Interpolated Linguistic Terms: Uncertainty Representation in Feedback Rule Based Fuzzy Systems

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Abstract

Rule Based Fuzzy Systems with Feedback suffer from the problem of an uncontrolled spread in uncertainty representation due to the intrinsic characteristics of classical fuzzy inference that not even type-2 uncertainty representation can address. This paper proposes a simple solution that allows type-1 uncertainty representation in this systems.

Keywords: *Interpolated Linguistic Terms; Uncertainty Representation and Propagation; Feedback Rule Based Fuzzy Systems; Rule Based Fuzzy Cognitive Maps.*

1. Introduction

When one must deal with feedback in rule based fuzzy systems, one must accept the fact that unless one defuzzifies the fuzzy variables (that belong to a feedback cycle) in each iteration, then the support set of the fuzzy value of those variables will inevitably spread until is defined in all the Universe of Discourse. As a consequence, the semantic meaning of those fuzzy variables will become useless. Since semantic meaning is the most important quality of a qualitative rule based system, its loss severely cripples the system. This problem can easily be observed in a rule based fuzzy system composed by a single fuzzy variable with its output connected to its input (i.e., where the antecedent and consequent of each rule in the rulebase is the same variable. Figure 1 shows a pictorial representation of how the support set spreads in an example of such system after just 2 iterations.

It is widely accepted that the support set of a given linguistic term can be used to represent its uncertainty [1], but the study and categorization of uncertainty by Klir [2] has shown that there is much more to be said regarding uncertainty than can be expressed by a support set, and in the last years, Mendel [3] has shown that type-2 Fuzzy Sets can be used to represent the several kinds of uncertainties that can occur in a rule based fuzzy system. However the problem of the uncontrolled support set spread occurs even when one uses type-2 fuzzy, and must be addressed as a problem closely related to uncertainty. Although we encourage the idea that type-2 fuzzy is a much more adequate approach to represent all kinds of uncertainties, in this paper we mainly use the widely accepted view that the support set of a linguistic fuzzy term represents uncertainty (at least some type of uncertainty). Therefore, since feedback causes uncontrolled and undesirable support set spread, one can say that, in terms of uncertainty, feedback in rule based fuzzy systems will make variables become more and more uncertain.

This problem is caused by the fact that the inference of a Fuzzy Rule Base results in a fuzzy set with uncertainty given by the union of the support set of the linguistic terms involved in the consequent of the active rules. If a single set is involved, then the resulting uncertainty is the uncertainty of that set; but if more than one set is involved, then there is not a smooth transition between the uncertainty of the involved sets, since even the smallest activation of another consequent will increase the uncertainty to the value represented by the union of both set's support. There is not a direct relation between the uncertainty of the result and the degree of activation of each linguistic term. Figure 2 shows a pictorial example of the idea we are trying to transmit. For example, if the only active consequent in Figure 2 is Large, i.e., $\mu(L)>0$ and $\mu(VL)=0$, then the resulting uncertainty is supp(L). But if both are active, i.e., $\mu(L)>0$, and $\mu(VL)>0$, then the resulting uncertainty will be given by supp(L) ∪supp(VL).

Therefore, one can say that "classical" rule based fuzzy inference is not particularly adapted to type-1 uncertainty propagation. In particular, one can say that in rule based fuzzy systems with feedback, uncertainty propagation application is inhibited by the fact that one must always end using crisp (defuzzified) variables as an input in following iterations.

Interpolated Linguistic Terms (ILT), an extension of formerly introduced Causal Output Sets (COS), were initially developed as a method to represent causal inference while avoiding combinatorial rule explosion in Fuzzy Rule Based Systems. They are extensively used in Rule Based Fuzzy Cognitive Maps (RB-FCM) [4][5][6][7][8][9][10], which are a tool to model and simulate Qualitative Systems Dynamics. RB-FCM can be essentially described as Rule Based Fuzzy Systems where we added fuzzy mechanisms to deal with feedback, different kinds of timing mechanisms, and different relations that can cope with the complexity and diversity of the dynamic qualitative systems they try to model. One of the design requisites of COS was the need to maintain a relation between support size and linguistic term activation levels, therefore, although not developed with uncertainty propagation as an objective, COS were automatically adapted to the task of representing variable uncertainty. ILT have been developed since they were named COS, and in this paper they are introduced as a tool that allows representation of type-1 uncertainty in Rule Based Fuzzy Systems with feedback, especially in RB-FCM.

