

# Qualitative Modelling of an Economic System using Rule-Based Fuzzy Cognitive Maps<sup>♦</sup>

João Paulo Carvalho

José A. B. Tomé

INESC-ID - Instituto de Engenharia de Sistemas e Computadores

IST – Instituto Superior Técnico

R. Alves Redol, 9, 1000-029 LISBOA, PORTUGAL

Phone: +351.21.3100262 Fax: +351.21.3145843

E-mail: joao.carvalho@inesc-id.pt

jose.tome@inesc-id.pt

**Abstract:** Truly qualitative modelling of qualitative dynamic systems is a delicate issue in a sense that even when one uses a “qualitative modelling tool” one often end up adopting quantitative model approaches in disguise. In this paper we use Rule Based Fuzzy Cognitive Maps to obtain and simulate a qualitative model of an Economic system<sup>1</sup>.

**Keywords:** Rule Based Fuzzy Cognitive Maps, Qualitative Modelling, Qualitative Dynamic Systems.

## 1. INTRODUCTION

Decision makers, whether they are social scientists, politicians or economists, usually face serious difficulties when approaching significant, real-world dynamic systems. Such systems are composed of a number of dynamic qualitative concepts interrelated in complex ways, usually including feedback links that propagate influences in complicated chains. Axelrod [1] work on Cognitive Maps (CMs) introduced a way to represent real-world qualitative dynamic systems, and several methods and tools have been developed to analyze the structure of CMs. However, complete, efficient and practical mechanisms to analyze and predict the evolution of data in CMs were not available for years due to several reasons. System Dynamics tools like those developed by J.W.Forrester [2] could have provided the solution, but since in CMs numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [4][5][6], were developed as a qualitative alternative approach to system dynamics. However, FCM are Causal Maps (a subset of Cognitive Maps that only allow basic symmetric and monotonic causal relations)[1], and, in most applications, a FCM is indeed a

man-trained Neural Network that is not Fuzzy in a traditional sense and does not explore usual Fuzzy capabilities. They do not share the properties of other Fuzzy systems and the causal maps end up being quantitative matrixes without any qualitative knowledge. Rule Based Fuzzy Cognitive Maps (RB-FCM) were introduced in [7][8][9][10][11][12][13][14] and are being developed as a tool that models and simulates real world qualitative system dynamics while avoiding the limitations of those approaches.

When one intends to model a qualitative dynamic system, even when using a proper qualitative modeling tool, one often end up making a direct adaptation of quantitative modeling techniques. One should try to avoid this modeling approach for one might risk ending up with a quantitative model in disguise. In this paper we present a didactic qualitative model of macro-economy in the Euro-area while showing how to avoid such approaches.

## 2. RULE BASED FUZZY COGNITIVE MAPS

This section provides a brief introduction to RB-FCM. Details are available in [7][8][9][10][11][12][13][14].

RB-FCM allow a representation of the dynamics of complex real-world qualitative systems with feedback, and the simulation of events and their influence in the system. They can be represented as fuzzy directed graphs with feedback, and are composed of fuzzy nodes (Concepts), and fuzzy links (Relations). RB-FCM are true cognitive maps (CM) since are not limited to the representation of causal relations. Concepts are fuzzy variables described by linguistic terms, and Relations are defined with fuzzy rule bases.

RB-FCM are essentially iterative fuzzy rule based systems where we added fuzzy mechanisms to deal with feedback, introduced timing mechanisms and new ways to deal with uncertainty propagation, and were we defined several kinds of Concept relations (Causal, Inference, Alternatives, Probabilistic, Opposition, Conjunction, etc.) to cope with the complexity and diversity of the dynamic qualitative systems we are trying to model. Among new contributions brought by RB-FCM, there is a new fuzzy operation - the Fuzzy Carry Accumulation -, which is essential to model the

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<sup>♦</sup> This work is partially supported by the FCT - Portuguese Foundation for Science and Technology under project POSI/SRI/47188/2002

mechanisms of qualitative causal relations (FCR – Fuzzy Causal Relations) while maintaining the simplicity and versatility of FCM.

