmanned spacecraft, and using various different guidance and control methods. In this context, this book addresses the use of Model Predictive Control (MPC) [2] for performing rendezvous manoeuvres, which is a widely successful optimal control strategy that naturally considers the system dynamics and can handle various operational constraints. The use of MPC for this purpose can grant more autonomy to the spacecraft and improve the optimality of the approach trajectories, when compared to the traditional techniques.

The literature for MPC applied to rendezvous is now quite considerable, and this topic remains an active area of research. Despite these facts, MPC has been tested in real spaceflight only once, to the best of the authors’ knowledge, by the
PRISMA mission [3]. Although this test was made in a formation flying context, not rendezvous, the underlying principles are identical. The main difficulty with the use of MPC for a real rendezvous mission is that it requires a considerable online computational effort, that can prove to be a challenge given the typically limited computing power available on board. Furthermore, there is not yet a standard strategy for granting robustness in the presence of disturbances possibly interfering with a rendezvous mission that is both feasible to implement in real time and maintains good operational performance, and thus more research into this topic is required.

1.1 Problem Formulation

A spacecraft rendezvous mission generally adheres to the following sequence of events: launch, phasing, far-range rendezvous, close-range rendezvous and mating [1]. The launch phase ends with orbital insertion, nearly coplanar with the target orbit and typically at a lower altitude and behind the target, and is completely out of the scope of this work. Phasing consists of small corrections to the orbit parameters, and of passive waiting that takes advantage of the different orbital periods, in order to allow the launched spacecraft to catch up the target. This phase can last a few days and does not require great precision, with correction manoeuvres being performed in open loop without the need for the use of MPC. The rendezvous process starts then with the far-range phase when relative navigation is possible which typically within a range of a few tens of kilometres with respect to the target. This phase ends and close-range rendezvous begins when the relative distance requires safety-critical manoeuvres, typically at a few kilometres. Thus, these two phases are the ones that can benefit from the use of MPC to perform the approach manoeuvres, with the latter phase being the focus of this book. The mating phase (docking or berthing) typically
has a very different set of requirements and constraints and thus is also out of scope for this book.

An on-board automatic control system for a spacecraft contains three sub-systems tasked with the execution of thrust manoeuvres: guidance, navigation and control (GNC) [1]. The guidance system generates the reference trajectory and spacecraft attitude; navigation provides state measurements and estimates; control commands the force and torque necessary to drive the spacecraft to the desired state. MPC can simultaneously handle both guidance and control functions and navigation is not considered in this book. Furthermore, because the translational and attitude control are typically decoupled in the far and close-range rendezvous phases [1], only translational control is addressed in this book. Finally, it is remarked that, while all real rendezvous and formation flying missions have been performed in circular or near-circular target orbits, there is the motive to consider elliptical target orbits, which imply an increased difficulty due to the dynamics becoming time-varying and, therefore, more complex. For example, for the future European Space Agency PROBA-3 mission, the spacecraft will be placed in a highly elliptical orbit in order to validate formation flying and rendezvous technology.

### 1.2 State of the Art

This book covers several different research areas, and thus we will address the essentials of the state of the art for these separately. MPC was first introduced in the 1960s and is now a very mature framework, with an extensive theoretical basis [2] and a vast history of successful applications, mostly in the process industry [4]. It remains an active area of research, with recent work being dedicated to the application of MPC to specific problems, including the rendezvous scenario [5]. Research has also been devoted to improving the real-time feasibility of MPC, with the design of new optimization algorithms that exploit the MPC problem structure, for example, [6–8] and with the further development of the popular Explicit MPC framework [9, 10]. New sub-fields of MPC have also emerged in recent years, such as Distributed MPC [11, 12], Hybrid MPC [13], Adaptive MPC [14], Stochastic MPC [15] or Neural Network MPC [16], among several others.

Concerning relative orbital mechanics, although the nonlinear dynamics can be easily derived from Newton’s laws [17], these differential equations do not have a closed-form solution, which limits their usefulness to an on-board and real-time environment. Thus, research into this topic is still active today, dedicated to determining approximated dynamics with a closed-form solution, necessary for any rendezvous strategy. A set of linearized equations for the relative motion represented in Cartesian coordinates in a local non-inertial frame of reference and for a circular orbit was first derived by Hill in 1878 [18]. They were first applied and solved in the context of orbital rendezvous in 1959, most famously by Clohessy and Wiltshire [19], although this solution is only accurate for near-circular orbits. The equations were extended to elliptic orbits by De Vries in 1963 [20], and simplified via a change of the indepen-
dent variable to the true anomaly. They were then solved and applied to spacecraft rendezvous in elliptic orbits by Tschauner and Hempel in 1965 [21], after whom the simplified equations became known. In 1998, Carter presented a simpler solution to the Tschauner–Hempel equations in the form of a state transition matrix [22], which is valid for any orbit eccentricity. Later, in 2002, Yamanaka and Ankersen introduced a computationally simpler state transition matrix [23], although this description is only valid for circular or elliptical orbits. Ankersen later complemented this solution by including a forced regime with a constant force discretization [24], which can be used for spacecraft control.

