Using Rule-Based Fuzzy Cognitive Maps to Model Dynamic Cell Behaviour in Voronoi Based Cellular Automata

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Abstract— **This paper focus on the use of Rule Based Fuzzy Cognitive Maps to represent cell behaviour in Voronoi Based Cellular Automata in order to model the dynamics of temporal and spatial propagation processes. As an application example, the proposed approach is applied to modelling and simulation of forest fire propagation.**

Keywords: Rule Based Fuzzy Cognitive Maps, Cellular Automata, Voronoi Regions, Forest Fire, Modelling.

I. Introduction

MODDELING and simulation of the dynamics of temporal and spatial propagation of human, economic or ecologic processes (like urban development, epidemics or forest fires) is a widely studied problem that can profit from a good integration with Geographic Information Systems (GIS). However, most dynamic models do not integrate well with GIS due to fundamental incompatibilities in conceptual representation of space and time [1]. Due to its spatial based structure, Cellular Automata (CA) provided a potential solution and are probably the most popular technique to model the dynamics of these processes, since they can predict complex global space pattern dynamic evolution using a set of simple local rules.

However, CA are usually associated to bi-dimensional matrixes of rectangular identical cells that are not the most adequate to model and tessellate a real world geographic area. CA extensions using Voronoi spatial models have been previously proposed to overcome this problem [2][3]. In these approaches one uses convex cells with different sizes and shapes that can provide a much more adequate terrain partition.

A different problem lies in the fact that, on regular CA, each cell has a finite set of possible states, and transition between states is a crisp function of present cell state and neighbour cells state. Crisp data modelling and crisp transition mechanisms have known limitations when one trying to model and simulate real-world processes where uncertainty and imprecision is present and cannot simply be ignored. Fuzzy set theory is an obvious known solution to this problem, and one that has been applied in some urban development models [4][5]. In these approaches, fuzzy variables and linguistic terms are used to represent uncertain and imprecise cell information, and the crisp transition rules are replaced with fuzzy "If-Then" rule-bases.

However, this simple approach might not be adequate to model imprecise and/or uncertain cell behaviour due to the fact that, in real world systems, each cell can be composed by several dynamic entities interrelated in complex ways, usually including feedback links that propagate influences in complicated chains and hinder fuzzy rule based capabilities to model uncertainty [6]. Therefore, in this paper we propose the use of dynamic Cognitive Maps to model cell behaviour in Voronoi-Region Based Cellular Automata. Fuzzy Cognitive Maps (FCM) [7] are the most popular Dynamic Cognitive Mapping technique, but in the proposed approach, one opted to model cell behaviour using Rule Based Fuzzy Cognitive Maps (RB-FCM) [8][9][10][11] due to their greater versatility.

To exemplify the proposed approach, one presents a model for a Forest Fire propagation simulator.

II. Cellular Automata

A. Regular Cellular Automata

One can describe automata as discrete entities with several inputs and internal states. State changes through time according to a set of rules that consider present automata state and external inputs. Formally, automata A can be represented by a set of states S, and a set of transition rules *T*:

$$A \sim (S, T) \tag{1}$$

Transition rules define next automata state, S_{t+1}, based on current state S_t, and current input I_t:

$$T : (S_t, I_t) \rightarrow S_{t+1},\ (S_t, S_{t+1} \in S) \tag{2}$$

A Cellular Automata is a system composed by several identical automata, physically organized as a 2 dimensional

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array of rectangular cells, where each cell is considered an automata, A, with a set of rules, T, which gets its inputs from its own state and from neighbouring cells states V:

$$A \sim (S,T,V) \tag{3}$$

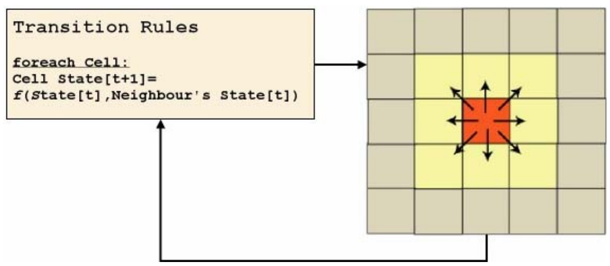

Fig. 1- Regular Cellular Automata

B. Irregular Cellular Automata: The Voronoi Spatial Model and Voronoi Based Cellular Automata

Regular CA have cells with identical shape and size. Since geographic features in nature are usually not distributed uniformly, regular spatial tessellation obviously limits modelling and simulation potential of regular CA. In order to overcome this limitation, several authors have extended the CA model to irregular cells. The most successful approaches use the Voronoi spatial model [2][3].