2. Interpolated Linguistic Terms (ILT)

Interpolated Linguistic Terms (ILT) are very simple and basic mechanisms that can be used as alternative representations for the fuzzy sets obtained by fuzzy rule base inference, and can be seen as a 2D interpolation of the linguistic terms involved in the consequents of that inference. One can show that ILT uncertainty is an interpolation of the uncertainty of the involved Linguistic terms, and that ILT can be defined and used as new linguistic terms of the fuzzy variable we are dealing with.

Before presenting a formal definition of an Interpolated Linguistic Term, we introduce in Figure 3 some terminology used in this paper, and the following a set of restrictions that are necessary to implement ILTs.

In order to be able to obtain an ILT on a rule based fuzzy system, one must enforce the following restrictions in the system:

i. The membership degree of all linguistic terms must be complementary, i.e., its sum must be 1 in every point of the variable Universe of Discourse (X):

$$\forall x \in \mathrm{X}, \forall (A_0, A_1, ..., A_n) \in \mathrm{F}(\mathrm{X}), \sum_{\mathrm{i}=0}^{\mathrm{n}} \mu_{A_i}(x) = 1$$

ii. All linguistic terms must have the same basic shape (trapezoidal, S, etc.), and their membership functions must cross with their neighbours when μ=0.5.

iii. The inference method must preserve both the shape and the centroid's x-coordinate of the consequent linguistic term; the Max-Dot method is an example of an adequate method.

iv. The fuzzy sets that result from the inference of the rule base must be summed. As a result one obtains a single fuzzy set, which we will call U.

These restrictions guarantee [8] that if there is a single antecedent, then a <u>maximum of 2 rules</u> is involved in each inference of U, and that the Area of U is univocally related with the area of the consequent terms involved in the inference of U. Therefore, if "larger" sets are used to represent terms that have a "larger" semantic meaning, then the Area of U will have a proper semantic meaning. For example, if the inference active terms are *High* and *Very_High*, and if $\mathrm{Area}_{High} < \mathrm{Area}_{Very\_High}$, then $\mathrm{Area}_{High} < \mathrm{Area}_U < \mathrm{Area}_{Very\_High}$. These restrictions are necessary for the implementation of ILT in Rule Based Fuzzy Cognitive Maps. For other applications, some might prove not necessary. With these restrictions in mind, one can present the following ILT definition:

Definition: An ILT, Interpolated Linguistic Term, is a fuzzy set that is univocally related with the active consequents of a rule based fuzzy inference. Given U, obtained respecting restrictions i. to iv., we call ILT_U (the Interpolated Linguistic Term of U), to the fuzzy set that respects the following conditions:

v. ILT_U and the term set of the fuzzy variable where U is defined must have the same shape.

vi. The x-coordinate of the centroid of ILT_U and the x-coordinate of U must be the same:

$$\mathrm{xC}_{ILT_U} = \mathrm{xC}_U \Leftrightarrow \left(\frac{\int_x \mu_{ILT_U}(x) \cdot x \; dx}{\int_x \mu_{ILT_U}(x) \; dx} \right) = \left(\frac{\int_x \mu_U(x) \cdot x \; dx}{\int_x \mu_U(x) \; dx} \right)$$

vii. U and ILT_U must have the same Area:

$$\mathrm{Area}_{ILT_U} = \mathrm{Area}_U \Leftrightarrow \int_x \mu_{ILT_U}(x) \; dx = \int_x \mu_U(x) \; dx$$

viii. ILT_U is normal, i.e.:
$\{\exists x \in X \mid \mu_{ILT_U}(x) = 1\} \Leftrightarrow ILT_{U1} \neq \varnothing \Leftrightarrow \mathrm{xtop}_{ILT_U} > 0$,

where ILT_{U1} represents the α-cut of ILT_U for α =1 and xtop is the size of ILT_{U1}.