There are 2 main classes of Concepts: **Levels**, that represent the absolute values of system entities (e.g., LInflation is Good); and **Variations**, that represent the change in value of a system entity in a given amount of time (e.g., VInflation increased very much).

### 3. A QUALITATIVE MACRO ECONOMIC MODEL

When we approached this problem, our primary goal was to show the capabilities and ease of use of RB-FCM to model the dynamics of qualitative real world systems. Even if the final model is rather complex and does not contain apparent flaws, it is not a complete model. We are not economic experts (even though we consulted some), and such a model would be even much more complex (in several ways). However, as we will see, we ended up obtaining rather interesting and surprising results.

Classic cognitive mapping techniques [1] were used as the first step to obtain the model we present: the concepts and relations were extracted after an economic expert analysis of “Short-term Tax Rate evolution in Europe” published in Portuguese newspaper Público [15]. Throughout the text, the author introduced several concepts, supporting its theories while explaining the relations between concepts using qualitative knowledge. The “classic” CM obtained was much simpler than the one we present here, which was expected, since the analysis of the dynamics of a much more complex model – like the one we ended up obtaining – would require several months of work using traditional quantitative econometric approaches. This first model used only the most important concepts (the ones really necessary to a short term analysis): Tax Rates, Inflation, Consumption, Oil Price, and Food Cost. Even with such a few concepts, a realistic model becomes rather difficult to analyse due to the complexity of the relations that affect the involved concepts. However, we wanted to show RB-FCM potential to deal with larger systems and long-term simulations, and these would require a more realistic model (more concepts and much more relations). We ended up adding 13 concepts to the original 5 (Figure 8).

At the end of this phase of the modelling process we obtained a classic Cognitive Map – basically a graph where the nodes were the Concepts and each edge represented an existing unknown relation between a pair of Concepts.

#### 3.1. Concept Modelling

The next step was refining the concepts to obtain a linguistic fuzzy representation for each one. This step consisted in defining the class(es) (Variation, Level) and the linguistic terms and membership functions for each concept. In dynamic systems, variations are much more important than

absolute values, therefore, most concepts are Variations, some are Levels, and a few key concepts like Inflation, Tax Rate, etc., are both Variations and Levels (the Level value of these concepts is actualized according to its Variation using a special LV relation [11]).

The linguistic terms of Levels must have a direct correspondence with the real world values. Therefore we allied common sense and expert consulting (using straight questions like “what do you consider a high value for Inflation?”, and receiving answers like “around 4%”) to define their membership functions. In the particular case of Levels that depend on LV relations, it was also necessary to define the real-world meaning of a certain amount of variation (e.g., a “Small” increase on inflation is around 0.3%). Figure 6 shows the linguistic terms of the Level concept LInflation.

Variation linguistic terms usually represent qualitative terms without a direct correspondence to absolute values. E.g., VInflation has 11 linguistic values ranging from “Huge Decrease” to “Huge Increase” (Figure 7). Linguistic terms of Variations can usually be represented by standard sets, which simplify and accelerate the modelling process [11].

#### 3.2. Qualitative Modelling of a Qualitative Dynamic System

The huge advantages of using Fuzzy Rule Bases (FRB) to define qualitative relations between Concepts has been largely discussed and proved [7][9][10][11]. The major drawback of rule-based fuzzy inference, the combinatorial explosion of the number of rules, is avoided in RB-FCM by the use of Fuzzy Causal Relations and the Fuzzy Carry Operation [8][9][10][11]. Another important feature of RB-FCM is the easy insertion and removal of Concepts and/or Relations, which also reduces the modelling complexity of FRB [8][9][10][11]. Therefore we have an adequate tool to model qualitative relations. However, the single fact of using linguistic rule bases to model relations does not guarantee the qualitative nature of the model. Let us see the example of Inflation modelling:

A pseudo-qualitative approach using FRB would try to closely map the widespread quantitative approaches: Inflation value is predicted by a weight averaged sum of several factors (Estimated Oil inflation, estimated Food price inflation, etc.). This method is highly dependent on the precision and validity of each factor real-world absolute value. In the model we present, we tried a completely different approach that uses rules that are independent from the real world absolute values. The model is based on a qualitative definition of inflation: Economics theory states that economic growth depends on inflation – without inflation there is no growth; In fact, the worst economic crisis (30’s for instance) are associated with deflation; Therefore, it is desirable and expected that all factors that affect inflation have a certain cost increase – If all factors

suffer a normal increase, then the inflation will maintain its normal and desired value. Therefore, one can state the following qualitative relation for each of those  $n$  factors:

“If factor <sub>$n$</sub>  has a normal increase, then Inflation will maintain”

This statement will be part of the fuzzy rule base of a causal relation. Since fuzzy causal effects are accumulative and their effect is a variation in the value of the consequent, then if all factors that cause inflation have the normally expected increase, Inflation will not vary. If some factors increase more than expected and the others maintain their value then inflation will somehow increase. If a factor increase less than normal, or even decreases, then its effect is a decrease in inflation (note that the final variation of Inflation is given by the accumulation of all causal variation effects – e.g., if some pull it down a bit, and one pulls it up a lot, in the end inflation still can maintain its normal value).

It is possible to build a completely qualitative and sound causal FRB to model each factor influence of Inflation, without ever referring to absolute values. If one intends to model inflation in South America, one can maintain the rule base. All that needs to be changed are the linguistic terms of the Level Concept associated to Inflation (for instance, normal inflation would become around 8 %, and so on...). Obviously some factors are more important than others (a large increase in food might cause a large increase in Inflation, but what is considered a large increase in Oil might only cause a small increase in Inflation – average Oil price increased over 20% in the last 2 years, but other factors had a normal increase, and therefore inflation had just a slight increase...). This “relative” importance is easily modelled as a causal effect in a FRB. Table I represents an example of a causal FRB. One can also mention the fact that oil price variation has a delayed effect in inflation. RB-FCM provide mechanisms to model these kinds of timing issues [12]

**TABLE I: FCR7+sl Food Cost, Inflation**

If Food Cost Decreases VeryMuch,	Inflation has aLarge Decrease
If Food Cost Decreases Much,	Inflation has a Large Decrease
If Food Cost Decreases,	Inflation has a Large Decrease
If Food Cost Decreases Few,	Inflation Decreases
If Food Cost Decreases VFew,	Inflation Decreases
If Food Cost Maintains,	Inflation Decreases
If Food Cost Increases VFew,	Inflation has a Small Decrease
If Food Cost Increases Few,	Inflation has a Very Small Decrease
If Food Cost Increases Normally,	Inflation Maintains
If Food Cost Increases M,	Inflation has a Small Increase
If Food Cost Increases VM,	Inflation Increases

This kind of qualitative approach was used throughout the model when causal relations were involved.

As we mentioned above, Variations usually have a standard set of linguistic terms. These allow the predefinition of certain common fuzzy causal relations (FCR). These FCR

are called macros [10][11] and were used to reduce the modelling effort.

The model includes other than causal relations. For instance: Oil price variation was modelled using a classic fuzzy inference rule base (FIRB) based on oil Offer/Demand (where Oil offer was decided in simulated periodic OPEP meetings); the Tax Rates were modelled considering that Banks were managed as a common business with profit in mind – for example, an increase in money demand would increase Tax Rates (this would be changed later (see 3.4)).

Regarding timing considerations, the system was modelled considering a one month period between iterations.

It is obviously impossible to detail every aspect of the system modelling in this paper. Figure 8 provides a graphic representation of the final RB-FCM model. The system consists of 18 concepts and around 400 fuzzy rules to express relations (most were automatically generated using macros). The system was described using RB-FCMsyntax (a dedicated language) – a complete description is available in [11]. Here are some guidelines regarding the description of relations in Figure 8: “FCR+” stands for a standard positive causal relation (an increase in the antecedent will cause an increase in consequent), and “FCR-“ a standard negative relation (increase causes a decrease). Several “+” or “-“ represent stronger effects. A “/” represents an attenuated effect. “sl” and “sr” represent biased effects (non symmetric causal relations. A “?” represents a relation which cannot be symbolically described (one must consult the FCR). A “d” represents a delay in the effect. FIR stands for Fuzzy Inference Relation. The number after FCR or FIR is the label for the complete description of the rule base.