More recent work focuses on formulating linearized models that include different perturbations. The Hill–Clohessy–Wiltshire equations were extended to include the \( J_2 \) perturbation in [25], and atmospheric drag in [26], although these remain valid only for near-circular target orbits. A dynamic model which includes \( J_2 \) and is valid for elliptical target orbits was later presented in [27], and another which also includes atmospheric drag was presented in [28].

While all the previously mentioned dynamic models are based on Cartesian coordinates, another approach commonly used for spacecraft rendezvous and formation control is based on the Keplerian orbital elements. Lagrange’s Planetary Equations and the Gauss Variational Equations [29] model the effect of, respectively, conservative and arbitrary local-frame perturbations, e.g. gravitational perturbations or a controlled thrust, on the orbital elements of a satellite. From these, linearized models of the relative dynamics of two satellites can be formulated in terms of Relative Orbital Elements (ROEs), rather than the relative position and velocity typically utilized. The ROEs may be arbitrarily defined from the target and chaser absolute orbital elements, where the most straightforward formulation is a simple difference of the elements of the two satellites [30–33]. The resulting linear equations present some advantages with respect to Cartesian-based models, such as remaining accurate for larger relative distances since they are linearized around the target orbital elements rather than the target position, readily include the target eccentricity and more easily allow the inclusion of higher order potential models and other perturbations. More recent dynamic models based on different ROE formulations have been presented by D’Amico et al. [34–36], which more easily allow for a stable and passively safe trajectory design, and include the \( J_2 \) perturbation and atmospheric drag [37].

The Yamanaka–Ankersen state transition matrix is considered to be the state-of-the-art solution for use in the design of rendezvous missions in elliptic orbits, due to its maturity, inclusion of the target eccentricity, relative simplicity when compared to other models and its representation in Cartesian coordinates, which are more intuitive than other state representations. Nevertheless, research to determine the different and improved linearized relative dynamic models continues. References [38, 39] present exhaustive surveys on the models currently available in the literature, and perform accuracy comparisons between the models, as well as runtime comparisons in the case of the latter.

Current state-of-the-art methods for rendezvous guidance and control rely on commanding the spacecraft to follow a sequence of waypoints, typically defined offline during mission analysis, using simple manoeuvres with analytical solutions, e.g.
two boosts or straight-line approaches, possibly with some limited mid-correction manoeuvring [1, 40, 41]. Such methods have a low computational burden compatible with the real-time environment they are required in, are robust and have great maturity and history of successful applications, at the sacrifice of optimality and autonomy, however.

The application of optimal control theory to rendezvous problems was pioneered in the 1950s by Lawden [42], in what culminated in the *primer vector* theory. Although technically the underlying theory is the calculus of variations and not optimal control, Lawden formulated first-order conditions for optimal spacecraft trajectories. Since then, several books on spacecraft optimal control have been published, consolidating the field, e.g. [43–45].

Despite the current relative *status quo*, research into the rendezvous guidance and control problem using different methods is vast, namely using optimal control methods. However, these works are based on indirect numerical methods (various forms of Pontryagin’s principle), which present several practical limitations [46]. Research on the application of direct methods, such as MPC or pseudospectral methods [47], has only begun in more recent decades, driven by the increasing available computational capability, and motivated by an increasing necessity for optimality and autonomy.

One of the first applications of MPC to the rendezvous problem was by Richards and How [48], where the basic formulations employed by most of the literature that followed were presented, namely Fixed-Horizon (FH) MPC and Variable-Horizon (VH) MPC, although the optimization formulations had been introduced earlier [49] but not in a rendezvous MPC context. These formulations explicitly minimize the fuel for the rendezvous manoeuvres and, in the case of the former, express the optimization problem as a linear program, that can allow for a feasible online computation time. Thus, current research is mostly dedicated to extending these formulations, for example, to grant robustness, in the presence of the many disturbances and perturbations interfering in a rendezvous mission, while ensuring convergence, constraint satisfaction and performance. A more in-depth state-of-the-art review for this topic is available along with Chap. 4 of this book, with special attention to robustness techniques.