ix. If *A* and *B* are the terms involved in the inference of *U*, then the size of ILT_{U1}, xtop_{ILTU}, is a function of *A* and *B*'s xtop and of *A*, B and *U*'s xC:

$$\text{xtop}_{ILT_U} = \min\{\text{xtop}_A, \text{xtop}_B\} + \left|\frac{\text{xC}_U - \text{xC}_A}{\text{xC}_B - \text{xC}_A} \times (\text{xtop}_A - \text{xtop}_B)\right|$$

x. If *A* and *B* are the terms involved in the inference of *U*, then the size of the inner base of ILT_U, bi_{ILTU}, is a function of *A* and *B*'s bi and of *A*, B and *U*'s xC:

$$\text{bi}_{ILT_U} = \min\{\text{bi}_A, \text{bi}_B\} + \left|\frac{\text{xC}_U - \text{xC}_A}{\text{xC}_B - \text{xC}_A} \times (\text{bi}_A - \text{bi}_B)\right|$$

End of Definition

From bi_{ILTU} one can derive the inner slope of ILT_U:

$$\text{si}_{ILT_U} = \frac{1}{\min\{\text{bi}_A, \text{bi}_B\} + \left|\frac{\text{xC}_U - \text{xC}_A}{\text{xC}_B - \text{xC}_A} \times (\text{bi}_A - \text{bi}_B)\right|}$$

The above conditions are quite obvious and natural: xtop_{ILTU} and bi_{ILTU} assume values that range between the respective values of *A* and *B*, according to the activation degree of the rules involving *A* and *B*. If only one rule is active, for instance the one involving *A*, then *U*, ILT_U and *A* are the same set.

If all above conditions are respected, ILT_U is unique [8]. For a given *U* and basic shape, since all equations have a single solution, then xtop, bi, xC and Area are unique and there is only one possible set; therefore it is impossible to obtain more than one *ILT* from a set *U*.

As we have seen above, the support set of *U* is the result of the union of suppA and suppB, but the support set of ILT_U, and therefore its **uncertainty**, is univocally related with the **uncertainty** of *A* and *B*. In RB-FCM, since it's guaranteed that "larger" causal linguistic terms have "larger" uncertainty [5][6][7][8], this property allows the resulting set to be shifted in the UoD without loosing its identity, the accumulation of the effect of several antecedent causes without exponentially increasing the size of the rule bases, and also a way to represent consequent uncertainty based on antecedent uncertainty.

Figure 4 shows graphic examples of ILT that result from the inference of two rules involving linguistic terms with different shapes.

One can say that the set of all possible ILT on a fuzzy linguistic variable are the result of an operation that transforms a finite set of *n* linguistic terms into a continuous "linguistic terms function". The equivalent crisp 2D operation would be the linear interpolation of a discrete number of points, hence the chosen name for the ILT.

ILT calculus involves much more complex operations than the usual fuzzy ruled based inference operations. However it's easy to find algorithms [8] that are fast enough to use ILT in Rule Based Cognitive Maps with a few hundred concepts without any loss in usability.

3. Using ILT to Represent Uncertainty in RB-FCM

ILT were developed with RB-FCM in mind, and as we have shown [5][6][8], are one of their fundamental mechanism. When combined with Fuzzy Causal Relations (FCR) – the fundamental relations between concepts in Cognitive Maps –, ILT also automatically allow a way to represent consequent uncertainty based on antecedent uncertainty:

- Since FCR rulebases only have a single antecedent (the causal accumulative effect of several causes is expressed using different rulebases and a special operation, the FCA);
- Since Causal variables linguistic terms respect the conditions presented in 2.;
- Since Causal variables linguistic term's support set represent the linguistic term uncertainty;
- ➢ Then the inference of a FCR always produces a set U that can be used to obtain ILT_U directly, and, as we have seen, suppILT_U (and therefore ILT_U uncertainty) is an interpolation of the uncertainty of the involved linguistic terms.

Since the resulting support set is not given by the union, but by a continuous function of the involved consequent support sets, and that this support set is based on the size of the support sets of the consequent linguistic terms where μ_{xC} is non-zero (a maximum of 2 linguistic terms), then the inevitable spread no longer occurs.

