Interpolated Linguistic Terms♦

João Paulo Carvalho
INESC-ID
IST - Instituto Superior Técnico
R. Alves Redol, 9, 1000-029 Lisboa, PORTUGAL
joao.carvalho@inesc-id.pt

José A. B. Tomé
INESC-ID
IST - Instituto Superior Técnico
R. Alves Redol, 9, 1000-029 Lisboa, PORTUGAL
jose.tome@inesc-id.pt

***Abstract* – Interpolated Linguistic Terms (ILT) are a valid alternative representation for the fuzzy sets obtained in the inference of a fuzzy rule base, and allow the propagation of the uncertainty defined in the linguistic term set of the consequent variable. ILT can be also be used as new linguistic terms of that consequent.**

***Keywords* –** Interpolated Linguistic Terms, Rule based fuzzy systems, Linguistic Term Set.

I. INTRODUCTION

The definition of the term set of a fuzzy variable (the set of its linguistic terms), is one of the most important issue in rule based fuzzy systems and qualitative reasoning [1]. Therefore, one would expect that the relation between the linguistic terms and the fuzzy sets that result from the inference of a fuzzy rule base should assume a similar importance (i.e., the resulting sets should be easily interpreted when compared to the linguistic terms of the variable). However, fuzzy rule based inference always results in a fuzzy set with a support set (and consequently its uncertainty /imprecision) given by the union of the support set of the linguistic terms involved in the consequent of the active rules. There is not a direct relation between the uncertainty of the result and the degree of activation of each linguistic term: If a single set is involved, then the resulting uncertainty is the uncertainty of that set; but if more than one set is involved, then there is not a smooth transition between the uncertainty of the involved sets, since even the smallest activation of another consequent will increase the uncertainty to the value represented by the union of both set's support. Fig. 1 shows a pictorial representation example of this issue: If rule base inference results in the activation of a single consequent, for example, Large (i.e., $\mu(L)>0$ and $\mu(VL)=0$), then the resulting uncertainty is given by the support set of Large, supp(L). But if both Large and Very_Large are activated (i.e., $\mu(L)>0$ and $\mu(VL)>0$), then the resulting uncertainty will always be given by $\text{supp}(L) \cup \text{supp}(VL)$.

This intrinsic characteristic of fuzzy rule based inference not only hinders the interpretation of the resulting fuzzy sets, but also hinders their use:

- as inputs to a new inference;
- to represent uncertainty propagation;
- if the size of the resulting support set is somehow important;
- as new linguistic terms of the consequent.

The first 3 restrictions became rather important issues when Rule Based Fuzzy Cognitive Maps (RB-FCM) [2][3][4][5][6][9] were developed, since:

1) RB-FCM are rule based systems with feedback where inference results (as well as uncertainty) need to be propagated;
2) The modelling of causal inference (while avoiding combinatorial rule explosion) depends on a method that needs to maintain a relation between support size and rule activation.

These restrictions led to the development of Interpolated Linguistic Terms (ILT). ILT are groundbreaking in a sense that they tamper the usual fuzzy rule based inference method. However, they are very simple and basic mechanisms, and can be used as <u>alternative representations for the fuzzy sets obtained by fuzzy rule base inference</u>. ILT can also be seen as a 2D interpolation of the linguistic terms involved in the consequents of that inference.

This paper introduces and provides a formal definition for ILT (including the most recent developments), and suggests future ILT application in areas that could profit from their properties.

II. INTERPOLATED LINGUISTIC TERMS (ILT)

Before presenting a formal definition of an Interpolated Linguistic Term, we need to introduce some terminology used in this paper (see Fig. 2), and a set of restrictions that are necessary to implement ILTs.