### 1.3 Contributions

The book starts by featuring a basic introduction to general MPC theory, with simple toy problems and experiments that demonstrate the capabilities of this control method, and that can serve as a practical tutorial for those uninitiated on this topic. It also contains an introduction to relative orbital mechanics, with a derivation of the approximated dynamic model, and featuring several simulations with the explanations required to fully understand the rendezvous dynamics and manoeuvres.

Our contributions to MPC applied to rendezvous include considering rendezvous manoeuvres in highly elliptical orbits, which is uncommon in the literature, especially with an eccentricity as high as that considered here. With this condition in mind, a
new approach for sampling the dynamics for the prediction horizon is proposed here, based on constant eccentric anomaly sampling intervals, which deals better with the fact that the dynamics are highly time-varying for such a highly elliptical orbit. We compare the Finite-Horizon and VH MPC formulations, regarding performance and computational complexity, and compare them with the traditional two-impulse transfer approach used in traditional rendezvous guidance algorithms.

A new method for formulating obstacle avoidance constraints and passive safety constraints is also presented, that relies on iterative linear optimization and which allows for the feasible inclusion of this constraint in a real-time application while maintaining optimality. Finally, the book contributes to new robustness techniques, with the use of a terminal quadratic controller for a more accurate and robust final braking manoeuvre, and the dynamic relaxation of the terminal constraint, in order to keep the control sparse and avoid the overcorrection of disturbances and waste of fuel. The latter techniques are demonstrated in an industrial high-fidelity simulation environment, considering the conditions of the ESA PROBA-3 rendezvous experiment (RVX).

1.4 Book Outline

In Chap. 2, the book covers general MPC theory, with a focus on MPC for linear system models, given the context of this book. The basic general formulation is discussed, and some specific techniques are also presented, such as reference tracking and the use of different cost functions. This chapter also features several simulations that show the effect of different cost functions and the choice of controller parameters, with some consideration also given to the computational performance.

The relative orbital dynamics between two satellites, that is crucial to understanding the design of a rendezvous mission, is presented in Chap. 3. The book introduces and derives linearized models of the relative dynamics, that will then be used for MPC in the next chapter. Several simulations of the relative motion between two satellites are also presented, both in the circular and elliptic orbit cases and the non-intuitive free-drift motions and thrust manoeuvres are explained.

Finally, in Chap. 4 the MPC framework is applied to the rendezvous problem. The book starts by considering the most naive approach and develops stepwise toward the ideal formulation. It then considers the presence of disturbances and provides a short literature review on robust techniques in MPC for rendezvous.

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34. S. D’Amico, Relative orbital elements as integration constants of Hill’s equations. DLR, TN, 05-08 (2005)
Chapter 2
Model Predictive Control

Model Predictive Control (MPC) is a control design method based on iterative online optimization [1]. The strategy is to obtain a control decision by solving an optimization problem which factors in future states of the system in a finite horizon, predicted using a (generally discrete) system model. Figure 2.1 illustrates this approach.

At each time step, the problem is solved with the most recent state measurement or estimate as to the initial condition for the prediction, and a control strategy for future steps within the prediction horizon is obtained. The first control value in the obtained sequence is applied to the plant, and the problem is solved again in the next time step, with an updated state and with the prediction horizon shifted forward. For this reason, this method is also known as Moving/Receding-Horizon Control.

Since MPC is formulated as an optimization problem, it allows for the inclusion of control and state constraints. The possibility to explicitly include constraints is a powerful tool and one of the major advantages of MPC in respect to other control methods since it allows to limit the control action and to model complex state restrictions, such as safety constraints. Furthermore, MPC naturally considers the system dynamics and can handle multivariate systems. It may also feature the use of nonlinear system models, that can generate better state predictions.

By definition, the MPC strategy requires that an optimization problem is solved online, at each time step. The computation time of the MPC problem depends on many factors, such as the order of the system model, linearity, the complexity of the control and state constraints and the length of the prediction horizon. The optimal control action must be computed and applied to the plant before the next sample, and thus the problem is required to be solved faster than the system sampling time, which makes its implementation infeasible in fast systems. This computational requirement is the major limitation for MPC, although modern technology and methods allow for MPC to be implemented in increasingly more complex systems, such as those in the aerospace industry.

Other major issues which are still the subject of current research are stability [2] and robustness [1]. Many stability proofs rely on imposing a terminal constraint and