However, in RB-FCM, fuzzy causal variables can have several antecedents whose effects are accumulated applying the FCA to the ILTs obtained from each antecedent. When this occurs, the accumulated resulting sets – ARS –, usually one reflecting positive accumulation and another reflecting negative accumulation, no longer provide an uncertainty indication that can be used in feedback systems [8]. In order to obtain an indication of variable uncertainty, we use the following procedure:

- Defuzzify the accumulated sets (using a special procedure that involves the total accumulated area [8]) obtaining a single xC_{ARS};
- Compute the single possible ILT_{ARS} for the given xC_{ARS} and linguistic terms where μ_{xCARS} is non-zero;

By definition, the resulting *ILT* provides an usable and expected indication of uncertainty for the point xC. However, since the computed *ILT* is based on a single point, it is possible to obtain the same *ILT* from different accumulated sets (the process of defuzzification is obviously not biunivocal), therefore uncertainty representation becomes independent from the number of antecedents, which is not sound, since with a greater number of accumulated effects, uncertainty should become larger. The solution lies in the use of interval type-2 fuzzy sets [3][11] to represent uncertainty in the obtained ILT. In order to make this possible it is necessary to find a method to compute the type-2 interval. The ideal solution would be the adaptation of the FCR, and the FCA to type-2 inference, and the use of the accumulated type-2 intervals in the ILT. However, although in the agenda for a needed future development, RB-FCM do not yet support type-2 sets and inference. The temporary solution consists in obtaining a unique interval type-2 ILT based on the inner slopes of the ARS – si_{ARS}. The si_{ARS} give an indication of the largest variations involved in the inference, and therefore, of the largest uncertainties involved. Due to the characteristics and properties of RB-FCM causal mechanisms and restrictions [8], the obtained type-2 ILT is unique and its uncertainty is related with antecedent uncertainty. Figure 6 shows a pictorial example of a type-2 ILT representing the expected variation obtained after the accumulated resulting sets (ARS+, ARS-) and linguistic terms presented in Figure 5. ARS+ results from the accumulation of 2 ILTs representing an increase in the variable value (1 antecedent variable caused a high increase and another a very small increase), and ARS- results from the accumulation of 2 ILTs representing a decrease (a third antecedent variable caused a small decrease and a fourth one caused a very small decrease). Note that since we are dealing with variable variations, the typical defuzzification problem of obtaining a result "in the middle" after fuzzy sets in the "left" and in the "right" is not relevant [8].

4. Using ILT to Represent Uncertainty in Generic Feedback Rule Based Fuzzy Systems

As we have seen above, in feedback rule based fuzzy systems, one must defuzzify the variables that belong to a feedback cycle in every iteration. If one fails to do it, then the support set of the fuzzy value of those variables will inevitably spread until is defined in all the UoD [8] and becomes useless (since one end up with maximum uncertainty in just a couple of iterations). In "classic" rule based fuzzy systems, variables and linguistic terms are not subject to the restrictions presented in 2., and one can always have several antecedents combined with different operations (and, or, etc.). Therefore, one cannot expect to obtain a set R that can be directly transformed into a ILT_R. However, as long as conditions i. and ii. are respected, then there is only a single *ILT* for a given couple of linguistic terms and a given xC [8], and it is always possible to obtain an *ILT* based on the defuzzified value of R (xC_R) and on the linguistic terms where μ_{xCR} is non-zero. Even knowing that this *ILT* does not necessarily reflects the propagated uncertainty, it reflects the uncertainty most expected value for the obtained xC (as long as one consider that the support of all involved linguistic terms represent its uncertainty), and provides a way to represent type-1 uncertainty propagation in feedback rulebased fuzzy systems without the inevitable spread that hinders inference results. Note that this does not means that uncertainty can not spread. It simply means that uncertainty spread is more dependent on the system properties than on the method used to infer the system, which is what happens when we use classical rule based inference (ideally uncertainty spread should only be dependent on the system, but that is not yet guaranteed with this simple approach).

Once again, this procedure is limited due to the fact that computed uncertainty is independent from the number of rule antecedents. Since type-2 fuzzy inference is the procedure of choice to represent uncertainty propagation in Rule Based Fuzzy Systems, the solution should lie once again in the development of type-2 ILTs. Note that this procedure is much simpler than the one proposed in 3., that also involves adapting FCR and FCA to type-2. When one must stick with type-1 fuzzy sets, ILTs alone must be considered a valid alternative to represent expectable uncertainty in feedback rulebased fuzzy systems, eliminating the need to defuzzify each variable in each iteration.