### 3.3. Simulation results<sup>2,3</sup>

The simulation of the system provided rather interesting results. The evolution of the system through time was rather independent from the initial values and the external effects. After a certain period of time, that could vary from a few months to several years (depending on a conjugation of external factors like a war, or a severe cut in oil production), economy would end up collapsing: deflation, negative growth, 0% tax-rates. Figure 1 represents one of those cases.

Initially one could think that there was a major flaw in the model (or in the RB-FCM mechanisms), but after a discussion and analysis of the results with an economics expert, the culprit was found: the model approached the economic situation before the creation of entities that control Interest Rates (like the U.S. Federal Reserve, the European Central Bank). The lack of these entities was the main cause

<sup>2</sup> Note: This section refers to an initial version of the system that did not include concept 17

<sup>3</sup> All Figures use the following scale: Oil price is in USD and must be divided by 2 (e.g., 60 represents 30 USD), and the other 3 Levels are in % and must be divided by 10 (e.g., 15 represents 1.5%)

to economic instability until 1930's. In fact, Economics was known in the 18<sup>th</sup> and 19<sup>th</sup> century as the “Dark Science”, because all theories indicated that economy was not sustainable. According to the simulation results, depression always comes after a growth period and due to an exaggerate increase in tax rates (the banks try to maximize their profit in a short period, and their greed cause an apparently avoidable crisis). Therefore, to support this theory, we introduced a simple model of the European Central Bank behaviour regarding interest rates.

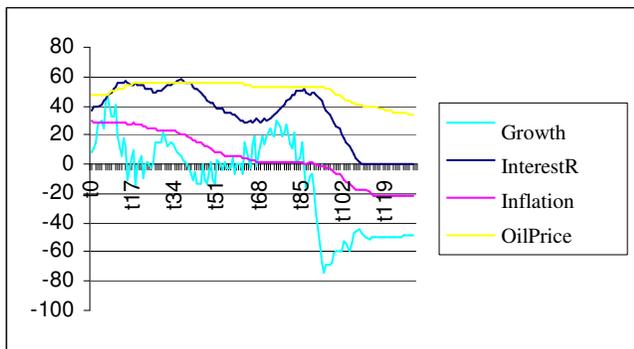


Figure 1 – Serious economic crisis: Negative growth and deflation

### 3.4. Modelling European Central Bank Influence

To simulate ECB influence, a Fuzzy Inference Subsystem (FISS) – a RB-FCM block used to model the process of decision making of system entities (FISS timing mechanisms are independent of the RB-FCM) – was added to the model (Figure 2). This FISS ended up as a simple FRB with 48 rules (each with 2 antecedents) [11]. These rules were designed to inhibit the greedy bank behaviour that was identified as the cause to the unavoidable crisis.

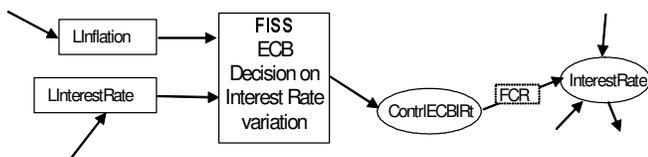


Figure 2 – FISS: ECB decision on Interest Rate variation

### 3.5. Complete Model Simulation

With the introduction of the ECB-FISS, system behaviour changed completely and serious crisis were avoided under normal circumstances (Figure 3). It is still possible to foul the system when simulating a long war (simulating external forces that cause social instability) (Figure 4), or when one imposes an Oil crisis (although in this cases the system usually recovers when the Oil price comes down by itself). One of the most interesting results was the fact that, under normal circumstances, the economic model stabilizes around the real-world BCE predicted ideal target value for inflation (slightly below 2%) and growth (averaging slightly above

2%). However, these values are not imposed anywhere in the model! They result from the system itself. Under certain external circumstances the system becomes chaotic, although stabilizing around a chaotic attractor (Figure 5) [5][6][13]. Another interesting result is that the evolution of the system “appears” to be “realistic”, in a sense that it has the chaotic look that is so characteristic of economic systems evolution, and that one expects in systems with such complexity. Quantitative models produce much cleaner “waves” that do not look natural (which does not means that they cannot approach a general trend). Detailed simulation results and analysis can be found in [11] or at <http://digitais.ist.utl.pt/uke/RBFCM.html>.