The Voronoi spatial model is a tessellation of space that is constructed by decomposing the entire space into a set of Voronoi regions around each spatial object. By definition, points in the Voronoi region of a spatial object are closest to the spatial object than to any other spatial object. The generation of Voronoi regions can be considered as 'expanding' spatial objects at a unique rate until these areas meet each other. The mathematical expression of the Voronoi region is defined as [12]:

$$V(p_i) = \{p \mid d(p,p_i) \le d(p,p_j), j \ne i, j = 1...n\} \tag{4}$$

In this equation, the Voronoi region of spatial object p_i, $V(p_i)$, is the region defined by the set of locations p in space where the distance from p to the spatial object p_i, $d(p, p_i)$, is less than or equal to the distance from p to any other spatial object p_j.

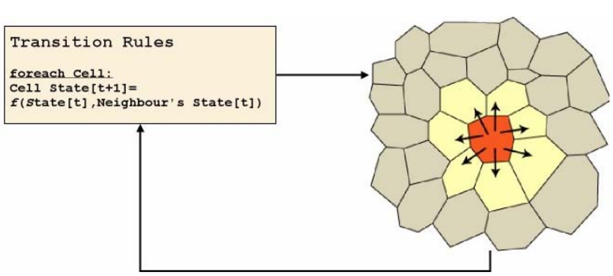

Fig. 2 - Voronoi Based Cellular Automata

Voronoi region boundaries are convex polygons. Points along a common boundary between Voronoi regions are equidistant to the corresponding spatial objects. Objects which share a common boundary are neighbours to each other in the Voronoi spatial model.

A Delaunay triangulation in which each edge of the triangle implies the neighbouring relations of two objects is a dual representation of the Voronoi spatial model that is useful in many calculations (Fig. 3). An edge of the Delaunay triangle must be associated with each pair of neighbours in the Voronoi spatial model.

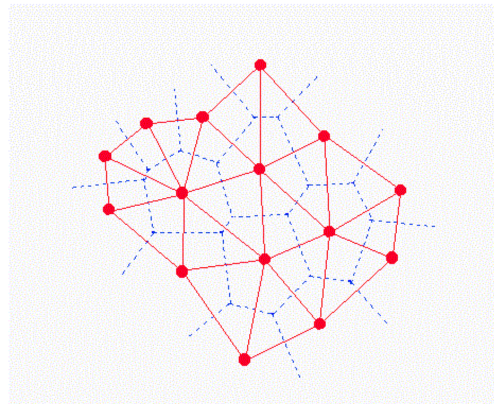

Fig. 3- Voronoi regions and Delaunay triangles

One immediate consequence of using Voronoi Regions as cells is that the number of neighbours is no longer constant, which means that state transition rules must be flexible to cope with this change.

On a Voronoi Based Cellular Automata (VB-CA), each cell is associated to a given relevant spatial object or feature of a relevant spatial object. Examples of features that can be used to create a space tessellation include *Altitude*, *Type of Vegetation*, *Humidity*, *Degree of Imunity*, *Income*, etc. The tessellation would mean, taking *Altitud*e as an example, that each point in each Voronoi region would have a more similar altitude to the centre of its region than to the centre of any of its neighbours. As a result one can obtain different Voronoi Space models for the same geographic area. Cellular Automata extensions where cell state transition uses different neighbours according to the input being considered would provide a huge advantage in what concerns modelling accuracy. To our knowledge, up to date, not a single CA versatile enough to take advantage of this feature has been proposed.

Another advantage of using Voronoi Regions is that they can be dynamic on a spatial sense, i.e., their size, shape and number of neighbours can change over time [2]. This means that CA can be developed to allow inference in each state to actualize GIS data, and use the new data to compute the Voronoi Spacial Model (and the new cells) for the next iteration.