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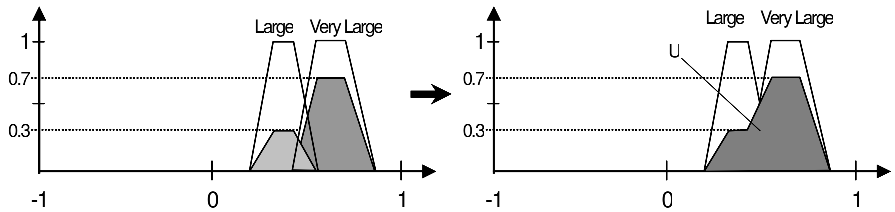
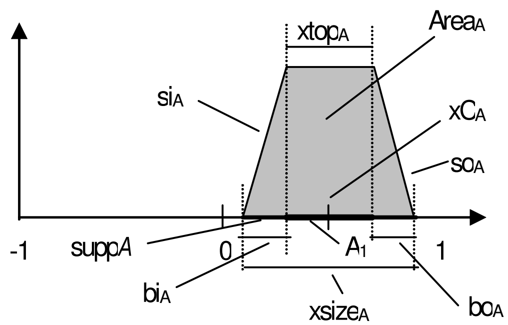


Fig. 1 – If both consequent terms are activated, then the support set of U is given by the union of the support sets of Large and Very Large



A – Linguistic term
xC – xcoordinate of centroid
supp – support set
A_1 – α-cut for α=1
xsize – size of supp
xtop – size of A_1
Area
bi – inner base
bo – outer base
si – inner slope
so – outer slope

Fig. 2 - Morphology of a Linguistic Term

In order to be able to obtain an ILT on a rule based fuzzy system, one must enforce the following restrictions in the system:

i. The membership degree of all linguistic terms must be complementary, i.e., its sum must be 1 in every point of the variable Universe of Discourse (X):

$$\forall x \in \mathrm{X}, \forall (A_0, A_1, \ldots, A_n) \in \mathrm{F}(\mathrm{X}), \sum_{i=0}^{n} \mu_{A_i}(x) = 1$$

ii. All linguistic terms must have the same basic shape (trapezoidal, triangular, S, etc.), and their membership functions must cross with their neighbours when μ=0.5.

iii. The inference method must preserve both the shape and the centroid's x-coordinate of the consequent linguistic term; the Max-Dot method is an example of an adequate method.

iv. The fuzzy sets that result from the inference of the rule base must be summed. As a result one obtains a single fuzzy set, which we will call U.

These restrictions guarantee [6] that if there is a single antecedent, then a maximum of 2 rules is involved in each inference of U, and that the Area of U is univocally related with the area of the consequent terms involved in the inference of U. Therefore, if "larger" sets are used to represent terms that have a "larger" semantic meaning, then the Area of U will have a proper semantic meaning. For example, if the inference active terms are *High* and *Very_High*, and if $\mathrm{Area}_{High} < \mathrm{Area}_{Very\_High}$, then $\mathrm{Area}_{High} < \mathrm{Area}_U < \mathrm{Area}_{Very\_High}$. These restrictions are necessary for the implementation of ILT in Rule Based Fuzzy Cognitive Maps. For other applications, some might prove not necessary.

With these restrictions in mind, one can present the following ILT definition:

Definition: An ILT, Interpolated Linguistic Term, is a fuzzy set that is univocally related with the active consequents of a rule based fuzzy inference. Given U, obtained respecting restrictions i. to iv., we call ILT_U (the Interpolated Linguistic Term of *U*), to the fuzzy set that respects the following conditions:

v. ILT_U and the term set of the fuzzy variable where *U* is defined must have the same shape.