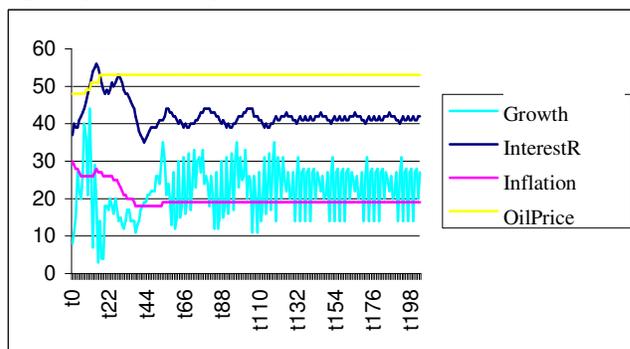


Figure 3 – Avoiding economic crisis trough ECB Interest Rate control

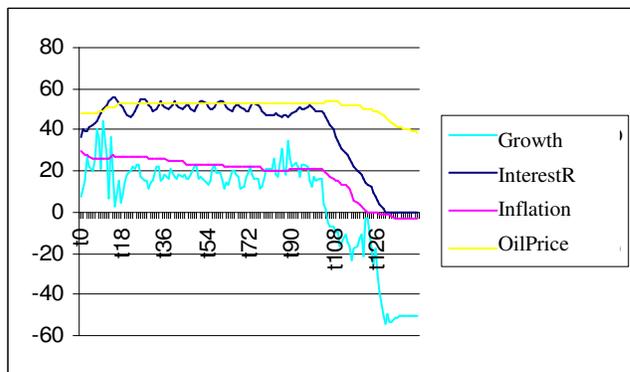
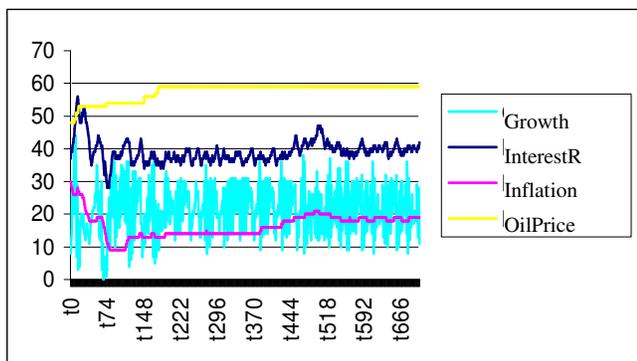


Figure 4 – Effect of War and Oil crisis on a stable economy

## 4. CONCLUSIONS, APPLICATIONS, AND FUTURE DEVELOPMENTS

In this paper we exemplified how one can maintain the qualitative nature of a qualitative system by avoiding quantitative modelling approach methods. As we have seen, true qualitative modelling techniques allow us to obtain results that look more realistic (plausible) than those obtained using quantitative approaches – where results almost never show the short term uncertainties that are so characteristic of qualitative real-world dynamic systems. In

the end, the results of the shown model are surprisingly realistic and accurate if we consider its necessary incompleteness. Even if the dream of obtaining a complete and realistic model of macro-economy is rather utopic (since we are not, and do not intend to be economic experts), these “simpler” models can always be rather useful when used in education, in management/economics training, as the inference engine of economic games, or as pseudo-AI engines in simulation and strategic recreational electronic games.