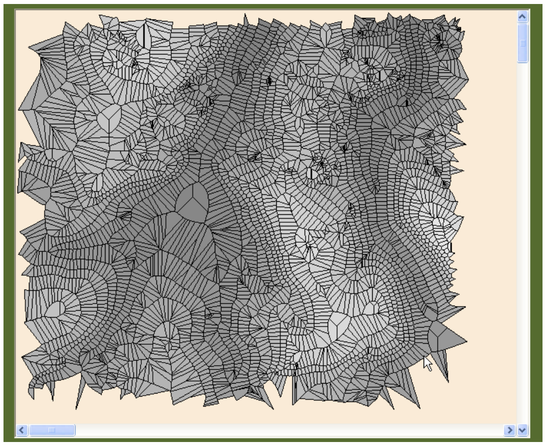

Fig. 4- Voronoi regions: on hilly terrain, the Voronoi space model allows a better modelling of terrain Slope, which, after Wind, is the most influent factor in fire propagation direction and velocity

C. *Fuzzy Cellular Automata*

Fuzzy Cellular Automata (FCA) differ from CA by using a set of fuzzy variables to define its state, and by using a Fuzzy "*IF…Then*" Rule Base (FRB) as the set of transition rules. The FRB infers the next automata state, based on current fuzzy state and current fuzzy inputs (fuzzy antecedents).

The use of Rule Based Fuzzy Systems to model cell state and cell state transition has the obvious and known advantages of both allowing an easy way to model expert knowledge, and a proper way to model expert and data uncertainty and imprecision [12]. It also allows an easier and more proper integration with Fuzzy GIS, which are being increasingly acknowledged as a solution to GIS modelling of real world specificities, like, for example, how to classify areas located in the transition zone between a rural and an urban area [14][15].

However, Rule Based Fuzzy Systems with Feedback suffer from the problem of an uncontrolled spread in uncertainty representation due to the intrinsic characteristics of classical fuzzy inference, that not even type-2 uncertainty representation can address [6]. This fact seriously hinders Fuzzy CA implementations, especially in what concerns cell state transitions. The best available solution to overcome this problem is to use Rule Based Fuzzy Cognitive Maps.

III. Rule Based Fuzzy Cognitive Maps

Qualitative Dynamic systems are composed of a number of dynamic qualitative concepts interrelated in complex ways, usually including feedback links that propagate influences in complicated chains. Axelrod [16] work on Cognitive Maps (CMs) introduced a way to represent real-world qualitative dynamic systems, and several methods and tools have been developed to analyze the structure of CMs. However, complete, efficient and practical mechanisms to analyze and predict the evolution of data in CMs were not available for years. System Dynamics tools like those developed by J.W.Forrester [17] could have provided the solution, but since in CMs numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [7], were developed as a qualitative alternative approach to system dynamics. However, FCM are Causal Maps (a subset of Cognitive Maps that only allow basic symmetric and monotonic causal relations)[2], and, in most applications, a FCM is indeed a man-trained Neural Network that is not Fuzzy in a traditional sense and does not explore usual Fuzzy capabilities. Rule Based Fuzzy Cognitive Maps (RB-FCM) were introduced in [2][9][10] [11][18] and were developed as a tool to model and simulate real world qualitative system dynamics while avoiding the limitations of previous approaches.

RB-FCM can be represented as fuzzy directed graphs with feedback, and are composed of fuzzy nodes (Concepts), and fuzzy links (Relations). RB-FCM are true cognitive maps (CM) since are not limited to the representation of causal relations. Concepts are fuzzy variables described by linguistic terms, and Relations are defined with fuzzy rule bases.

RB-FCM are essentially iterative fuzzy rule based systems with fuzzy mechanisms to deal with feedback, timing mechanisms and new ways to deal with uncertainty propagation, and have several kinds of Concept relations (Causal, Inference, Alternatives, Probabilistic, Opposition, Conjunction, etc.) to cope with the complexity and diversity of the dynamic systems they can model. Among new contributions brought by RB-FCM, there is a new fuzzy operation – the Fuzzy Carry Accumulation –, which is essential to model the mechanisms of qualitative causal relations (FCR – Fuzzy Causal Relations) while maintaining the simplicity and versatility of FCM. FCR are commutative and associative, and their properties allow concepts and/or relations to be added or removed from the system without increasing the system complexity more than necessary, since there is no combinatorial explosion of rules. A FCR rule involves at most, two concepts (one antecedent and one consequent). Whenever a concept has several antecedents, there are independent rule bases for each antecedent, and the combined causal effect of all antecedents is accumulated in the consequent by application of the Fuzzy Carry Accumulation operation.

There are two main classes of Concepts in RB-FCM: $\boxed{\text{Levels}}$, that represent the absolute values of system entities (e.g., LInflation is Good); and Variations, that represent the change in value of a system entity in a given amount of time (e.g., VInflation increased very much).