vi. The x-coordinate of the centroid of ILT_U and the x-coordinate of *U* must be the same:

$$\mathrm{xC}_{ILT_U} = \mathrm{xC}_U \Leftrightarrow \left(\frac{\int_x \mu_{ILT_U}(x) \cdot x \; dx}{\int_x \mu_{ILT_U}(x) \; dx} \right) = \left(\frac{\int_x \mu_U(x) \cdot x \; dx}{\int_x \mu_U(x) \; dx} \right)$$

vii. *U* and ILT_U must have the same Area:

$$\mathrm{Area}_{ILT_U} = \mathrm{Area}_U \Leftrightarrow \int_x \mu_{ILT_U}(x) \; dx = \int_x \mu_U(x) \; dx$$

viii. ILT_U is normal, i.e.:

$$\{\exists x \in X \mid \mu_{ILT_U}(x) = 1\} \Leftrightarrow ILT_{U1} \neq \varnothing \Leftrightarrow \mathrm{xtop}_{ILT_U} > 0,$$

where ILT_{U1} represents the α-cut of ILT_U for α =1 and xtop is the size of ILT_{U1}.

ix. If *A* and *B* are the terms involved in the inference of *U*, then the size of ILT_{U1}, xtop_{ILTU}, is obtained according to the following function of *A* and *B*'s xtop, and of *A*, B and *U*'s xC:

$$\mathrm{xtop}_{ILT_U} = \min\{\mathrm{xtop}_A, \mathrm{xtop}_B\} + \left| \frac{\mathrm{xC}_U - \mathrm{xC}_A}{\mathrm{xC}_B - \mathrm{xC}_A} \times (\mathrm{xtop}_A - \mathrm{xtop}_B) \right|$$

x. If *A* and *B* are the terms involved in the inference of *U*, then the size of the inner base of ILT_U, bi_{ILTU}, is obtained according to the following function of A and B's bi, and of A, B and U's xC:

$$\mathrm{bi}_{ILT_U} = \min\{\mathrm{bi}_A, \mathrm{bi}_B\} + \left|\frac{\mathrm{xC}_U - \mathrm{xC}_A}{\mathrm{xC}_B - \mathrm{xC}_A} \times (\mathrm{bi}_A - \mathrm{bi}_B)\right|$$

End of Definition

From $\mathrm{bi}_{ILT\mathrm{U}}$ one can derive the inner slope of ILT_U:

$$\mathrm{si}_{ILT_U} = \frac{1}{\min\{\mathrm{bi}_A, \mathrm{bi}_B\} + \left|\frac{\mathrm{xC}_U - \mathrm{xC}_A}{\mathrm{xC}_B - \mathrm{xC}_A} \times (\mathrm{bi}_A - \mathrm{bi}_B)\right|}$$

The above conditions are quite obvious and natural: $\mathrm{xtop}_{ILT\mathrm{U}}$ and $\mathrm{bi}_{ILT\mathrm{U}}$ assume values that range between the respective values of A and B, according to the activation degree of the rules involving A and B. If only one rule is active, for instance the one involving A, then U, ILT_{U} and A are the same set.

If all above conditions are respected, $\underline{ILT_U\text{ is unique}}$ [6]: since all equations have a single solution for a given U and basic shape, then xtop, bi, xC and Area are unique and there is only one possible set; therefore it is impossible to obtain more than one *ILT* from a set *U*.

As we have seen above, the support set of U is the result of the union of suppA and suppB, but the support set of ILT_U, and therefore its uncertainty, is univocally related with the uncertainty of A and B. Since in RB-FCM it is guaranteed that "larger" causal linguistic terms have "larger" uncertainty [3][4][5][6][9], this property allows the resulting set to be shifted in the UoD without loosing its identity, and allow the accumulation of the effect of several antecedent causes without exponentially increasing the size of the rule bases.

Fig. 3 shows some pictorial examples of ILT that result from the inference of two rules involving linguistic terms with different shapes. In the figure it is possible to observe that the resulting ILT shape, uncertainty and position in the UoD are univocally related to the active inference rules and consequent linguistic terms, and that it is very easy to infer its semantic meaning when comparing it to the linguistic terms of the consequent. One can also observe that any ILT can be defined as a new linguistic term of the consequent.