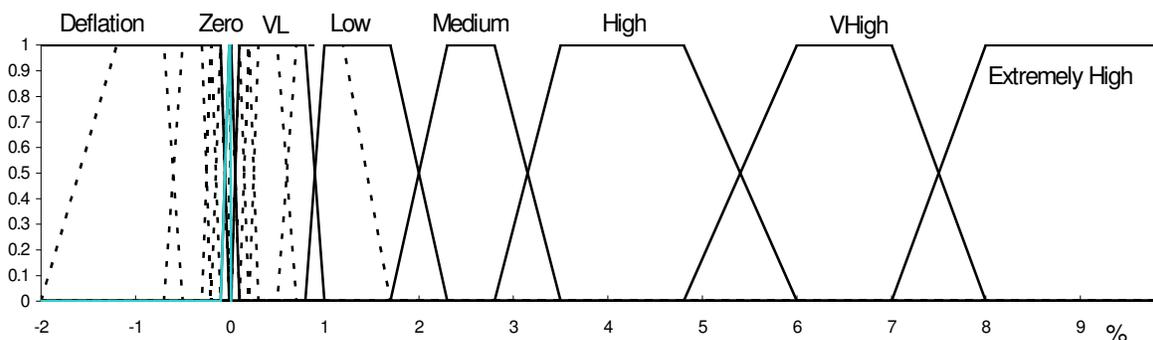


**Figure 5 – Pseudo-stability: chaotic attractor**

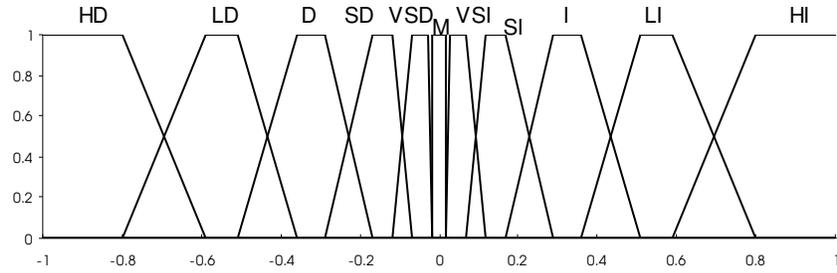
### 5. REFERENCES

[1] Axelrod, R., “The Structure of Decision: Cognitive Maps of Political Elites”, Princeton University Press, 1976  
 [2] Forrester, J.W., several papers available online at <http://sysdyn.mit.edu/sd-intro/home.html>  
 [3] Zadeh, L., “Fuzzy Sets and Applications: Selected Papers, Wiley-Interscience”, 1987  
 [4] Kosko, B., “Fuzzy Cognitive Maps”, International Journal of Man-Machine Studies, 1986  
 [5] Kosko, B., “Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence”, Prentice-Hall International Editions, 1992