IV. Dynamic Cognitive Map Based Cellular Automata

Dynamic Cognitive Map Based Cellular Automata

(DCM-CA) try to take advantage of CA strengths on modelling the dynamics of spatial propagation and of Dynamic Cognitive Map strengths to model and simulate system dynamics. On a DCM-CA, cell dynamics (i.e. states and state transitions) are modelled and simulated using Dynamic Cognitive Maps (like FCM or RB-FCM). Cells should also, for obvious reasons, be based on Voronoi Regions. The integration of Voronoi Based CA with Dynamic Cognitive Maps is possible due to the fact that either FCM and RB-FCM have the capability to include or remove new inputs without the need of major changes in existing knowledge (rule bases in RB-FCM), or major loss in usability or performance. On a "classic" rule based fuzzy system, this approach would not be possible, since the simple fact of having a different number of neighbour cells would imply major changes in the cell rule base (and one does not need to mention the problem of rule number explosion as new antecedents are added to a rule base). Choice between FCM or RB-FCM depends essentially on the kind of system we are modelling: FCM are simpler to implement but do not have the versatility and potential of RB-FCM to model more complex systems. If the cell behaviour can be modelled using only symmetric and monotonic causal relations, FCM are an obvious choice. If cell dynamics are more complex and need a true cognitive map to properly model its behaviour (like systems where one combines human expert qualitative knowledge with hard crisp mathematical equations), then one must recur to RB-FCM. Since RB-FCM provide a more versatile solution, the rest of the paper will only focus on RB-FCM Cellular Automata (RB-FCM-CA).

Conventional RB-FCM-CA uses a single RB-FCM to model cell behavior. Cell state is defined by the values of the RB-FCM Level Concepts. However, real world temporal and spatial propagation processes can be very complex. In some systems one might need to use more than one RB-FCM per cell to model cell behavior.

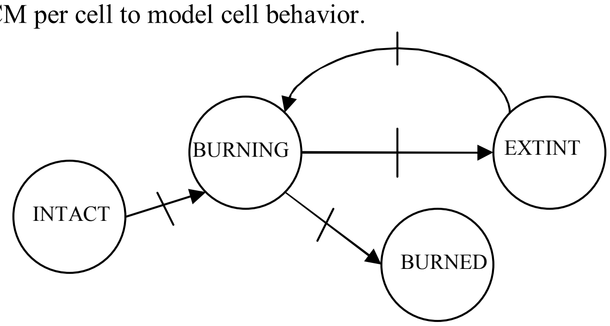

Fig. 5- RB-FCM-CA Cell Meta-states: Each meta-state contains a RB-FCM that is simulated when its meta-state is active. Transition between meta-states occurs when the active RB-FCM reaches certain conditions

A possible approach to model these systems is to define several cell meta-states, where each meta-state is a RB-FCM. Transition between meta-states occurs when, during simulation, the RB-FCM reaches certain conditions. Meta-states can be represented using state diagrams (Fig. 5).

V. Application Example: Modelling and Simulation of Forest Fire Propagation

A. Forest Fire Modelling

In order to test the proposed approach, a model to simulate forest fire propagation was developed.

Forest fire propagation is a complex spatial and temporal dynamic process that depends on a large number of uncertain and imprecise factors such as wind, slope, type of vegetation (also known as fuel), temperature, humidity, etc., being the first two the most important by a large margin.

Since slope is the most important non-variable factor in forest fire propagation [19], our model creates an altitude based Voronoi tessellation. The Voronoi regions are computed based on the orography of the area being modelled, which is obtained from a GIS database.

Each Voronoi region was associated with a cell in a CA. Besides *Height*, each cell has attributes for *Fuel*, *Local Slope*, *Dominant Wind Azimuth*, *Temperature* and *Moisture*. These attributes are present in common GIS, and can be Crisp or Fuzzy, according to the GIS nature. Temperature, wind and/or moisture can have real time values as long as sensors to measure and transmit real time data exist. Each cell is also associated with a parameter named *Ignition Factor*, which is computed based on the other cell attributes.

Due to forest fire propagation complexity, four different cell Meta-states were defined: *INTACT*, *BURNING*, *EXTINCT*, and *BURNED*. Fig. 5 represents a state diagram for transitions between the four Meta-states. Three of the Meta-states are associated to RB-FCMs: *INTACT*, *EXTINT* and *BURNING* (the first two sharing the same RB-FCM). State *BURNED* is a non-dynamic final state, and therefore does not need to be modeled by an RB-FCM.