One can say that the set of all possible ILT on a fuzzy linguistic variable is the result of an operation that transforms a finite set of *n* linguistic terms into a continuous "linguistic terms function". The equivalent crisp 2D operation would be the linear interpolation of a discrete number of points, hence the chosen name of "Interpolated" Linguistic Terms.

ILT calculus extends the usual fuzzy ruled based inference operations and involves much more complex operations. However it's easy to find algorithms that are fast enough to use ILT in Rule Based Cognitive Maps with a few hundred concepts without any loss in usability [6]. Therefore, their complexity should not be an issue in other applications.

III. APLICATIONS

The goals of this paper are not only to introduce ILT and existing applications, but also to expand their scope, proposing new areas where their properties could be useful and trying to stimulate cooperation and captivate interest within researchers from those areas. In this section we present current ILT applications and suggest several areas where they could be applied.

A. Rule Based Fuzzy Cognitive Maps

ILT were developed with RB-FCM in mind, and are their fundamental mechanism. The use of ILT in RB-FCM:

- Allow the existence of concepts with a large number of causal antecedents (which translates to rule bases with a large number of antecedents) without the burden of the usual combinatorial rule explosion;
- Provide the flexibility of adding or removing antecedents without changing the whole rule base, therefore adding a needed flexibility unheard in rule based fuzzy systems.

Details about ILT use in RB-FCM (where they were formerly called COS – Causal Output Sets), can be found in [3][4][5][6].

B. Qualitative Reasoning and Uncertainty Representation and Propagation in Feedback Fuzzy Rule Based Systems

In feedback rule based fuzzy systems, one must defuzzify the variables that belong to a feedback cycle in every iteration. If one fails to do it, then the support set of the fuzzy value of those variables will inevitably spread until is defined in all the UoD [6] and becomes useless (one gets maximum uncertainty). This problem can easily be observed in a system composed by a single fuzzy variable with its output connected to its input. Fig. 4 shows how uncertainty spreads in such system. In such a system (and in most systems where feedback occurs), this fact limits uncertainty propagation, since one must always end using crisp variables as an input in following iterations to avoid the problem. ILT are being developed to minimize (or even eliminate) this problem. Detailed information regarding this topic was published in [9].

C. Chained Rule Based Fuzzy Systems

What was said in the previous section regarding deffuzification in every iteration to avoid the problem of uncontrolled uncertainty spread, can also apply to Rule Based FS without feedback where we have a sequential chain of fuzzy variables. In Fig. 5 we present a simple fuzzy system consisting in 3 fuzzy variables A, B and C, and 2 fuzzy rule bases. In this system, if we do not defuzzify each fuzzy variable one risks ending up without any useful information in system output. Notice that C uncertainty spreads over the entire UoD. If the system had more variables then one would also start loosing information regarding the centroid of the fuzzy variables, because the fuzzy value off each following variable tend to a rectangular shape. In the end the system would be useless. Notice that this is not a particular case. It can happen in any fuzzy rule based system. Therefore we must give up a lot of fuzziness to obtain reliable results in chained systems. The use of ILT with slight modifications [9] minimizes this problem (Fig. 6).

D. Fuzzy Numbers and Fuzzy Arithmetic

A fuzzy number is, by definition, a convex and normal fuzzy set. An ILT can always be, by definition also, a fuzzy number. Therefore one can <u>directly</u> and automatically obtain a fuzzy number from a fuzzy set *U* that results from fuzzy rule based inference when using ILT. This simple fact creates a bridge between rule based fuzzy inference and fuzzy arithmetic. In fact, albeit being two topics under the same fuzzy "umbrella", these are rather different and incompatible fields (rule based fuzzy inference is not used in fuzzy arithmetic) that could profit from each other strengths.