Pictorial examples and results of the application of the proposed methods to several feedback rule based fuzzy systems can be found at http://digitais.ist.utl.pt/uke/ILT

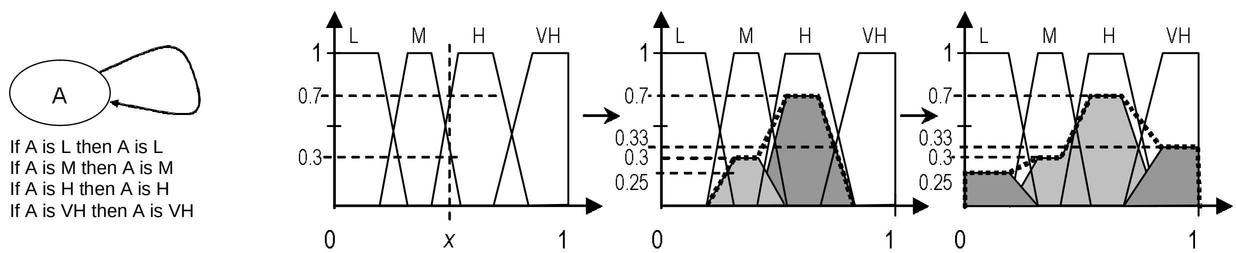

Figure 1 – Uncertainty Spread in Feedback Rule Based Fuzzy Systems

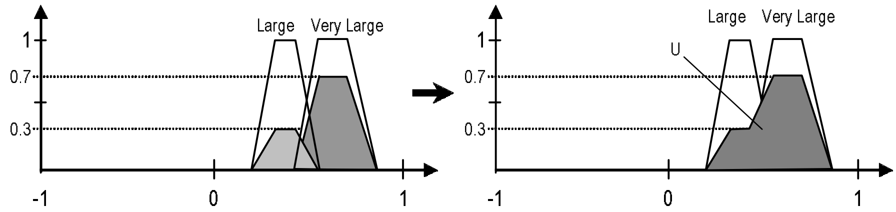

Figure 2 – The support set of U is the union of the support sets of Large and Very Large