B. State Transition and Fire Propagation in CA Models

CA Forest Fire propagation models usually use quite simple methods to simulate propagation between cells. In most approaches, e.g. [20][21], whenever a cell starts burning, it is calculated a time interval during which the cell will be in the *BURNING* (active) state. The time interval is computed based on cell characteristics, and can be more or less accurate depending on the model. When the time interval ends, the cell goes into state *BURNED*, and neighbour cells are ignited and enter the *BURNING* state.

Even assuming a perfectly accurate model to calculate burning time, this propagation model has many flaws and is unable to model fire propagation properly even though wind is used to model fire propagation direction and combustion. In fact, the model basically ignores wind effect in what regards fire propagation:

- In strong winds it is perfectly plausible that fire propagates to neighbour cells faster than complete combustion occurs;
- Under wind absence and an unfavourable slope it is

perfectly plausible that fire can extinguish before propagating to other cells.

Other flaw in these propagation models is that the possibility of fire re-ignition is never considered.

One can say that these models are not hindered by the approaches used to model the dynamic process of fire combustion, but because of cellular automata limitations to cope with real-world complex systems.

In the proposed RB-FCM-CA approach, state transition and cell propagation are handled independently and take into consideration a number of parameters much closer to reality.

C. State Transition and Fire Propagation in RBFCM-CA

Initial state for all cells in the RB-FCM-CA is *INTACT*. In order for a target cell to ignite, one or more Voronoi neighbor cells must be in state *BURNING*, and geographic conditions must be favoring fire propagation to target cell. Fig. 6 shows the RB-FCM associated to meta-state *INTACT*.

The RB-FCM uses a variable number of inputs depending on the number of burning neighbors.

Fire Propagation Velocity from each burning neighbor ("source") is calculated using a chain of FIR (Fuzzy Inference Relations [9][10]) that includes fuzzy concepts such as Radiation and Convection, which use as inputs the most important factors related with fire propagation[19]:

- A measure relating wind strength, wind direction and orientation of source/current cell axis (given by the edge of the Delaunay triangle associated to both cells);
- Slope between current cell and source cell;
- Source local temperature;
- Source combustion intensity.

The above inputs can be either crisp or fuzzy according to the GIS system being used. Crisp inputs are fuzzified using relevant linguistic terms defined by experts. Fig. 7 represents linguistic terms for variable Slope. All FIR rule bases were also defined by experts. Currently, 133 rules are being used to infer Propagation Velocity.