E. Granularity and Complexity Reduction in Rule Based Fuzzy Systems

As we have seen, an ILT maintains all the fundamental characteristics of a fuzzy linguistic term. Therefore, an ILT can always be used as a new linguistic term of the fuzzy variable where it was defined. One can imagine using ILT to reduce and optimize the term set, defining and using new linguistic terms when necessary (one could even speak of "dynamic term sets"). This optimization could help to minimize the problem of combinatorial rule explosion and allow a practical implementation of more complex rule based fuzzy systems. This is obviously just a far-fetched idea without any serious research involved yet, and is presented here as such.

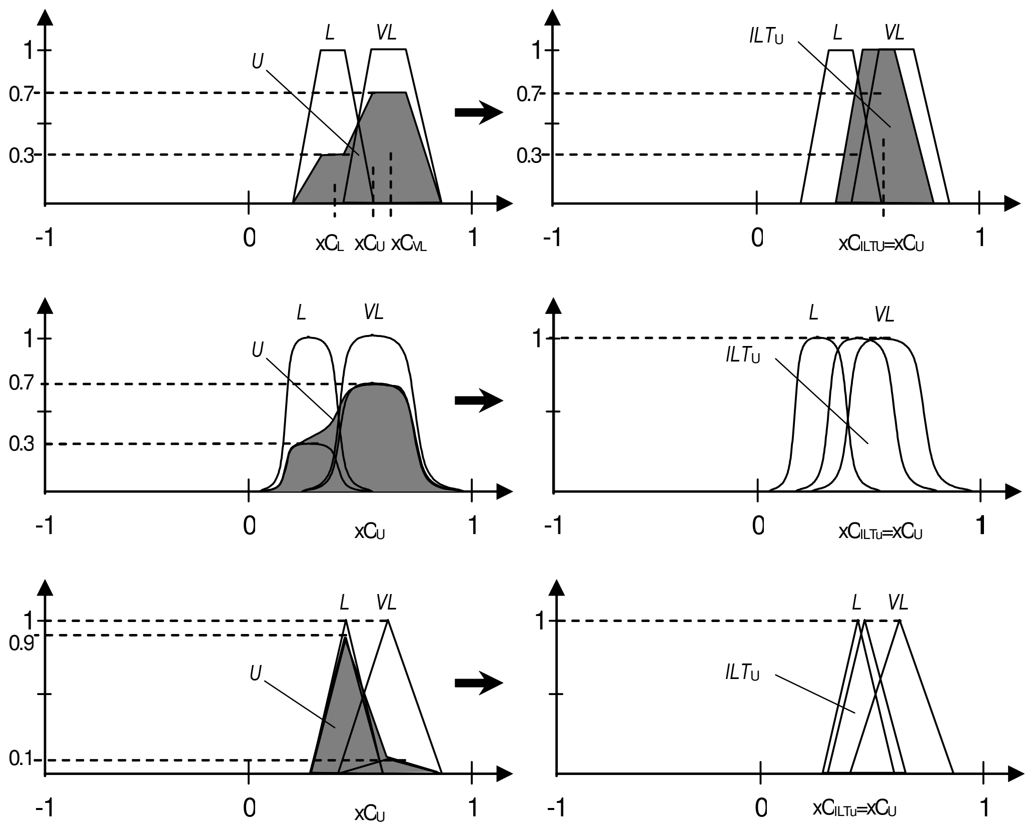


Fig. 3 – Examples of ILT for different term shapes and rule activations

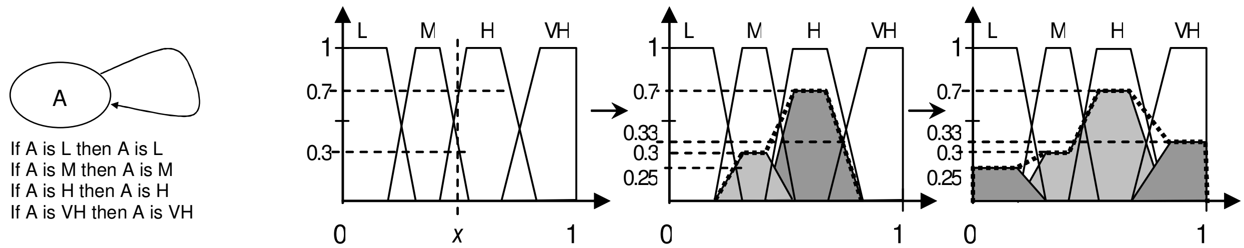


Fig. 4 - Uncertainty Spread in a simple Feedback Rule Based Fuzzy System

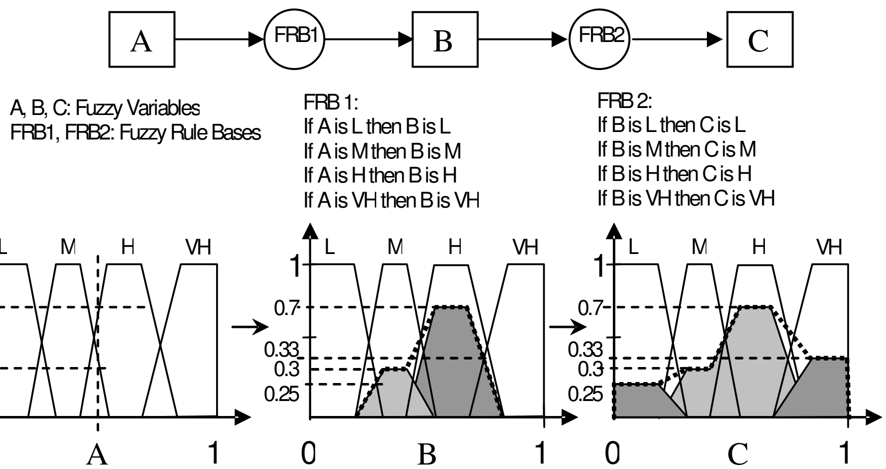


Fig. 5 – Example of uncertainty spread in a simple chained Rule Based Fuzzy System

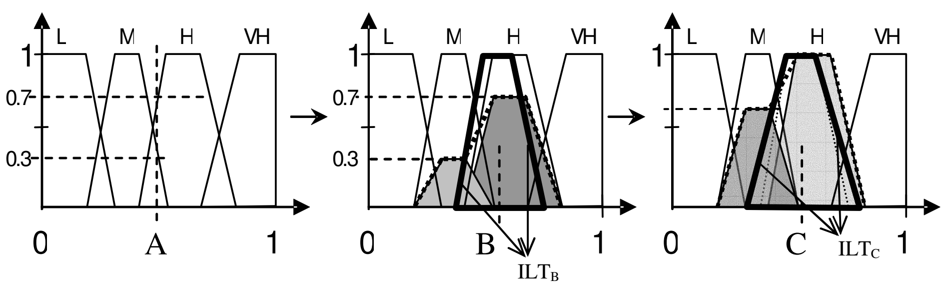


Fig. 6 – Minimizing the uncertainty spread using ILT

IV. CONCLUSIONS AND FUTURE DEVELOPMENTS

Interpolated Linguistic Terms are essential in RB-FCM. Without the use of ILT it would not be possible to model, implement and simulate the complex dynamic qualitative feedback systems associated with true real world cognitive maps. They also provide an invaluable tool to represent and propagate uncertainty in those systems. However, we feel that ILT potential spreads over this niche, and we suggest other areas and applications where they could prove useful.

Still, one must not forget that the fuzzy systems where ILT are currently applicable must abide to the restrictions presented in II. – even if some of these restrictions are due to RB-FCM properties and can be eliminated without any loss when applied to other systems. One must also not forget that some suggested applications aren't but suggestions; therefore, a lot of future work should be done to prove ILT applicability in those areas.

On a more formal level, future ILT developments involve restriction elimination in order to spread their application domain, and also the development of non-linear interpolation methods to obtain the ILT.

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