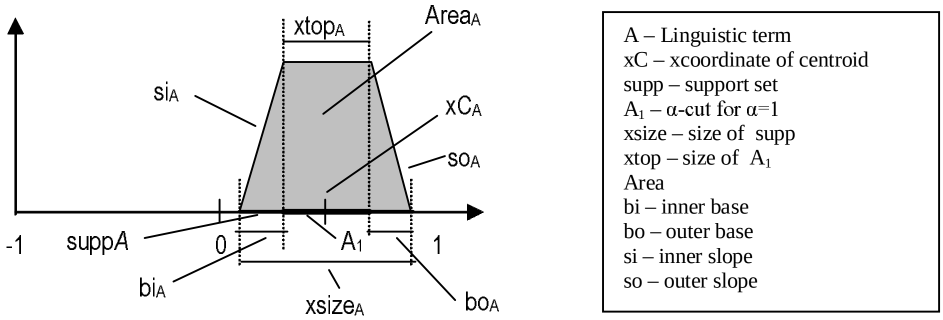

Figure 3 – Morphology of a Linguistic Term

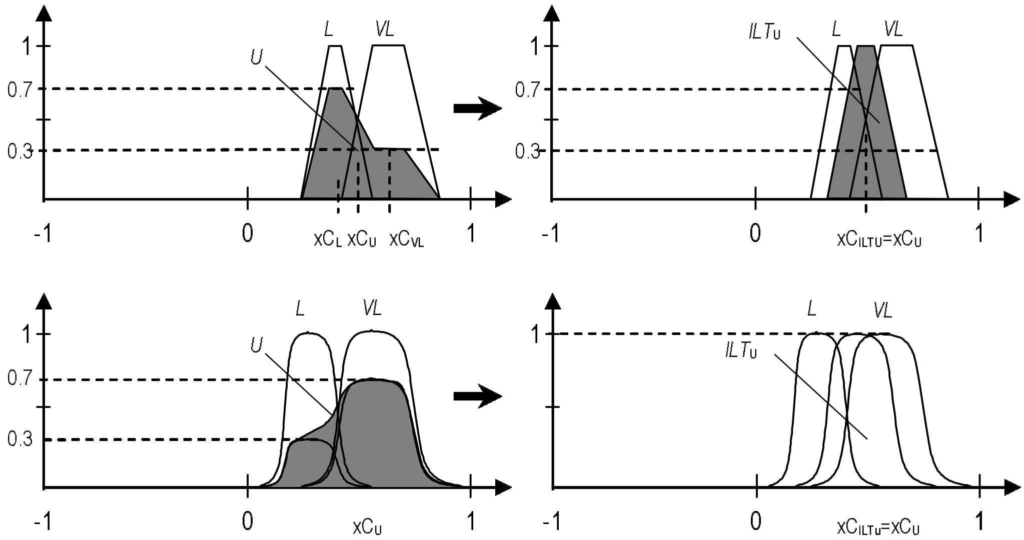

Figure 4 – Examples of ILT for different term shapes and rule activations

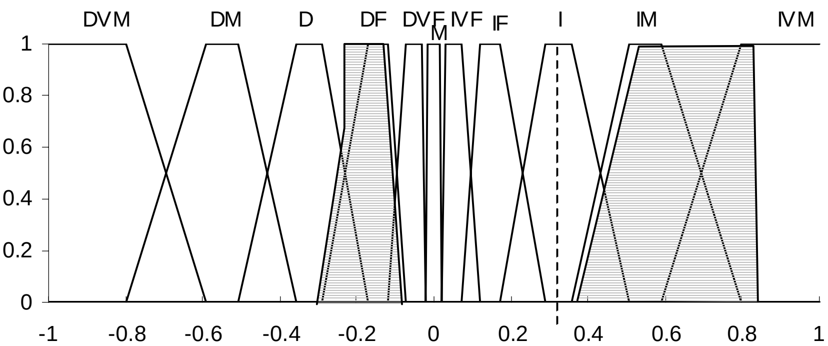

Figure 5 – Linguistic Terms and sample ARS of a Causal Variable in a RB-FCM

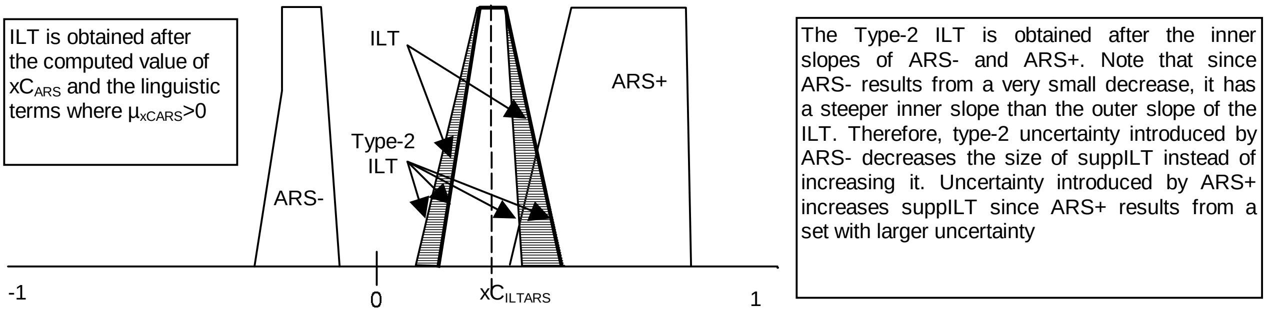

Figure 6 – Type-2 ILT obtained after ARS- and ARS+ given the linguistic terms of Figure 5

5. Conclusions and Future Developments

This paper proposes a simple solution to solve the problem of uncertainty uncontrolled spread in feedback rulebased fuzzy systems. This type-1 uncertainty related problem cannot be solved by the use of type-2 fuzzy sets since it is directly related with "classical" rulebased fuzzy inference procedures that are also used in type-2 inference. The proposed solution is the use of Interpolated Linguistic Terms. Although being a limited solution, it provides an effective method to eliminate the need to defuzzify the fuzzy variables belonging to a feedback loop in each iteration, while allowing the representation of an expected uncertainty value. Since type-2 fuzzy logic is arguably the best method to represent and deal with uncertainty in rulebased fuzzy systems, in order to eliminate the weaknesses of the proposed type-1 procedure, we propose the future development and adaptation of ILTs to type-2 according to the guidelines herein proposed. Results and conclusions regarding these developments can be found online at: http://digitais.ist.utl.pt/uke/ILT.

Other future developments involve ILT restriction elimination (some of the restrictions come from the fact that ILT were developed with RB-FCM in mind and might not be necessary in generic systems), and the development of non-linear interpolation methods to obtain the ILT.

6. References

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