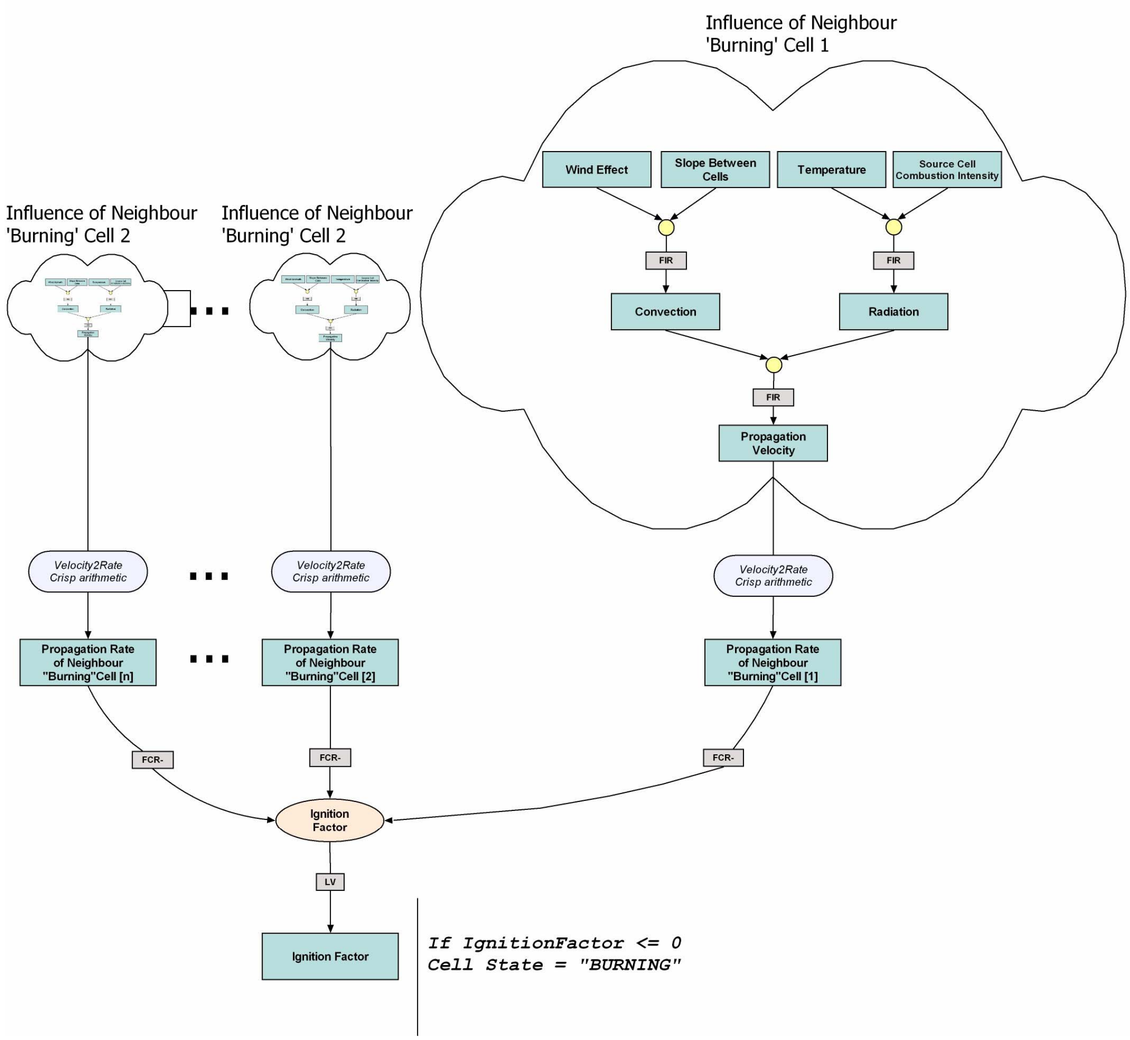

Fig. 6 -RB-FCM for cell Meta-states INTACT and EXTINT (Cell's Ignition Mechanism)

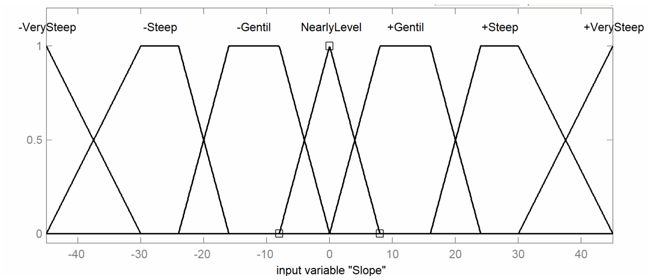

Fig. 7 –Linguistic terms for input Slope

The fire Propagation Rate from each source is calculated based on the inferred Propagation Velocity and on data from the involved Voronoi regions, using the following arithmetic equation:

$$\Pr opRate = \frac{4 \cdot (\Delta T * V \Pr op) \cdot AreaO \cdot Front}{Dist \cdot (AreaO + AreaD) \cdot \left(\sqrt{AreaD} + \sqrt{AreaO}\right)}, \tag{5}$$

where ΔT is the iteration interval, VProp is the Fire Propagation Velocity from source cell, AreaO is the area of the source Voronoi region, AreaD is the area of the current Voronoi region, Dist is the distance between the centre of both cell Voronoi regions, and Front is the size of the frontier between both cells.

Propagation Rate from each burning neighbor is accumulated using an inverse FCR (Fuzzy Causal Relation). One must note that FCR are accumulative and their inference mechanism is independent from the number of antecedent neighbors. The absolute value of Ignition Factor will decrease as the Propagation Rate effect from all neighbor burning areas accumulates as time passes. Transition to *BURNING* state will occur when Ignition Factor reaches zero.

Fig. 8 shows the RB-FCM associated to meta-state *BURNING*. In our approach, contrary to other CA approaches, cell characteristics are not kept constant while the cell is burning. Other approaches attribute a given crisp [20][21][22][24][25][26] or fuzzy [10][19] value to factors such as Temperature or Humidity, and these values will be unchanged while the cell burns. However, this not the most realistic approach, since the burning process will locally cause those values to change, and these changes can affect the burning process itself. Using a RB-FCM allows us to model those local changes and provide a more realistic model. Scenarios like fire extinguishing itself, or drastic variations in fire intensity are possible with this approach. Causal influences between concepts are modeled using Fuzzy Causal Relation Rule bases (FCRBs), and concept variations are defined by linguistic terms such as “Increase_much”, “Decrease_Very_Few” or “Maintains”.