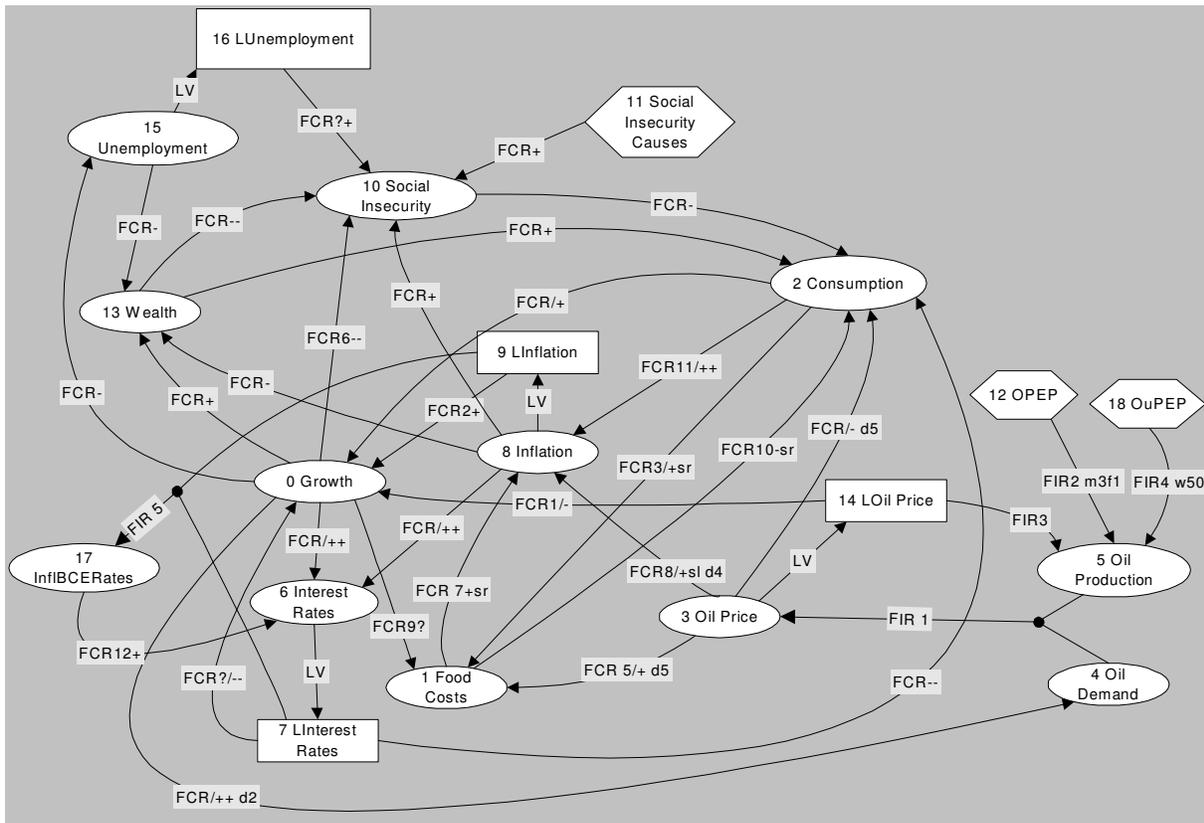
[6] Kosko, B., “Fuzzy Engineering”, Prentice-Hall International Editions, 1997  
 [7] Carvalho, J.P., Tomé, J.A., “Rule Based Fuzzy Cognitive Maps and Fuzzy Cognitive Maps - A Comparative Study”, Proceedings of the 18th International Conference of the North American Fuzzy Information Processing Society, NAFIPS99, New York  
 [8] Carvalho, J.P., Tomé, J.A., “Rule Based Fuzzy Cognitive Maps - Fuzzy Causal Relations”, Computational Intelligence for Modelling, Control and Automation, Edited by M. Mohammadian, 1999  
 [9] Carvalho, J.P., Tomé, J.A., “Fuzzy Mechanisms For Causal Relations”, Proceedings of the Eighth International Fuzzy Systems Association World Congress, IFSA'99, Taiwan  
 [10] Carvalho, J.P., Tomé, J.A., “Rule Based Fuzzy Cognitive Maps – Qualitative Systems Dynamics”, Proceedings of the 19th International Conference of the North American Fuzzy Information Processing Society, NAFIPS2000, Atlanta  
 [11] Carvalho, J.P., “Mapas Cognitivos Baseados em Regras Difusas: Modelação e Simulação da Dinâmica de Sistemas Qualitativos”, PhD thesis, Instituto Superior Técnico, Universidade Técnica de Lisboa, Portugal 2002  
 [12] Carvalho, J.P., Tomé, J.A., “Rule Based Fuzzy Cognitive Maps - Expressing Time in Qualitative System Dynamics”, Proceedings of the 2001 FUZZ-IEEE, Melbourne, Australia 2001  
 [13] Carvalho, J.P., Tomé, J.A., “Issues on the Stability of Fuzzy Cognitive Maps and Rule-Based Fuzzy Cognitive Maps”, Proceedings of the 21st International Conference of the North American Fuzzy Information Processing Society, NAFIPS2002, New Orleans  
 [14] Carvalho, J.P., Tomé, J.A., “Using Interpolated Linguistic Term to Express Uncertainty in Rule Based Fuzzy Cognitive Maps”, Proceedings of the 22nd International Conference of the North American Fuzzy Information Processing Society, NAFIPS2003, Chicago  
 [15] Neves, A., “Taxas de Juro podem descer já este mês”, Público 2001/8/10



**Figure 6 – Level “Inflation” linguistic terms. Dotted linguistic terms represent the variation degrees of Inflation {Huge\_Decrease, Large\_Decrease, Decrease, Small\_Decrease, VS\_Decrease, Maintain, VS\_Increase, ..., Huge\_Increase}**



**Figure 7 – Variation “Inflation” linguistic terms: {Huge Decrease, Large Decrease,...,Huge Increase}. x-scale values are normalized values. There is no relation no real world values**



**Figure 8 – RB-FCM: A qualitative model of economy. Concept 17, representing a simple FISS was added later**