One of the most important concepts in the *BURNING* RB-FCM, is Local Convection. Although this concept is represented, for simplicity reasons, as an external input, it depends on local cell wind (which can change during the burning process) and on cell slope. Sudden variations in wind intensity and direction (which sometimes are a consequence of the burning process) associated to the slope of the burning area, are one of the main causes to the above mentioned scenarios.

Transition from the *BURNING* meta-state to the *EXTINT* meta-state occurs whenever cell Combustion Intensity falls to zero before the entire cell is burned. As it was mentioned above, this situation can occur due to unfavorable conditions to fire propagation. This state is different from a state where the entire cell has burned due to the fact that nothing prevents it from start burning again. Therefore this state is modeled using the same RB-FCM of state *INTACT*. This possibility is ignored in all known CA Fire Propagation Models, like for instance [20][24][25][26], which represents an obvious flaw in those models.

Transition from the *BURNING* meta-state to the *BURNED* meta-state occurs obviously when the entire cell area is burned. This state is a non-dynamic final state, which is not associated to any RB-FCM.

VI. Results

A complete modelling and simulation software application was developed to test the proposed RB-FCM-CA approach. The application creates Voronoi tessellations of the input geographic data, allows input of crisp and fuzzy relevant data, allows tuning of fuzzy variables’ linguistic terms and membership functions and allows tuning of all model rule bases.

Fig. 9 presents a screen shot of the application where a fire propagation simulation example is shown. The modelled area consists in 258Ha of hilly terrain. The simulation shows fire propagation 120 min after fire ignition under strong wind conditions. Each different shade represents fire propagation after 20 min.

Although the system has not been optimized or compared to real fires (due to lack of data), obtained results show that simulated fire propagation behaves as expected by experts, and that complex fire behaviour, like temporary fire extinction and consequent re-ignition can occur in several scenarios. Other CA based proposed systems are not able to exhibit this behaviour.

VII. Conclusion and Future Developments

The proposed RB-FCM-CA based approach has the theoretical advantage of allowing a more proper and versatile modelling of temporal and spatial propagation processes. This is accomplished without the need to recur to complex mathematical knowledge while allowing modelling of uncertain and imprecise knowledge.

The proposed application example, although not completely optimized or compared to real scenarios, behaves as predicted and shows very interesting and promising results.

Future theoretical developments include the simultaneous use of different feature Voronoi regions (see section B.).

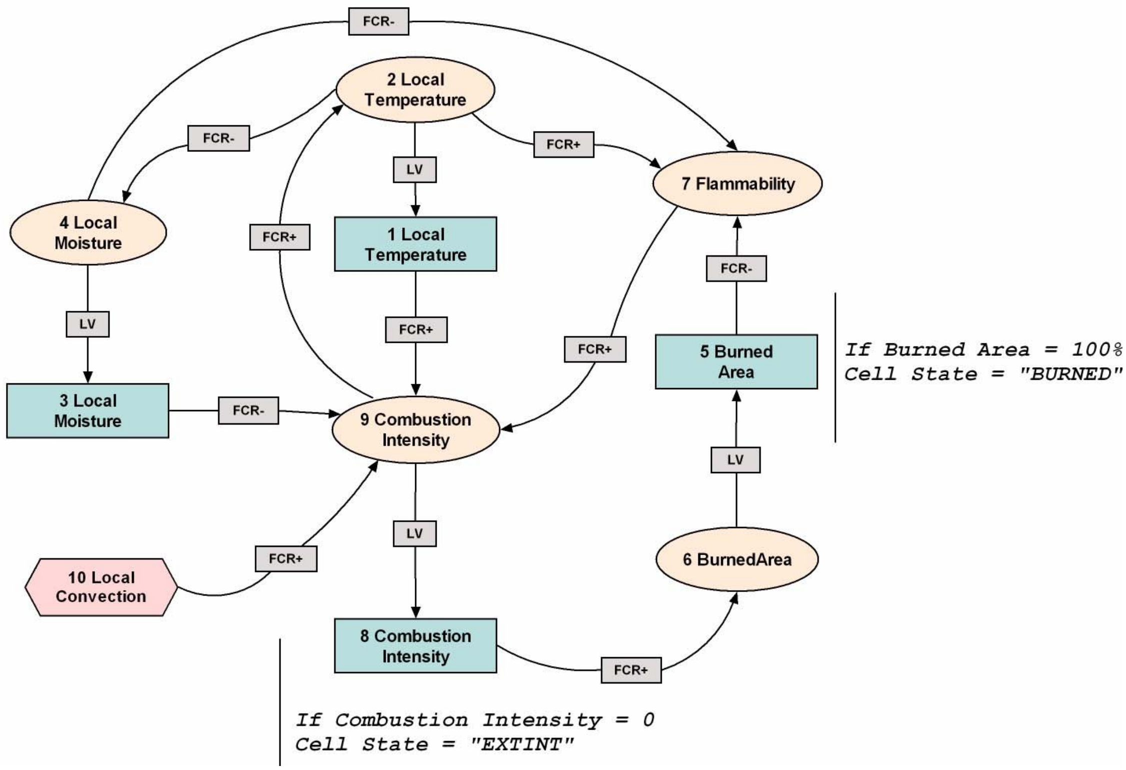

Fig. 8 - RB-FCM for cell Meta-state BURNING (Fire Evolution)

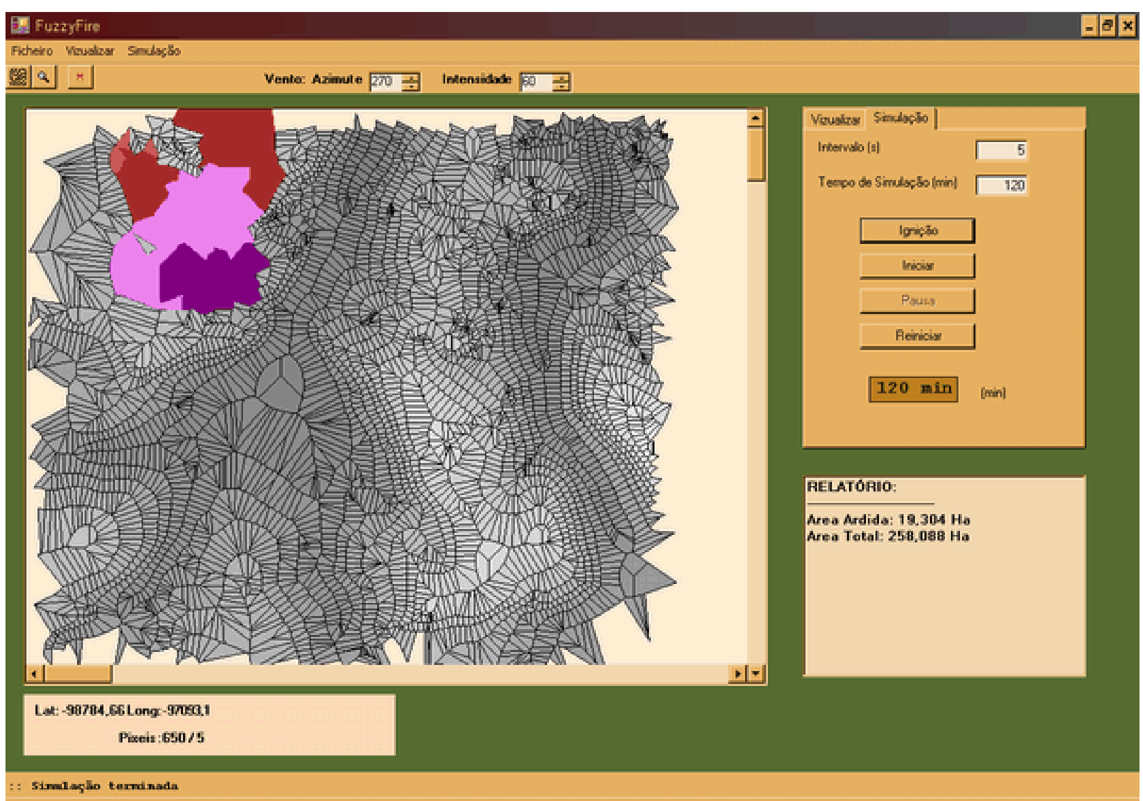

Fig. 9 – Example of simulation of Fire propagation

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